# PHYS 3446 – Lecture #19

*Thursday April 23, 2015*
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- Accelerators
- Particle Physics
- Project
- .

# Synchroton Accelerators

- For very energetic particles, relativistic effects must be taken into account
- For relativistic energies, the equation of motion of a charge q under magnetic field B is*

$$m\gamma\frac{d\vec{v}}{dt} = m\gamma\vec{v}\times\vec{\varpi} = q\frac{\vec{v}\times\vec{B}}{c} \Rightarrow \omega = \frac{qB}{\gamma mc}$$

- For v ~ c, the resonance frequency ($\nu$ or f) becomes

$$f = \nu = \frac{\varpi}{2\pi} = \frac{1}{2\pi}\left(\frac{q}{m}\right)\frac{1}{\gamma}\frac{B}{c}$$

- Thus for high energies, either B or $\nu$ should increase
- Machines with constant B but variable $\nu$ are called synchro-cyclotrons
- Machines with variable B independent of the change of $\nu$ are called synchrotrons

# Synchroton Accelerators

- Electron synchrotrons, B varies while $\nu$ is held constant
- Proton synchrotrons, both B and $\nu$ vary
- For v ~ c, the frequency of motion can be expressed

$$f = \frac{1}{2\pi}\frac{v}{R} \approx \frac{c}{2\pi R} = \frac{1}{2\pi}\left(\frac{q}{m}\right)\frac{1}{\gamma}\frac{B}{c}$$

