PHYS 3446 – Lecture #4

Thursday, January 29, 2015
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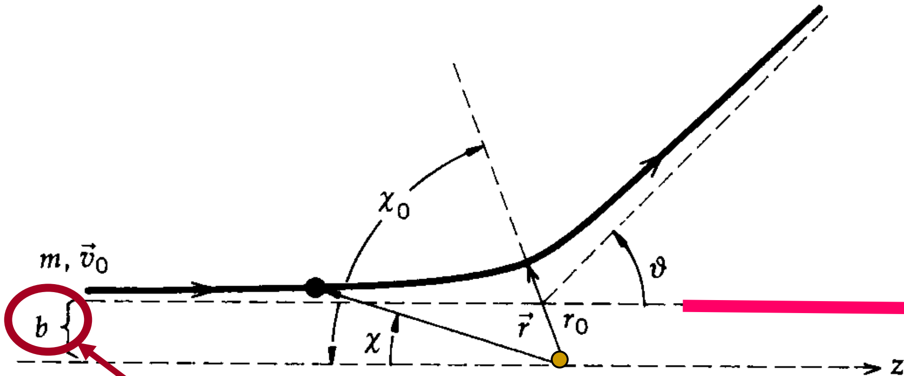
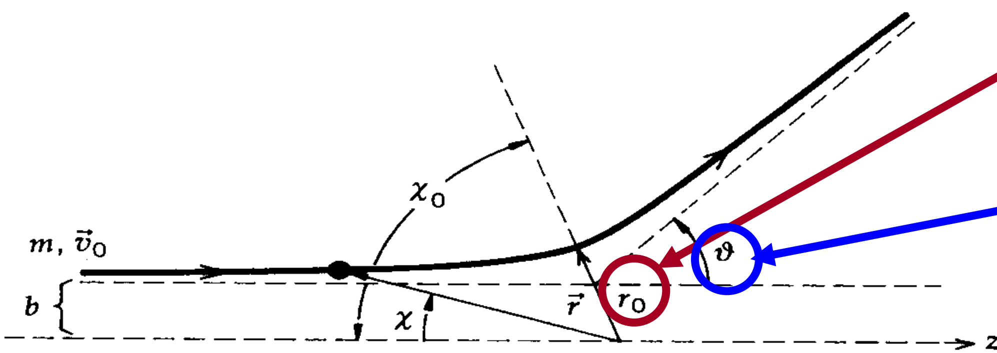
1. Rutherford Scattering with Coulomb force
2. Scattering Cross Section
3. Differential Cross Section of Rutherford Scattering
4. Measurement of Cross Sections

Rutherford Scattering with EM Force 2



If no EM no deflection Unless direct hit

Impact parameter=distance of closest approach if there were no Coulomb force



Distance of closest approach with EM

Asymptotic scattering angle: *

$\theta = \pi - 2\chi_0$

The distance vector **r** is always in the same direction as the force so the net torque (**rxF**) is 0 and angular momentum is conserved **dL/dt=0** (**L**=**rxp**) is conserved. ➔ The magnitude of angular momentum is ***L=mvb***.

Rutherford Scattering with EM Force 2b

- Far from target E=KE PE=qV V~0

$$E=\frac{1}{2}mv_0^2=\text{constant}>0 \qquad \Rightarrow \text{v}_0=\sqrt{\frac{2E}{m}}$$

$$l=m\sqrt{2E/m}\cdot b=b\sqrt{2mE}\Rightarrow b^2=l^2/2mE$$

- From angular momentum defn. , we obtain an equation of motion , where χ = angle between –x axis and radial vector"

L=Iw w=dχ/dt

$$d\chi/dt=l/mr^2$$

- From energy conservation, we obtain another equation of motion

E=KE+E(rot)+PE PE aka V(r)

$$E=\frac{1}{2}m\left(\frac{dr}{dt}\right)^2+\frac{1}{2}mr^2\left(\frac{d\chi}{dt}\right)^2+V(r) \Rightarrow \frac{dr}{dt}=\pm\sqrt{\frac{2}{m}\left(E-V(r)-\frac{l^2}{2mr^2}\right)}$$

