PHYS 3446 – Lecture #3

Tueday, Jan. 27 2015
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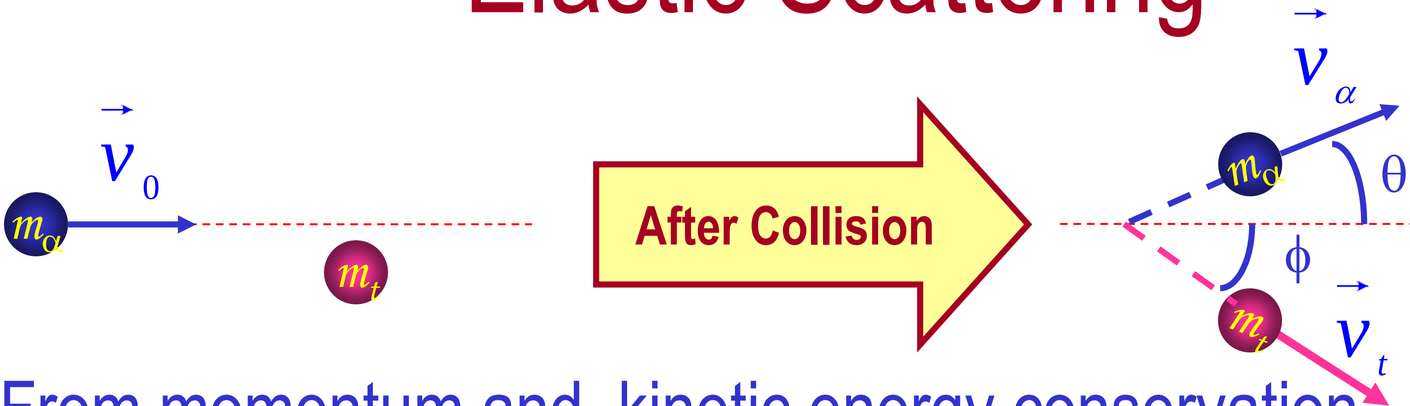
1. Rutherford Scattering with Coulomb force
2. Scattering Cross Section
3. Differential Cross Section of Rutherford Scattering
4. Measurement of Cross Sections

Elastic Scattering



- From momentum and kinetic energy conservation

$$v_0^2 = v_\alpha^2 + \frac{m_t}{m_\alpha} v_t^2$$ *****Eq. 1.2**

$$v_t^2 \left(1 - \frac{m_t}{m_\alpha}\right) = 2\vec{v}_\alpha \cdot \vec{v}_t$$ *****Eq. 1.3**

Analysis Case 1

$$v_t^2\left(1-\frac{m_t}{m_\alpha}\right)=2\vec{v}_\alpha\cdot\vec{v}_t$$

- If $m_t \ll m_\alpha$,
  - change of momentum of alpha particle is negligible

Analysis Case 2

- If $m_t \gg m_\alpha$,
  - alpha particle deflected backwards

Rutherford Scattering with EM Force 1

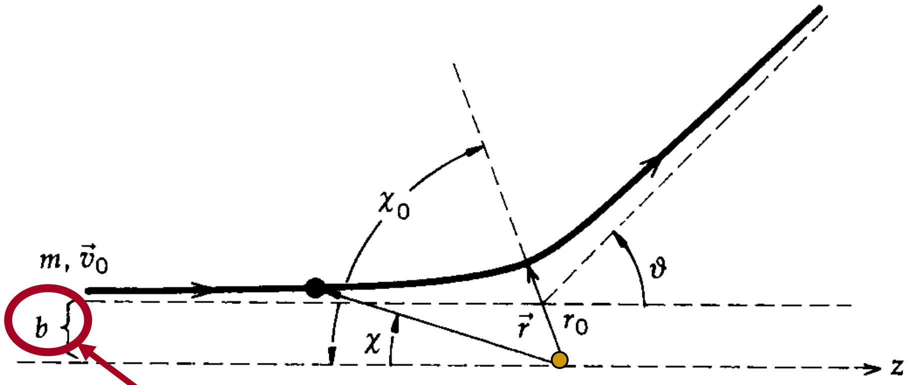
- Need to take into account the EM force between the α and the atom
- The Coulomb force is a central force->conservative force
- The Coulomb potential energy between particles with Ze and Z'e electrical charge, separated by a distance r is