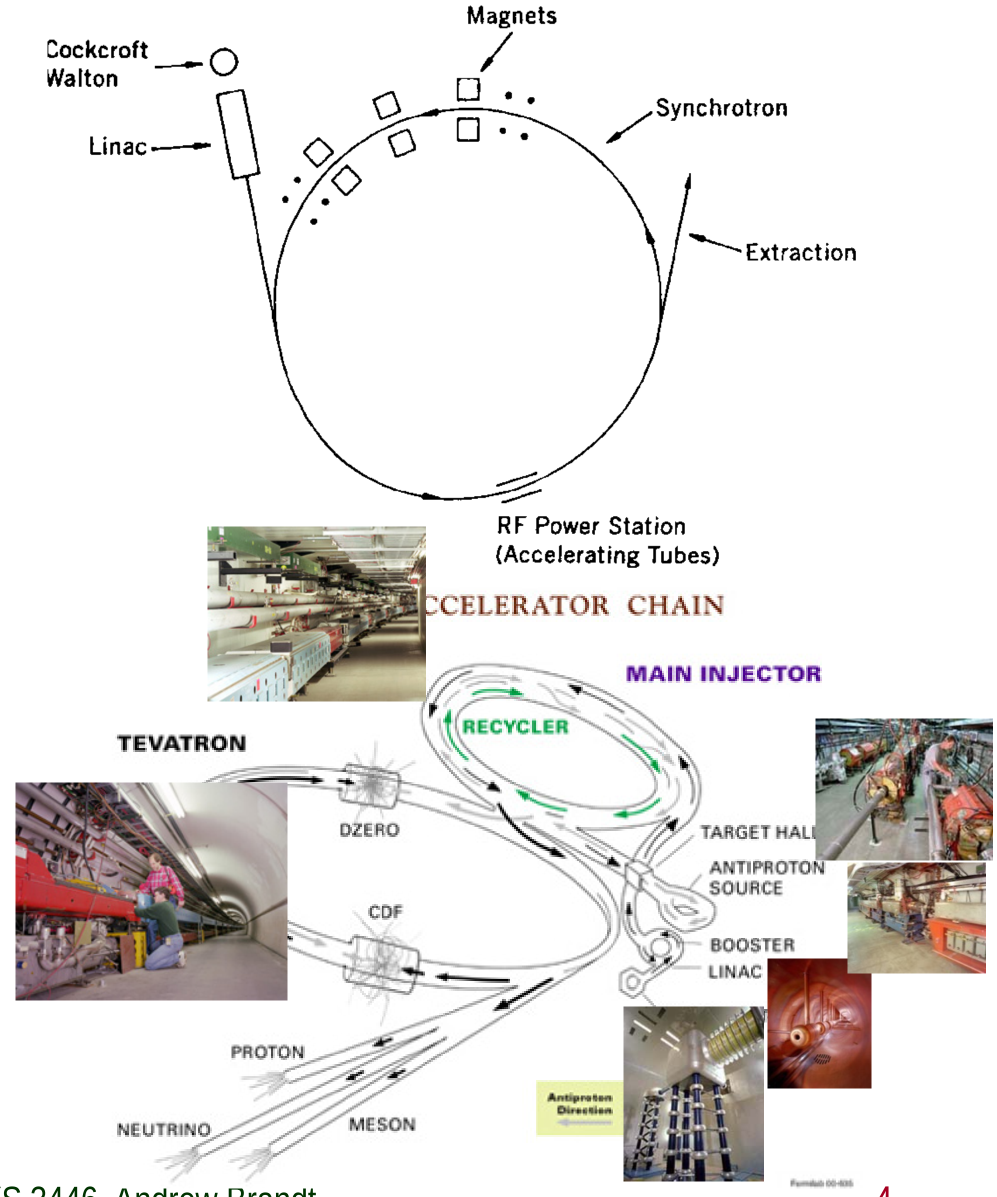
- with p=$\gamma$mc and q=e

$$R(m) = \frac{pc}{qB} \approx \frac{p(GeV/c)}{0.3B(Tesla)}$$

- For magnetic field strength of 2 Tesla, one needs a radius of 50 m to accelerate an electron to 30 GeV/c.

# Synchroton Accelerators

- Synchrotons use magnets arranged in a ring-like fashion.
- Multiple stages of accelerations are needed before reaching the GeV scale
- RF power stations are located through the ring to pump electromagnetic energy into the particles



# Particle Physics

- What are elementary particles?
  - Particles that make up matter in the universe
  - Cannot be broken into smaller pieces
  - Cannot have extended size
- The notion of "elementary particles" has changed from 1930's through present
  - In the past, people thought protons, neutrons, pions, kaons, $\rho$-mesons, etc. were elementary particles
- What changed?
  - The increasing energies of accelerators allow the probing of smaller distance scales, revealing sub-structure
- What is the energy needed to probe 0.1 fm?
  - From de Broglie Wavelength, we obtain

$$P = \frac{\hbar}{\lambda} = \frac{\hbar c}{\lambda c} = \frac{197\text{fm} - MeV}{0.1\text{fm}\ c} \approx 2000 MeV/c$$

# Forces and Their Relative Strengths

- Classical forces:
  - Gravitational: every particle is subject to this force, including massless ones
  - Electromagnetic: only those with electrical charges
  - What are the ranges of these forces?
    - Infinite!!
  - What does this tell you?
    - Their force carriers are massless!!
  - What are the force carriers of these forces?
    - Gravity: graviton (not seen…yet)
    - Electromagnetism: Photons

# Forces and Their Relative Strengths

- What other forces?
  - Strong force
    - Holds nucleus together
  - Weak force
    - Responsible for nuclear beta decay
  - What are their ranges?
    - Very short
  - What does this imply?
    - Their force carriers are massive (especially true for weak force)
- All four forces can act at the same time!!!

# Relative Strengths of Forces

- The strengths can be obtained from the potential
- Considering two protons separated by a distance r:

  Magnitude of Coulomb and gravitational potential are

$$V_{EM}\left(r\right)=\frac{e^2}{r} \quad \xrightarrow{\text{Fourier x-form}} \quad V_{EM}\left(r\right)=\frac{e^2}{q^2}$$

$$V_g\left(r\right)=\frac{G_N m^2}{r} \quad \xrightarrow{\text{Fourier x-form}} \quad V_g\left(r\right)=\frac{G_N m^2}{q^2}$$

  – q: magnitude of the momentum transfer
- What do you observe?
  – The absolute values of the potential  decreases quadrati with increasing momentum transfer
  – The relative strength is independent of momentum transfer

$$\frac{V_{EM}}{V_g}=\frac{e^2}{G_N m^2}=\left(\frac{e^2}{\hbar c}\right)\frac{1}{\left(mc^2\right)^2}\frac{\hbar c^5}{G_N}=\left(\frac{1}{137}\right)\frac{1}{1GeV^2}\frac{10^{39}GeV^2}{6.7}\sim 10^{36}$$