Centrifugal barrier

Effective potential

Eq. 1.17

Rutherford Scattering with EM Force 3

(or not 2b)

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - V(r) - \frac{l^2}{2mr^2}\right)}$$

$$\Rightarrow b^2 = l^2/2mE$$

$$\frac{dr}{dt} = -\frac{l}{mrb}\sqrt{r^2\left(1 - \frac{V(r)}{E}\right) - b^2}$$ Eq. 1.18

-sign since r decreases as alpha particle approaches target

substitute 1.18 in the following
$d\chi/dt = L/mr^2 = d\chi\,/dr^{*}dr/dt$ gives

$$d\chi = -\frac{b\,dr}{r\left[r^2\left(1 - \frac{V(r)}{E}\right) - b^2\right]^{1/2}}$$ Eq. 1.19

Rutherford Scattering with EM Force 4

- What happens at the DCA?
  - Radial velocity 0 when radial distance is minimum at $r=r_0$
    $$\left.\frac{dr}{dt}\right|_{r=r_0} = 0$$
  - Consider the case where the alpha particle is incident on the z-axis, it would reach the DCA, stop, and reverse direction!
  - From Eq. 1.18, we can obtain
    $$r_0^2\left(1-\frac{V(r_0)}{E}\right)-b^2=0$$
  - This allows us to determine DCA for a given potential and χ_0 (asymptotic scattering angle)

- Substituting 1.19 into the scattering angle equation gives:

$$\theta = \pi - 2\chi_0 = \pi - 2b\int_{r_0}^{\infty}\frac{dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 5

- For a Coulomb potential

$$V(r) = \frac{ZZ'e^2}{r}$$

- DCA can be obtained for a given impact parameter b,

$$r_0 = \frac{ZZ'e^2/E}{2}\left(1+\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}\right)$$

- And the angular distribution becomes

$$\theta = \pi - 2b\int_{r_0}^{\infty} \frac{dr}{r\left[r^2\left(1-\frac{ZZ'e^2}{rE}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 6

- Replace the variable 1/r=x, and performing the integration, we obtain

$$\theta = \pi + 2b\cos^{-1}\left(\frac{1}{\sqrt{1+4b^2E^2/(ZZ'e^2)^2}}\right)$$

- This can be rewritten

$$\frac{1}{\sqrt{1+4b^2E^2/(ZZ'e^2)^2}} = \cos\left(\frac{\theta-\pi}{2}\right)$$

- Solving this for b, we obtain

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

Rutherford Scattering with EM Force 7

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- From the solution for b, we can learn the following
  1. For fixed b and E
     - The scattering is larger for a larger value of Z or Z' (large charge in projectile or target)
       - Makes perfect sense since Coulomb potential is stronger with larger Z.
       - Results in larger deflection.
  2. For fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - If particle has low energy, its velocity is smaller
       - Spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Makes perfect sense also, since as the incident particle is closer to the nucleus, it feels stronger Coulomb force.