$$V(r) = \frac{ZZ'e^2}{r}$$

- Since the total energy is conserved,

$$E = \frac{1}{2}mv_0^2 = \text{constant} > 0 \Rightarrow \mathrm{v}_0 = \sqrt{\frac{2E}{m}}$$

Rutherford Scattering with EM Force 2



Impact parameter=distance of closest approach if there were no Coulomb force

- The distance vector **r** is always the same direction as the force throughout the entire motion, so the net torque (**rxF**) is 0.
- Since there is no net torque, the angular momentum (**l=rxp**) is conserved. ➔ The magnitude of the angular momentum is ***l=mvb***.
- From the energy relation, we obtain

$$l = m\sqrt{2E/m} \cdot b = b\sqrt{2mE} \Rightarrow b^2 = l^2/2mE$$

- From angular momentum defn. , we obtain an equation of motion , where χ = angle between –x axis and radial vector" $d\chi/dt = l/mr^2$
- From energy conservation, we obtain another equation of motion

$$E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}mr^2\left(\frac{d\chi}{dt}\right)^2 + V(r)$$

$$\Rightarrow \frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - V(r) - \frac{l^2}{2mr^2}\right)}$$

Centrifugal barrier

Effective potential

Eq. 1.17

Rutherford Scattering with EM Force 3

- Rearranging the terms, we obtain

Eq. 1.18

$$\frac{dr}{dt} = -\frac{l}{mrb}\sqrt{r^2\left(1-\frac{V(r)}{E}\right)-b^2}$$

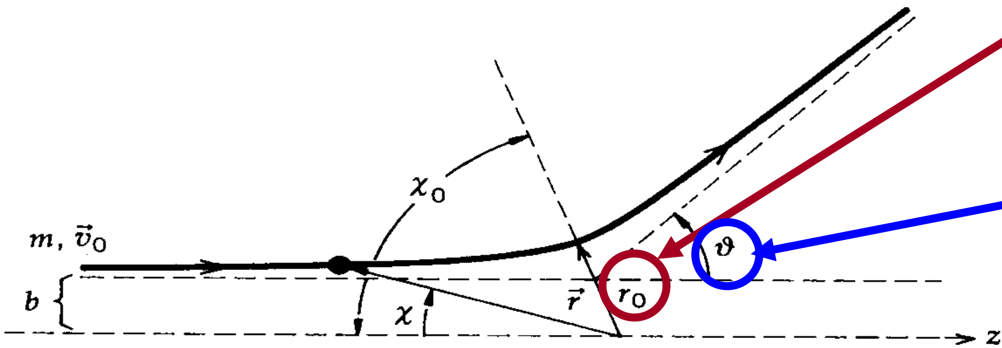
-sign since r decreases as alpha particle approaches target

- and

$$d\chi = -\frac{bdr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Eq. 1.19

Using 1.18 in the following $d\chi/dt=L/mr^2=d\chi/dr*dr/dt$ gives



Distance of closest approach

Asymptotic scattering angle: $\theta = \pi - 2\chi_0$

Rutherford Scattering with EM Force 4

- What happens at the DCA?
  - Kinetic energy goes to 0. $$\left.\frac{dr}{dt}\right|_{r=r_0} = 0$$
  - Consider the case where the alpha particle is incident on the z-axis, it would reach the DCA, stop, and reverse direction!
  - From Eq. 1.18, we can obtain $$r_0^2\left(1-\frac{V(r_0)}{E}\right)-b^2=0$$
  - This allows us to determine DCA for a given potential and χ_0.
- Substituting 1.19 into the scattering angle equation gives:

$$\theta = \pi - 2\chi_0 = \pi - 2b\int_{r_0}^{\infty}\frac{dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 5