# Relative Strengths

- Using Yukawa potential form, the magnitudes of strong and weak potential can be written as

$$V_S(r) = \frac{g_S^2}{r} e^{-\frac{m_\pi c^2 r}{\hbar c}} \xrightarrow{\text{Fourier x-form}} V_S(r) = \frac{g_S^2}{q^2 + m_\pi^2 c^2}$$

$$V_W(r) = \frac{g_W^2}{r} e^{-\frac{m_W c^2 r}{\hbar c}} \xrightarrow{\text{Fourier x-form}} V_W(r) = \frac{g_W^2}{q^2 + m_W^2 c^2}$$

  - $g_W$ and $g_s$: coupling constants or effective charges
  - $m_W$ and $m_\pi$: masses of force mediators
- The values of the coupling constants can be estimated from experiments

$$\frac{g_S^2}{\hbar c} \approx 15 \qquad \frac{g_W^2}{\hbar c} \approx 0.004$$

# Relative Strengths

- We could think of $\pi$ as the strong force mediator w/ $m_\pi \approx 140 MeV/c^2$
- From observations of beta decays, $m_W \approx 80 GeV/c^2$
- However there still is an explicit dependence on momentum transfer
  - Since we are considering two protons, we can replace the momentum transfer, q, with the mass of protons

$$q^2c^2 = m_p^2c^4 \approx 1GeV$$

# Relative Strengths

- The relative strength between the Strong and EM potentials is

$$\frac{V_S}{V_{EM}} = \frac{g_S^2}{\hbar c}\frac{\hbar c}{e^2}\frac{q^2}{q^2 + m_\pi^2 c^2} = \frac{g_S^2}{\hbar c}\frac{\hbar c}{e^2}\frac{m_p^2 c^4}{m_p^2 c^4 + m_\pi^2 c^2}$$

$$\approx 15 \times 137 \times 1 \approx 2 \times 10^3$$

- And that between EM and weak potentials is

$$\frac{V_{EM}}{V_W} = \frac{e^2}{\hbar c}\frac{\hbar c}{g_W^2}\frac{q^2 + m_W^2 c^2}{q^2} = \frac{e^2}{\hbar c}\frac{\hbar c}{g_W^2}\frac{m_p^2 c^4 + m_W^2 c^2}{m_p^2 c^4}$$

$$\approx \frac{1}{137} \times \frac{1}{0.004} \times 80^2 \approx 1.2 \times 10^4$$

$$\frac{V_S}{V_W} = 2.4 \times 10^7$$

# Interaction Time

- The ranges of forces also affect interaction time
  - Typical time for Strong interaction ~$10^{-24}$sec
    - What is this?
    - A time that takes light to traverse the size of a proton (~1 fm)
  - Typical time for EM force ~$10^{-20}$ – $10^{-16}$ sec
  - Typical time for Weak force ~$10^{-13}$ – $10^{-6}$ sec
- The forces have different characteristic energy scales, which are used along with their interaction type to classify elementary particles

# Elementary Particles

- Prior to the quark model, all known elementary particles were divided in four groups depending on the nature of their interactions

| *Particle* | *Symbol* | *Range of Mass Values* |
|---|---|---|
| Photon | $\gamma$ | $\lesssim 2 \times 10^{-16}$ eV$/c^2$ |
| Leptons | $e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau$ | $\lesssim 3$ eV$/c^2$ – 1.777 GeV$/c^2$ |
| Mesons | $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0,$ $\rho^+, \rho^-, \rho^0, \ldots$ | 135 MeV$/c^2$ – few GeV$/c^2$ |
| Baryons | $p, n, \Lambda^0, \Sigma^+, \Sigma^-, \Sigma^0, \Delta^{++},$ $\Delta^0, N^{*0}, Y_1^{*+}, \Omega^-, \ldots$ | 938 MeV$/c^2$ – few GeV$/c^2$ |

# Elementary Particles

- How do these particles interact??
  - All particles, including photons and neutrinos, participate in gravitational interactions
  - Photons can interact electromagnetically with any particles with electric charge
  - All charged leptons participate in both EM and weak interactions
  - Neutral leptons do not have EM couplings
  - All hadrons (Mesons and baryons) responds to the strong force and appear to participate in all the interactions

