1. Use the CRC to find the mass of these isotopes

We note that $^8Be^4$, $^{12}C^6$, $^{56}Fe^{26}$ and $^{208}Pb^{82}$ are all even–even nuclei. Therefore, we consider only the lower sign in the last term in Eqs. (3.11) and (3.13) in evaluating the B.E. and the B.E. per nucleon. Putting in the numbers, we obtain the table:

Table 3.1.

A	Z	B.E.$_{\text{calculated}}$ (MeV)	B.E.$_{\text{tables}}$ (MeV)	$\frac{B}{A} = -\frac{\text{B.E.}}{A}$ (MeV)
8	4	−58.99	−56.50	7.37
12	6	−93.09	−92.20	7.76
56	26	−495.48	−492.30	8.85
208	82	−1621.68	−1635.80	7.80

2.

As we have already seen in Problem 2.2,

$$\text{B.E. of } ^4He^2 = -28.29\,\text{MeV}. \qquad (3.14)$$

Therefore, we see from Table 3.1 that the difference in the B.E. of $^8Be^4$ and that of two $^4He^2$ nuclei is given by

$$\begin{aligned}\Delta &= -56.50\,\text{MeV} - 2 \times (-28.29)\,\text{MeV} \\ &= (-56.50 + 56.58)\,\text{MeV} = 0.08\,\text{MeV}.\end{aligned} \qquad (3.15)$$

As a result, we see that $^8Be^4$ can decay into two $^4He^2$ nuclei, thereby releasing 80 keV of energy:

$$^8Be^4 \rightarrow {}^4He^2 + {}^4He^2 + 80\,\text{keV}. \qquad (3.16)$$

3.

3.4 From the *CRC Handbook*, we know that

$$
\begin{aligned}
M(^1\mathrm{H}^1) &= 1.0078\,\mathrm{amu}, & m_n &= 1.0087\,\mathrm{amu},\\
M(^{14}\mathrm{N}^7) &= 14.0031\,\mathrm{amu}, & M(^{15}\mathrm{N}^7) &= 15.0001\,\mathrm{amu},\\
M(^{16}\mathrm{N}^7) &= 16.0061\,\mathrm{amu}, & M(^{15}\mathrm{O}^8) &= 15.0030\,\mathrm{amu},\\
M(^{16}\mathrm{O}^8) &= 15.9949\,\mathrm{amu}.
\end{aligned} \tag{3.17}
$$

Using these values and the conversion between "amu" and "MeV" units, we can calculate the binding energy of the last neutron in $^{15}\mathrm{N}^7$

$$
\begin{aligned}
\mathrm{B.E.} &= -(M(^{14}\mathrm{N}^7) + m_n - M(^{15}\mathrm{N}^7))c^2\\
&= -(14.0031 + 1.0087 - 15.0001)\,\mathrm{amu} \times c^2\\
&\approx -0.0117 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 = -10.8985\,\mathrm{MeV}.
\end{aligned} \tag{3.18}
$$

Similarly, the binding energy of the last proton in $^{15}\mathrm{O}^8$ is

$$
\begin{aligned}
\mathrm{B.E.} &= -(M(^{14}\mathrm{N}^7) + M(^1\mathrm{H}^1) - M(^{15}\mathrm{O}^8))c^2\\
&= -(14.0031 + 1.0078 - 15.0030)\,\mathrm{amu} \times c^2\\
&\approx -0.0079 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 = -7.3588\,\mathrm{MeV}.
\end{aligned} \tag{3.19}
$$

Furthermore, the binding energy for the last neutron in $^{16}\mathrm{N}^7$ is given by

$$
\begin{aligned}
\mathrm{B.E.} &= -(M(^{15}\mathrm{N}^7) + m_n - M(^{16}\mathrm{N}^7))c^2\\
&= -(15.0001 + 1.0087 - 16.0061)\,\mathrm{amu} \times c^2\\
&\approx -0.0027 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 \approx -2.5150\,\mathrm{MeV}.
\end{aligned} \tag{3.20}
$$

Finally, the binding energy of the last neutron in $^{16}\mathrm{O}^8$ is

$$
\begin{aligned}
\mathrm{B.E.} &= -(M(^{15}\mathrm{O}^8) + m_n - M(^{16}\mathrm{O}^8))c^2\\
&= -(15.0030 + 1.0087 - 15.9949)\,\mathrm{amu} \times c^2\\
&\approx -0.0168 \times 931.5\,\mathrm{MeV}/c^2 \times c^2 = -15.6492\,\mathrm{MeV}.
\end{aligned} \tag{3.21}
$$

4.

$$Q = T_D + T_\alpha = (M(A,Z) - M(A-4, Z-2) - M(4,2))c^2$$

for a) $^{208}Po^{84} \rightarrow {}^{204}Pb^{82} + \alpha$

$$Q = (207.9812 - 203.9730 - 4.0026)u \times 931.5 \frac{MeV}{u}$$

$$= \boxed{5.216 \text{ MeV}}$$

$$T_\alpha = \frac{A-4}{A} Q = \frac{204}{208} \times 5.216 = \boxed{5.11 \text{ MeV}}$$

$$T_D = Q - T_\alpha = 5.216 - 5.11 = \boxed{0.11 \text{ MeV}}$$

b)