- For a Coulomb potential

$$V(r) = \frac{ZZ'e^2}{r}$$

- DCA can be obtained for a given impact parameter b,

$$r_0 = \frac{ZZ'e^2/E}{2}\left(1+\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}\right)$$

- And the angular distribution becomes

$$\theta = \pi - 2b\int_{r_0}^{\infty}\frac{dr}{r\left[r^2\left(1-\frac{ZZ'e^2}{rE}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 6

- Replace the variable 1/r=x, and performing the integration, we obtain

$$\theta = \pi + 2b\cos^{-1}\left(\frac{1}{\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}}\right)$$

- This can be rewritten

$$\frac{1}{\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}} = \cos\left(\frac{\theta-\pi}{2}\right)$$

- Solving this for b, we obtain

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

Rutherford Scattering with EM Force 7

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- From the solution for b, we can learn the following
  1. For fixed b and E
     - The scattering is larger for a larger value of Z or Z' (large charge in projectile or target)
       - Makes perfect sense since Coulomb potential is stronger with larger Z.
       - Results in larger deflection.
  2. For fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - If particle has low energy, its velocity is smaller
       - Spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Makes perfect sense also, since as the incident particle is closer to the nucleus, it feels stronger Coulomb force.

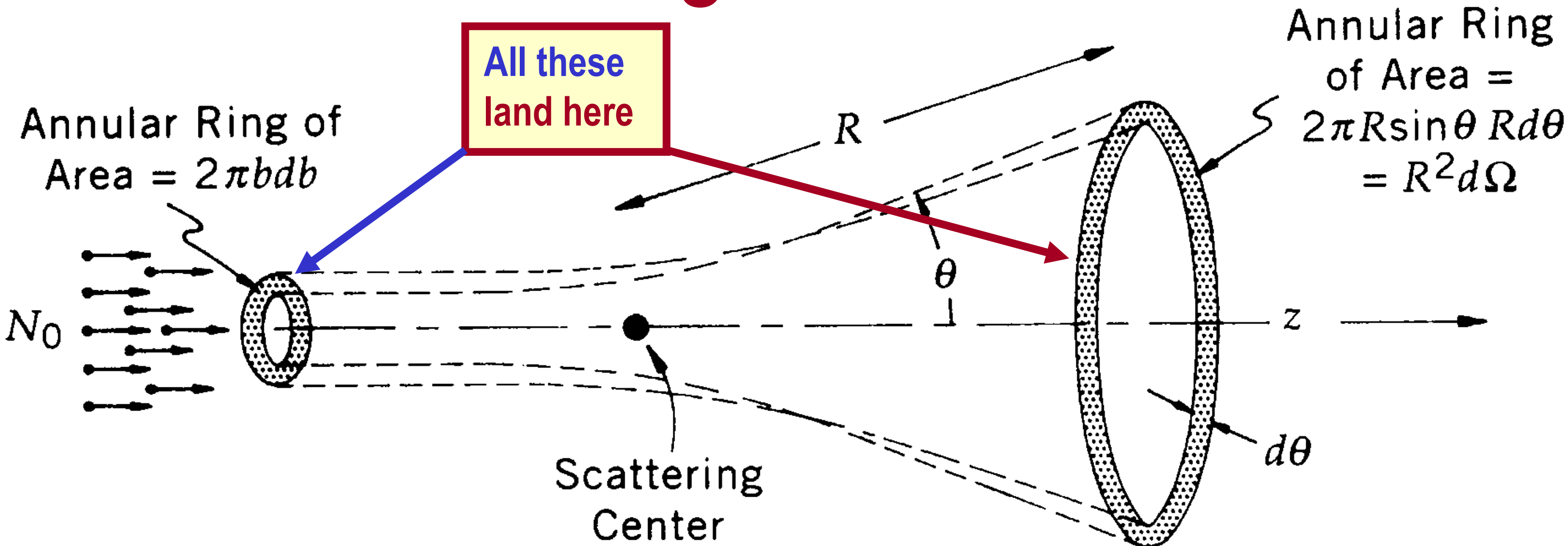
What do we learn from scattering?