# Bosons, Fermions,and Antiparticles (Oh My)

- Bosons
  - All have integer spin angular momentum
  - All mesons are bosons
- Fermions
  - All have half-integer spin angular momentum
  - All leptons and baryons are fermions
- All particles have anti-particles
  - What are anti-particles?
    - Particles that have same mass as the normal particle but with opposite quantum numbers
  - What is the anti-particle of
    - A $\pi^0$?
    - A neutron?
    - A $K^0$?
    - A Neutrino?
    - An electron

# Elementary Particles: Bosons and Fermions

- All particles can be classified as bosons or fermions
  - Bosons follow Bose-Einstein statistics
    - Quantum mechanical wave function is symmetric under exchange of any pair of bosons

$$\Psi_B\left(x_1, x_2, x_3, \ldots x_i \ldots x_n\right) = \Psi_B\left(x_2, x_1, x_3, \ldots x_i \ldots x_n\right)$$

    - $x_i$: space-time coordinates and internal quantum numbers of particle i
  - Fermions obey Fermi-Dirac statistics
    - Quantum mechanical wave function is anti-symmetric under exchange of any pair of Fermions

$$\Psi_F\left(x_1, x_2, x_3, \ldots x_i \ldots x_n\right) = -\Psi_F\left(x_2, x_1, x_3, \ldots x_i \ldots x_n\right)$$

    - Pauli exclusion principle is built into the wave function
      - For $x_i = x_j$, $\Psi_F = -\Psi_F$

# Quantum Numbers

- When can an interaction occur?
  - If it is kinematically allowed
  - If it does not violate any recognized conservation laws
    - Eg. A reaction that violates charge conservation will not occur
  - In order to deduce conservation laws, a full theoretical understanding of forces are necessary
- Since we do not have full theory for all the forces
  - Many of general conservation rules for particles are based on experiment
- One easily observed conservation law is lepton number conservation
  - While photon and meson numbers are not conserved

# Baryon Numbers

- Can the decay $p \to e^{+} + \pi^{0}$ occur?
  - Kinematically??
    - Yes, proton mass is a lot larger than the sum of the two masses
  - Electrical charge?
    - Yes, it is conserved
- But this decay does not occur (<$10^{40}$/sec)
  - Why?
    - Must be a conservation law that prohibits this decay
  - What could it be?
    - An additive and conserved quantum number, Baryon number (B)
    - All baryons have B=1
    - Anti-baryons? (B=-1)
    - Photons, leptons and mesons have B=0
- Since proton is the lightest baryon, it does not decay.

# Lepton Numbers

- Quantum number of leptons
  - All leptons carry $\mathcal{L}$=1 (particles) or $\mathcal{L}$=-1 (antiparticles)
  - Photons or hadrons carry $\mathcal{L}$=0
- Lepton number is a conserved quantity
  - Total lepton number must be conserved
  - Lepton numbers by species must be conserved
  - This is an empirical law necessitated by experimental observation (or lack thereof)
- Consider the decay $e^{-} + e^{-} \rightarrow \pi^{-} + \pi^{-}$
  - Does this decay process conserve energy and charge?
    - Yes
  - But it hasn't been observed, why?
    - Due to the lepton number conservation

# Lepton Number Assignments

| Leptons (anti-leptons) | $\mathcal{L}_e$ | $\mathcal{L}_\mu$ | $\mathcal{L}_\tau$ | $\mathcal{L}=\mathcal{L}_e+\mathcal{L}_\mu+\mathcal{L}_t$ |
|---|---|---|---|---|
| $e^-$ $(e^+)$ | 1 (-1) | 0 | 0 | 1 (-1) |
| $\nu_e \ (\bar{\nu}_e)$ | 1 (-1) | 0 | 0 | 1 (-1) |
| $\mu^- \ (\mu^+)$ | 0 | 1 (-1) | 0 | 1 (-1) |
| $\nu_\mu \ (\bar{\nu}_\mu)$ | 0 | 1 (-1) | 0 | 1 (-1) |
| $\tau^- \ (\tau^+)$ | 0 | 0 | 1 (-1) | 1 (-1) |
| $\nu_\tau \ (\bar{\nu}_\tau)$ | 0 | 0 | 1 (-1) | 1 (-1) |

