PHYS 3446 – Lecture #11

Tuesday, February 31 2015
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- Nuclear Models
- Alpha Decay
- Beta Decay

Nuclear Radiation: Alpha Decay

- Represents the disintegration of a parent nucleus to a daughter through emission of a He nucleus
- Reaction equation is

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}He^{2}$$

- α-decay is a spontaneous fission of the parent nucleus into two daughters typically of asymmetric mass
- Assuming parent at rest, from energy conservation

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha$$

- Can be re-organized as

$$T_D + T_\alpha = \left(M_P - M_D - M_\alpha\right)c^2 = \Delta M c^2$$

Nuclear Radiation: Alpha Decay

- Since electron masses cancel, we could use atomic mass expression

$$T_D + T_\alpha = \left(M(A,Z) - M(A-4, Z-2) - M(4,2)\right)c^2 \equiv Q$$

- This is the definition of the disintegration energy or Q-value
  - Difference of rest masses of the initial and final states
  - Q value is equal to the sum of the final state kinetic energies
- For non-relativistic particles, KE are

$$T_D = \frac{1}{2} M_D v_D^2 \qquad T_\alpha = \frac{1}{2} M_\alpha v_\alpha^2$$

Nuclear Radiation: Alpha Decay

- Since the parent is at rest, from momentum conservation

$$M_D v_D = M_\alpha v_\alpha \Rightarrow v_D = \frac{M_\alpha}{M_D} v_\alpha$$

- If $M_D > M_\alpha$; $v_D < v_\alpha$, then $T_D < T_\alpha$
- In general, the relationship between KE and Q-value is

$$T_D + T_\alpha = \frac{1}{2} M_D v_D^2 + \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_D \left(\frac{M_\alpha}{M_D} v_\alpha \right)^2 + \frac{1}{2} M_\alpha v_\alpha^2$$

$$T_D + T_\alpha = Q = T_\alpha \frac{M_\alpha + M_D}{M_D}$$

$$T_\alpha = \frac{M_D}{M_\alpha + M_D} Q$$

Eq. 4.8

- This means that T_α is unique for a given nuclei
- Direct consequence of 2-body decay of a parent at rest

Nuclear Radiation: Alpha Decay

- KE of the emitted α must be positive
- Thus for an α-decay to occur, it must be an exothermic process $\Delta M \geq 0, \quad Q \geq 0$
- For massive nuclei, the daughter's KE is

$$T_D = Q - T_\alpha = \frac{M_\alpha}{M_\alpha + M_D} Q = \frac{M_\alpha}{M_D} T_\alpha < T_\alpha$$

- Since $M_\alpha / M_D \approx 4/(A-4)$, we obtain

$$T_\alpha \approx \frac{A-4}{A} Q \qquad T_D \approx \frac{4}{A} Q$$

where

$$T_\alpha = \frac{M_D}{M_\alpha + M_D} Q$$

Nuclear Radiation: Alpha Decay

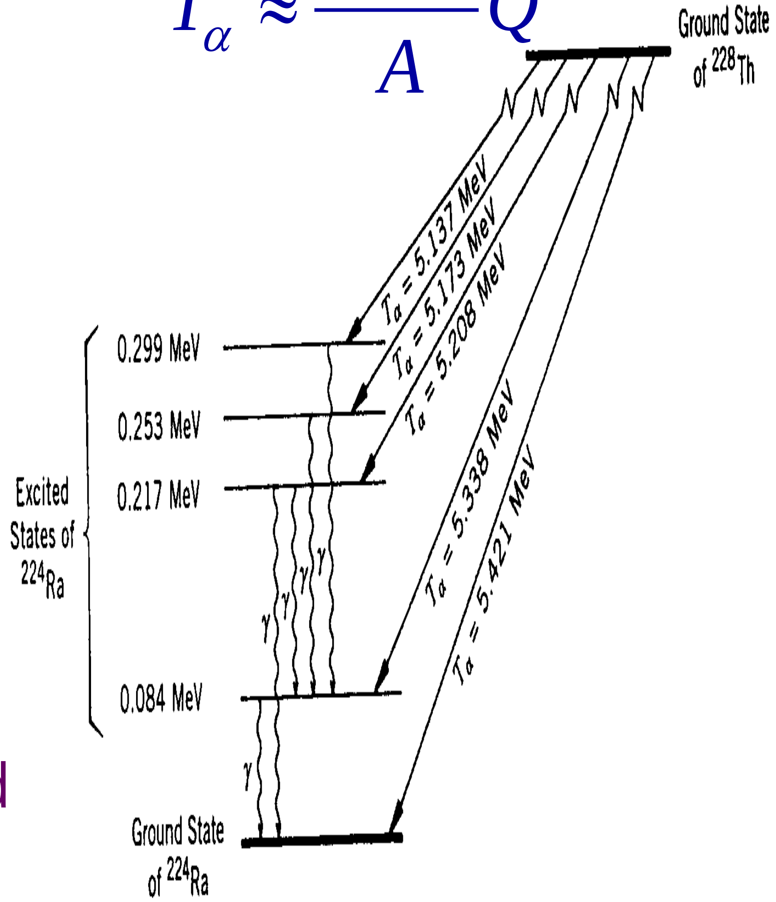
- Most energetic α-particles produced alone
  - Parent nucleus decays to the ground state of a daughter and produces an α-particle whose KE is $T_\alpha \approx \frac{A-4}{A}Q$
- Less energetic ones accompanied by delayed photons
  - Indicates quantum energy levels
  - Parent decays to an excited state of the daughter after emitting an α

