PHYS 3446 – Lecture #10

Thursday, Feb 26, 2015

*Dr. **Brandt***

- Early Nuclear models

Nuclear Models: Liquid Droplet Model

- This was the earliest phenomenological model and had success in describing the binding energy of a nucleus
- Nucleus is essentially spherical with radius proportional to $A^{1/3}$.
  - Densities are independent of the number of nucleons
- Led to a model that envisions the nucleus as an incompressible liquid droplet
  - In this model, nucleons are equivalent to molecules
- Quantum properties of individual nucleons are ignored

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- An early attempt to incorporate quantum effects
- Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume

Nuclear Models: Fermi Gas Model

- An early attempt to incorporate quantum effects
- Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume
  - The nucleons occupy quantized (discrete) energy levels
  - Nucleons are moving inside a spherically symmetric well with the range determined by the radius of the nucleus
  - Depth of the well is adjusted to obtain correct binding energy

Nuclear Models: Fermi Gas Model

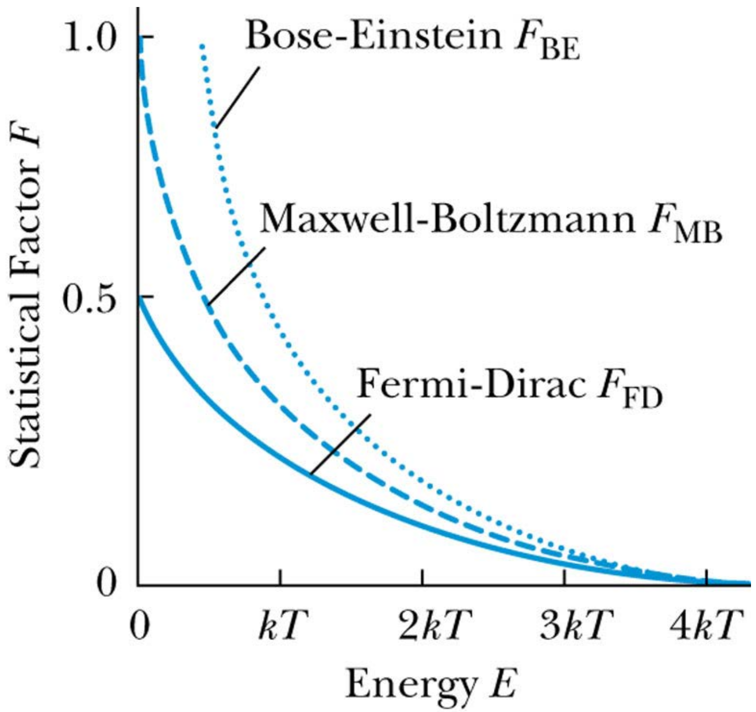
- Nucleons are Fermions (spin ½ particles) so
  - Obey Pauli exclusion principle
  - Any given energy level can be occupied by at most two identical nucleons – opposite spin projections
- For greater stability, the energy levels fill up from the bottom to the Fermi level
  - Fermi level: Highest, fully occupied energy level (E_F)
- Binding energies are given as follows:
  - BE of the last nucleon= E_F since no fermions above E_F

Classical and Quantum Distributions

Table 9.2 Classical and Quantum Distributions

Distributors	Properties of the Distribution	Examples	Distribution Function
Maxwell-Boltzmann	Particles are identical but distinguishable	Ideal gases	$F_{MB} = A\exp(-\beta E)$
Bose-Einstein	Particles are identical and indistinguishable with integer spin	Liquid ^{4}He, photons	$F_{BE} = \dfrac{1}{B_2\exp(\beta E) - 1}$
Fermi-Dirac	Particles are identical and indistinguishable with half-integer spin	Electron gas	$F_{FD} = \dfrac{1}{B_1\exp(\beta E) + 1}$

Quantum Distributions



- The normalization constants for the distributions depend on the physical system being considered.
- Because bosons do not obey the Pauli exclusion principle, more bosons can fill lower energy states.
- Three graphs converge at high energies – the classical limit.

→ Maxwell-Boltzmann statistics may be used in the classical limit.

Fermi-Dirac Statistics

$$F_{\mathrm{FD}} = \frac{1}{\exp(\beta(E - E_{\mathrm{F}})) + 1}$$

- E_F is called the **Fermi energy**.
- When $E = E_F$, the exponential term is 1.

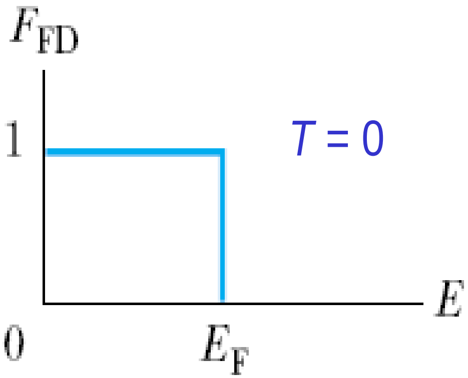
$\longrightarrow$ $F_{FD} = 1/2$

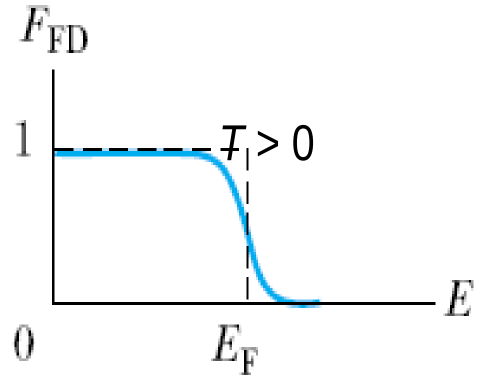
- In the limit as T $\rightarrow$ 0,

$$F_{\mathrm{FD}} = \begin{cases} 1 & \text{for } E < E_{\mathrm{F}} \\ 0 & \text{for } E > E_{\mathrm{F}} \end{cases}$$