$$Q = (M(^{243}Am^{95}) - M(^{239}Np^{93}) - M(^4He^2))c^2$$

$$= (243.0614 - 239.0529 - 4.0026) \text{ amu} \times c^2$$

$$= .0059 \times 931.5 \frac{MeV}{c^2} \times c^2$$

$$\boxed{Q = 5.496 \text{ MeV}}$$

$$T_\alpha = \frac{M(^{239}Np^{93})}{M(^{239}Np^{93}) + M(^4He^2)} \cdot Q$$

$$= \frac{239.0529}{239.0529 + 4.0026} \times 5.496$$

$$\boxed{T_\alpha = 5.405 \text{ MeV}}$$

$$T_D = Q - T_\alpha$$

$$= (5.496 - 5.405) = 0.091$$

$$\boxed{T_D = 0.091 \text{ MeV}}$$

c) $M(^{241}Am^{95}) = 241.0568 \text{ amu} = 224.5 \text{ GeV}/c^2$

$M(^{243}Am^{95}) = 243.0614 \text{ amu} = 226.4 \text{ GeV}/c^2$

d) see ABOVE

5.

We know that $^{23}\text{Na}^{11}$ is stable. The isotope $^{22}\text{Na}^{11}$ has one less neutron, while $^{24}\text{Na}^{11}$ has one extra neutron relative to $^{23}\text{Na}^{11}$. Consequently, a proton in $^{22}\text{Na}^{11}$ can undergo an inverse β decay to yield

$$^{22}\text{Na}^{11} \rightarrow {}^{22}\text{Ne}^{10} + e^+ + \nu_e, \tag{4.37}$$

where $^{22}\text{Ne}^{10}$ is a naturally occurring stable isotope of $^{20}\text{Ne}^{10}$. Similarly, the extra neutron in $^{24}\text{Na}^{11}$ can undergo a β decay to yield

$$^{24}\text{Na}^{11} \rightarrow {}^{24}\text{Mg}^{12} + e^- + \bar{\nu}_e, \tag{4.38}$$

where $^{24}\text{Mg}^{12}$ is stable.

6.

An α particle with 10 MeV of kinetic energy is clearly nonrelativistic. Outside the nuclear potential well, all its energy is kinetic and we can therefore identify

$$T_\alpha^{(\text{outside})} = E = 10\,\text{MeV}. \tag{4.41}$$

On the other hand, inside the nuclear well, the α particle feels the potential of the well and we have

$$E = T_\alpha^{(\text{inside})} + V(r) = T_\alpha^{(\text{inside})} - U_0, \tag{4.42}$$

where the potential depth of the nuclear well (if we assume a square-well potential) is about $U_0 = 40\,\text{MeV}$ (as discussed in Sec. 4.3 of the text). Therefore, using Eq. (4.41) and conservation of energy, we obtain

$$\begin{aligned} E = T_\alpha^{(\text{inside})} - U_0 &= 10\,\text{MeV} \\ T_\alpha^{(\text{inside})} = 10\,\text{MeV} + U_0 &= (10 + 40)\,\text{MeV} = 50\,\text{MeV}. \end{aligned} \tag{4.43}$$

In other words, the α particle has more kinetic energy inside the nucleus than outside.

Since the α particle is nonrelativistic both inside and outside the potential well, we can calculate its momentum as

$$\begin{aligned} p^{(\text{outside})} &= \sqrt{2m_\alpha T_\alpha^{(\text{outside})}} = \sqrt{2m_\alpha c^2 E} \times \frac{1}{c} \\ &\approx \sqrt{2 \times 3728\,\text{MeV} \times 10\,\text{MeV}} \times \frac{1}{c} \\ &\approx 273\,\text{MeV}/c, \\ p^{(\text{inside})} &= \sqrt{2m_\alpha T_\alpha^{(\text{inside})}} = \sqrt{2m_\alpha c^2 T_\alpha^{(\text{inside})}} \times \frac{1}{c} \\ &\approx \sqrt{2 \times 3728\,\text{MeV} \times 50\,\text{MeV}} \times \frac{1}{c} \\ &\approx 610\,\text{MeV}/c, \end{aligned} \tag{4.44}$$

where we have used the value of m_α given in Eq. (4.9).

The corresponding de Broglie wavelength of the α particle inside the nuclear well is

$$\begin{aligned} \lambda^{(\text{inside})} &= \frac{h}{p^{(\text{inside})}} = \frac{2\pi\hbar c}{p^{(\text{inside})}c} \\ &\approx \frac{6 \times 197\,\text{MeV} - \text{F}}{610\,\text{MeV}} \approx 1.9\,\text{F} \\ &= 1.9 \times 10^{-13}\,\text{cm}, \end{aligned} \tag{4.45}$$

where we have used the value of the momentum from Eq. (4.44).

From Eq. (2.16) of the text, we can obtain

$$\begin{aligned} R_{^{12}\text{C}} &= 1.2 \times 10^{-13} A^{\frac{1}{3}}\,\text{cm} = 1.2 \times (12)^{\frac{1}{3}} \times 10^{-13}\,\text{cm} \\ &\approx 2.76 \times 10^{-13}\,\text{cm} > \lambda^{(\text{inside})}, \\ R_{^{238}\text{U}} &= 1.2 \times (238)^{\frac{1}{3}} \times 10^{-13}\,\text{cm} \\ &\approx 7.44 \times 10^{-13}\,\text{cm} > \lambda^{(\text{inside})}. \end{aligned} \tag{4.46}$$

We therefore conclude that the α particle can be contained inside either of these nuclei.