$$ {}^{A}X^{Z} \rightarrow {}^{A-4}Y^{*Z-2} + {}^{4}He^{2} $$

  - Daughter then subsequently de-excite by emitting a photon

$$ {}^{A-4}Y^{*Z-2} \rightarrow {}^{A-4}Y^{Z-2} + \gamma $$

  - Difference in the two Q values correspond to photon energy



Nuclear Radiation: α-Decay Example

- ^{240}Pu94 decay reaction is

$$^{240}Pu^{94} \rightarrow {}^{236}U^{92} + {}^{4}He^{2}$$

- α particles observed with 5.17MeV and 5.12 MeV
- Since $Q = \frac{A}{A-4} T_{\alpha}$
- We obtain the two Q-values

$$Q_1 \approx \frac{240}{236} 5.17 MeV = 5.26 MeV \qquad Q_2 \approx \frac{240}{236} 5.12 MeV = 5.21 MeV$$

- Which yields a photon energy of $E_{\gamma} = \Delta Q = Q_1 - Q_2 = 0.05 MeV$
- Consistent with experimental measurement, 45KeV
- Indicates the energy level spacing of order 100KeV for nuclei
  – Compares to order 1eV spacing in atomic levels

Nuclear Radiation: β-Decays

- Three kinds of β-decays
  - Electron emission
    - Nucleus with large N_n
    - Atomic number increases by one
    - Nucleon number stays the same

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$$

  - Positron emission (inverse β-decay)
    - Nucleus with many protons
    - Atomic number decreases by one
    - Nucleon number stays the same

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+}$$

  - Electron capture

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1}$$

    - Similar to positron emission
    - A K-shell electron is absorbed converting a proton into a neutron
    - Causes cascade X-ray emission from the transition of remaining atomic electrons

Nuclear Radiation: β-Decays

- You can treat nucleus reaction equations algebraically
  - The reaction is valid in the opposite direction as well
  - Any particle moved “across the arrow” becomes its anti-particle

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1}$$

- For β-decay: ΔA=0 and |ΔZ|=1

Nuclear Radiation: β-Decays

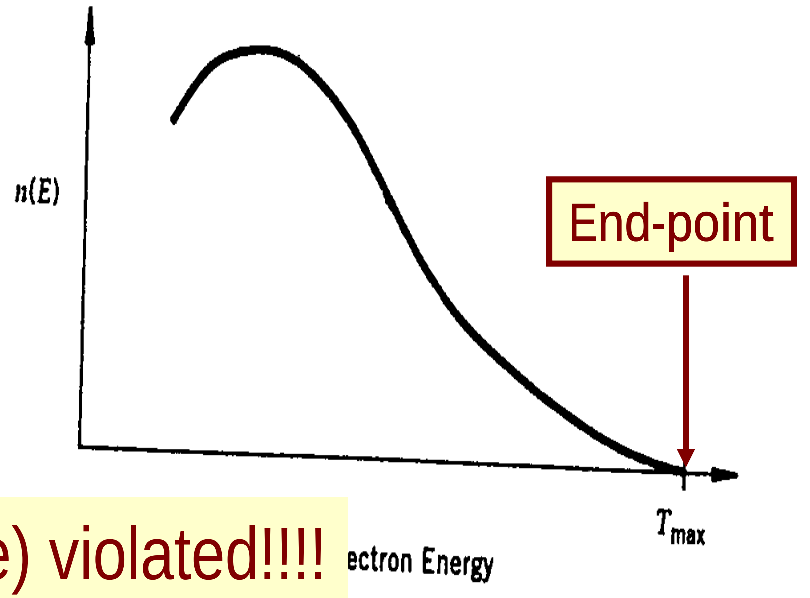
- Initially assumed to be 2-body decay $^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$
- From energy conservation

$$E_X = E_Y + E_{e^-} = E_Y + T_e + m_e c^2$$

- Since lighter electron carries most of the KE

$$T_e = \left(E_X - E_Y - m_e c^2\right) = \left(m_X - m_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Results in a unique Q value as in α-decay.
- In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above



Energy conservation is (seems to be) violated!!!!