# Lepton Number Conservation

- Can the following decays occur?

| Decays | $\mu^- \to e^- + \gamma$ | $\mu^- \to e^- + e^+ + e^-$ | $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ |
|---|---|---|---|
| $\mathcal{L}_e$ | $0 \to 1+0$ | $0 \to 1-1+1$ | $0 \to 1-1+0$ |
| $\mathcal{L}_\mu$ | $1 \to 0+0$ | $1 \to 0+0+0$ | $1 \to 0+0+1$ |
| $\mathcal{L}_\tau$ | $0 \to 0+0$ | $0 \to 0+0+0$ | $0 \to 0+0+0$ |
| $\mathcal{L}=\mathcal{L}_e+\mathcal{L}_\mu+\mathcal{L}_\tau$ | $1 \to 1+0$ | $1 \to 1-1+1$ | $1 \to 1-1+1$ |

– Case 1: $\mathcal{L}$ is conserved but $\mathcal{L}_e$ and $\mathcal{L}_\mu$ not conserved

– Case 2: $\mathcal{L}$ is conserved but $\mathcal{L}_e$ and $\mathcal{L}_\mu$ not conserved

– Case 3: $\mathcal{L}$ is conserved, and $\mathcal{L}_e$ and $\mathcal{L}_\mu$ are also conserved

total $\mathcal{L}$ not especially useful

# Quantum Number Summary

- Baryon Number
  - An additive and conserved quantum number, Baryon number (B)
  - All baryons have B=1
  - Anti-baryons? (B=-1)
  - Photons, leptons and mesons have B=0
- Lepton Number
  - Quantum number assigned to leptons
  - All leptons carry $\mathcal{L}$=1 (particles) or $\mathcal{L}$=-1 (antiparticles)
  - Photons or hadrons carry $\mathcal{L}$=0
  - Total lepton number must be conserved
  - Lepton numbers by species must be conserved

# Strangeness

- From cosmic ray shower observations
  - K-mesons and $\Sigma$ and $\Lambda^0$ baryons are produced strongly w/ large x-sec
    - But their lifetime typical of weak interactions (~$10^{-10}$ sec)
    - Are produced in pairs – a K with a $\Sigma$ or a K with a $\Lambda^0$
  - Gave an indication of a new quantum number
- Consider the reaction $\pi^- + p \to K^0 + \Lambda^0$
  - $K^0$ and $\Lambda^0$ subsequently decay
  - $\Lambda^0 \to \pi^- + p$ and $K^0 \to \pi^+ + \pi^-$
- Observations about $\Lambda^0$
  - Always produced w/ $K^0$ never with just a $\pi^0$
  - Produced with a $K^+$ but not with a $K^-$

$$\pi^- + p \to K^+ + \pi^- + \Lambda^0$$

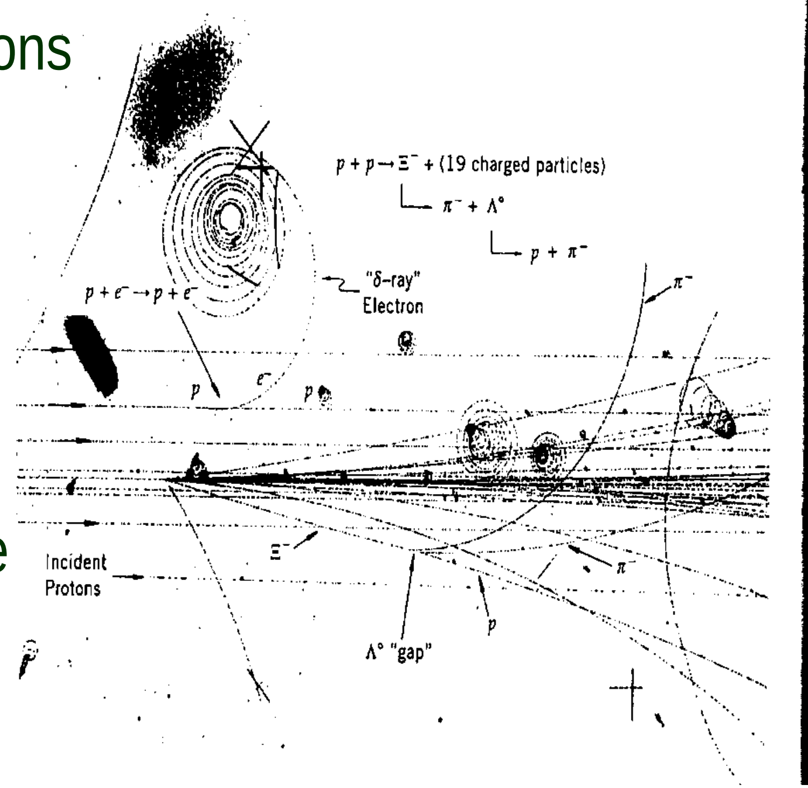
$$\pi^- + p \not\to K^- + \pi^+ + \Lambda^0 \qquad \pi^- + p \not\to \pi^- + \pi^+ + \Lambda^0$$