- Scattering of a particle in a potential is completely determined when we know both
  - The impact parameter, b, and
  - The energy of the incident particle, E

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- For a fixed energy, the deflection is defined by
  - The impact parameter, b.
- What do we need to perform a scattering experiment?
  - Incident flux of beam particles with known E
  - Device that can measure number of scattered particles at various angle, θ.
  - Measurements of the number of scattered particles reflect
    - Impact parameters of the incident particles
    - The effective size of the scattering center
- By measuring the scattering angle θ, we can learn about the potential or the forces between the target and the projectile

Scattering Cross Section



- N_0: The number of particles incident on the target foil per unit area per unit time.
- Any incident particles entering with impact parameter b and b+db will scatter to the angle θ and θ-dθ.
- In other words, they scatter into the solid angle dΩ (=2πsinθdθ).
- So the number of particles scattered into the solid angle dΩ per unit time is $2\pi N_0 b db$.
- Note:have assumed thin foil, and large separation between nuclei—why?

Scattering Cross Section

- For a central potential
  - Such as Coulomb potential
  - Which has spherical symmetry
- The scattering center presents an effective transverse cross-sectional area of

$$\Delta\sigma = 2\pi b db$$

- For the particles to scatter into θ and $\theta+d\theta$

Scattering Cross Section

- In more generalized cases, $\Delta\sigma$ depends on both θ & ϕ.

$$\Delta\sigma(\theta,\phi) = bdbd\phi = -\frac{d\sigma}{d\Omega}(\theta,\phi)d\Omega = -\frac{d\sigma}{d\Omega}(\theta,\phi)\sin\theta d\theta d\phi$$

Why negative?

Since the deflection and change of b are in opposite direction!!

- With a spherical symmetry, ϕ can be integrated out:

$$\Delta\sigma(\theta) = -\frac{d\sigma}{d\Omega}(\theta)2\pi\sin\theta d\theta = 2\pi bdb$$

reorganize

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta}$$

Differential Cross Section

What is the dimension of the differential cross section?

Area!!

Scattering Cross Section

femptobarn

- For a central potential, measuring the yield as a function of θ (the differential cross section) is equivalent to measuring the entire effect of the scattering
- So what is the physical meaning of the differential cross section?

⇒ Measurement of yield as a function of specific experimental variables

⇒ This is equivalent to measuring the probability of occurrence of a physical process in a specific kinematic phase space

- Cross sections are measured in the unit of barns:

$$1\ \text{barn} \equiv 10^{-24}\text{cm}^2$$

Where does this come from?

picobarn

Cross sectional area of a typical nucleus!

nanobarn

Cross Section of Rutherford Scattering

- The impact parameter in Rutherford scattering is

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- Thus,

$$\frac{db}{d\theta} = -\frac{1}{2}\frac{ZZ'e^2}{2E}\operatorname{cosec}^2\frac{\theta}{2}$$

- Differential cross section of Rutherford scattering is

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta} = \left(\frac{ZZ'e^2}{4E}\right)^2 \operatorname{cosec}^4\frac{\theta}{2} = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

what happened? can you say "trig identity?" sin(2x)=2sin(x) cos(x)

plot/applets

Measuring Cross Sections

$$dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$

Number of detected particles/sec

Projectile particle flux

Density of the target particles

Scattering cross section

Detector acceptance

- This is a general expression for any scattering process, independent of the theory
- This gives an observed counts per second