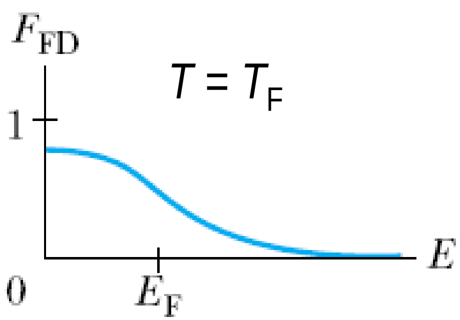
- At $T = 0$, fermions occupy the lowest energy levels.
- Near $T = 0$, there is little chance that thermal agitation will kick a fermion to an energy greater than E_F.

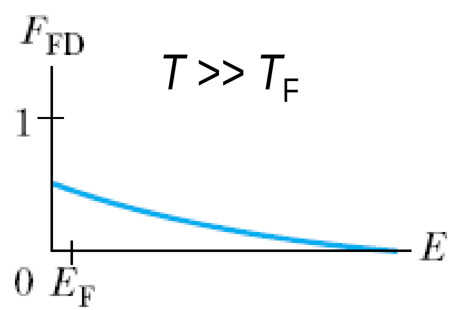
Fermi-Dirac Statistics





- As the temperature increases from $T = 0$, the Fermi-Dirac factor “smears out”.
- **Fermi temperature**, defined as $T_F \equiv E_F / k$.   .

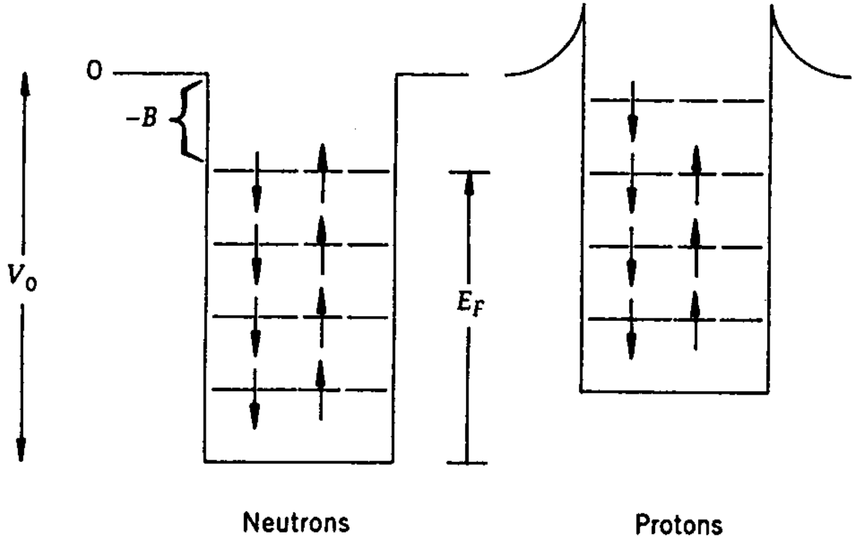




- When $T >> T_F$, F_{FD} approaches a decaying exponential.

Nuclear Models: Fermi Gas Model

- Experimental observations show BE is charge independent
- If the well depth is the same for p and n, BE for the last nucleon would be charge dependent for heavy nuclei (Why?)
  - Since there are more neutrons than protons, neutrons would have higher E_F
- E_F must be the same for protons and neutrons. How do we make this happen?
  - Make protons move to a shallower potential well
- What happens if this weren't the case?
  - Nucleus would be unstable
  - All neutrons at higher energy levels would undergo a β-decay and transition to lower proton levels



Atomic Shell Model Review

- Orbits and energy levels an electron can occupy are labeled by
  - Principle quantum number: n
    - n can only be integer
  - For given n, energy degenerate orbital angular momentum: ℓ
    - The values are given from 0 to $n-1$ for each n
  - For any given orbital angular momentum, there are $(2\ell+1)$ sub-states: m_ℓ
    - $m_\ell = -\ell, -\ell+1, \ldots, 0, 1, \ldots, \ell-\ell, \ell$
    - Due to rotational symmetry of the Coulomb potential, all these sub-states are degenerate in energy
  - Since electrons are fermions w/ intrinsic spin angular momentum $\hbar/2$,
    - Each of the sub-states can be occupied by two electrons
  - So the total number of state is $2(2\ell+1)$

Nuclear Models: Shell Model

- Nuclei are observed to have magic numbers just like inert atoms
  - Atoms: Z=2, 10, 18, 36, 54
  - Nuclei: N=2, 8, 20, 28, 50, 82, and 126 and Z=2, 8, 20, 28, 50, and 82
  - Magic Nuclei: Nuclei with either N or Z a magic number → Stable
  - Doubly magic nuclei: Nuclei with both N and Z magic numbers → Particularly stable
- Could explain the stability of nucleus—but can we obtain these magic numbers with a simple model?