# Strangeness

- Further observation of cross section measurements
  - The cross section for reactions $\pi^- + p \rightarrow K^+ + \pi^- + \Lambda^0$ and $\pi^- + p \rightarrow K^0 + \Lambda^0$ with 1GeV/c pion momenta are ~ 1mb
    - Whereas the total pion-proton scattering cross section is ~ 30mb
    - The interactions are strong interactions
  - $\Lambda^0$ at v~0.1c decays in about 0.3cm
    - Lifetime of $\Lambda^0$ baryon is

$$\tau_{\Lambda^0} \approx \frac{0.3cm}{3\times10^9 cm/s} = 10^{-10}\sec$$

    - The short/intermediate lifetime of these strange particles indicate weak decay



# Strangeness

- Strangeness quantum number
  - Murray Gell-Mann and Abraham Pais proposed a new additive quantum number that are carried by these particles
  - Conserved in strong interactions
  - Violated in weak decays
  - S=0 for all ordinary mesons and baryons as well as photons and leptons
  - For any strong associated-production reaction w/ the initial state S=0, the total strangeness of particles in the final state should add up to 0.

# Strangeness

- Based on experimental observations of reactions and w/ an arbitrary choice of $S(K^0)=1$, we obtain
  - $S(K^+)=S(K^0)=1$ and $\Sigma(K^-)=\Sigma(\overline{K}^0)=-1$
  - $S(\Lambda^0)=S(\Sigma^+)=S(\Sigma^0)=S(\Sigma^-)=-1$
  - Does this work for the following reactions?
  - $\pi^- + p \to K^+ + \pi^- + \Lambda^0$
  - $\pi^- + p \to K^0 + \Lambda^0$
- For strong production reactions $K^- + p \to \Xi^- + K^+$ and $\overline{K}^0 + p \to \Xi^0 + K^+$
  - cascade particles $S(\Xi^-) = S(\Xi^0) = -2$ if $S(\overline{K}^0) = S(K^-) = -1$

# More on Strangeness

- Let's look at the reactions again

$$\pi^{-} + p \rightarrow K^{0} + \Lambda^{0}$$

  - This is a strong interaction
    - Strangeness must be conserved
    - S: 0 + 0 → +1 -1
- How about the decays of the final state particles?
  - $\Lambda^{0} \rightarrow \pi^{-} + p$ and $K^{0} \rightarrow \pi^{+} + \pi^{-}$
  - These decays are weak interactions so S is not conserved
  - S: -1 → 0 + 0 and +1 → 0 + 0
- A not-really-elegant solution
  - S only conserved in Strong and EM interactions → Unique strangeness quantum numbers cannot be assigned to leptons
- Leads to the hypothesis of strange quarks

# Isospin Quantum Number

- Strong force does not depend on the charge of the particle
  - Nuclear properties of protons and neutrons are very similar
  - From the studies of mirror nuclei, the strengths of p-p, p-n and n-n strong interactions are essentially the same
  - If corrected by EM interactions, the x-sec between n-n and p-p are the same
- Since strong force is much stronger than any other forces, we could imagine a new quantum number that applies to all particles
  - Protons and neutrons are two orthogonal mass eigenstates of the same particle like spin up and down states

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathrm{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Isospin Quantum Number

- Protons and neutrons are degenerate in mass because of some symmetry of the strong force
  - Isospin symmetry ➔ Under the strong force these two particles appear identical
  - Presence of Electromagnetic or Weak forces breaks this symmetry, distinguishing p from n
- Isospin works just like spin
  - Protons and neutrons have isospin ½ ➔ Isospin doublet
  - Three pions, $\pi+$, $\pi-$ and $\pi^0$, have almost the same masses
  - X-sec by these particles are almost the same after correcting for EM effects
  - Strong force does not distinguish these particles ➔ Isospin triplet

$$\pi^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \pi^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \pi^- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

# Isospin Quantum Number

- This QN is found to be conserved in strong interactions
- But not conserved in EM or Weak interactions
- Isospin no longer used, replaced by quark model

# Quantum Numbers

- Baryon Number
  - An additive and conserved quantum number, Baryon number (B)
  - This number is conserved in strong interactions and EM but not necessarily in weak interactions
- Lepton Number
  - Quantum number assigned to leptons
  - Lepton numbers by species and the total lepton numbers must be conserved (EM+EW)
- Strangeness Numbers
  - Conserved in strong interactions
  - But violated in weak interactions
- Isospin Quantum Numbers
  - Conserved in strong interactions
  - But violated in weak and EM interactions

# Quantum Number Conservation

- Some quantum numbers are conserved in strong interactions but not in electromagnetic and weak interactions
  – Inherent reflection of underlying forces
- Understanding conservation or violation of quantum numbers in certain situations is important for formulating quantitative theoretical framework

# Weak Interactions

- Three types of weak interactions
  - Hadronic decays: Only hadrons in the final state

$$\Lambda^0 \to \pi^- + p$$

  - Semi-leptonic decays: both hadrons and leptons are present

$$n \to p + e^- + \bar{\nu}_e$$

  - Leptonic decays: only leptons are present

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

# Symmetry

- When is a quantum number conserved?
  - When there is an underlying symmetry in the system
  - When the quantum number is not affected by changes in the physical system
- **Noether's theorem**: If there is a conserved quantity associated with a physical system, there exists an underlying invariance or symmetry principle responsible for this conservation.
- Symmetries provide critical restrictions in formulating theories

# Symmetries in Lagrangian Formalism?

- Consider an isolated non-relativistic physical system of two particles interacting through a potential that only depends on the relative distance between them
  - EM and gravitational force
- The total kinetic and potential energies of the system are: $T = \frac{1}{2} m_1 \dot{\vec{r}}_1^{\,2} + \frac{1}{2} m_2 \dot{\vec{r}}_2^{\,2}$ and $V = V\left(\vec{r}_1 - \vec{r}_2\right)$
- The equations of motion are then

$$m_1 \ddot{\vec{r}}_1 = -\vec{\nabla}_1 V\left(\vec{r}_1 - \vec{r}_2\right) = -\frac{\partial}{\partial \vec{r}_1} V\left(\vec{r}_1 - \vec{r}_2\right)$$

$$m_2 \ddot{\vec{r}}_2 = -\vec{\nabla}_2 V\left(\vec{r}_1 - \vec{r}_2\right) = -\frac{\partial}{\partial \vec{r}_2} V\left(\vec{r}_1 - \vec{r}_2\right)$$

where $\frac{\partial}{\partial \vec{r}_i} V\left(\vec{r}_1 - \vec{r}_2\right) = \hat{x}\frac{\partial}{\partial x_i} V + \hat{y}\frac{\partial}{\partial y_i} V + \hat{z}\frac{\partial}{\partial z_i} V$

# Symmetries in Lagrangian Formalism

- If we perform a linear translation of the origin of coordinate system by a constant vector $-\vec{a}$
  - The position vectors of the two particles become

$$\vec{r}_1 \to \vec{r}_1 - \vec{a} \qquad \vec{r}_2 \to \vec{r}_2 - \vec{a}$$

  - But the equations of motion do not change since $-\vec{a}$ is a constant vector
  - This is due to the invariance of the potential V under the translation

$$V' = V\left(\vec{r}\,'_1 - \vec{r}\,'_2\right) = V\left(\vec{r}_1 - \vec{a} - \vec{r}_2 + \vec{a}\right) = V\left(\vec{r}_1 - \vec{r}_2\right)$$