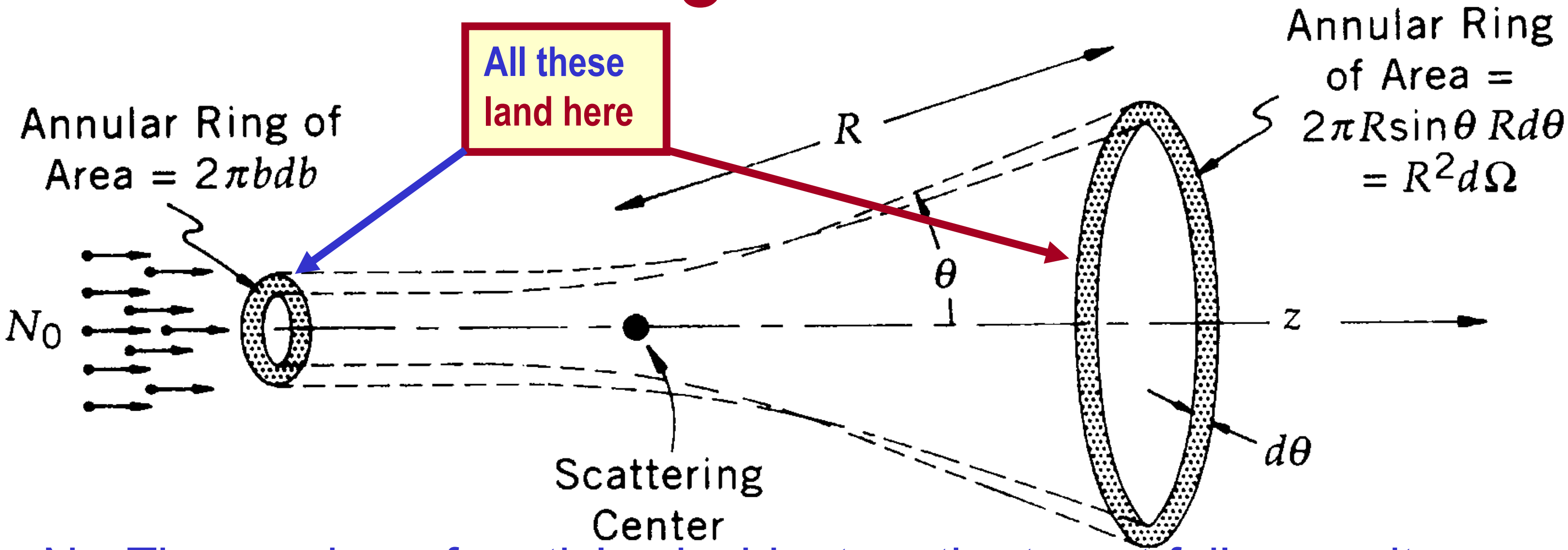
What do we learn from scattering?

- Scattering of a particle in a potential is completely determined when we know both
  - The impact parameter, b, and
  - The energy of the incident particle, E

$$b = \frac{ZZ'e^2}{2E} \cot\frac{\theta}{2}$$

- What do we need to perform a scattering experiment?
  - Incident flux of beam particles with known E
  - Device that can measure number of scattered particles at various angle, θ.
  - Measurements of the number of scattered particles reflect
    - Impact parameters of the incident particles
    - The effective size of the scattering center
- By measuring the scattering angle θ, we can learn about the potential or the forces between the target and the projectile

Scattering Cross Section



- N_0: The number of particles incident on the target foil per unit area per unit time.
- Any incident particles entering with impact parameter b and b+db will scatter to the angle θ and θ-dθ.
- In other words, they scatter into the solid angle dΩ ($=2\pi\sin\theta d\theta$).
- So the number of particles scattered into the solid angle dΩ per unit time is $2\pi N_0 b db$.

Scattering Cross Section

- For a central potential the scattering center presents an effective transverse cross-sectional area of

$$\Delta\sigma = 2\pi b\,db$$

for the particles to scatter into θ and $\theta+d\theta$

Scattering Cross Section

- In more generalized cases, $\Delta\sigma$ depends on both θ & ϕ.

$$\Delta\sigma(\theta,\phi) = bdbd\phi = -\frac{d\sigma}{d\Omega}(\theta,\phi)d\Omega = -\frac{d\sigma}{d\Omega}(\theta,\phi)\sin\theta d\theta d\phi$$

Why negative?

Since the deflection and change of b are in opposite direction!!

- With a spherical symmetry, ϕ can be integrated out:

$$\Delta\sigma(\theta) = -\frac{d\sigma}{d\Omega}(\theta)2\pi\sin\theta d\theta = 2\pi bdb$$

reorganize

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta}$$

Differential Cross Section

What is the dimension of the differential cross section?

Area!!

Scattering Cross Section

femptobarn

- So what is the physical meaning of the differential cross section?

$\Rightarrow$ Measurement of the probability of occurrence of a physical process given certain input variables

- Cross sections are measured in the unit of barns:

$$1\ \text{barn} \equiv 10^{-24}\text{cm}^2$$

nanobarn

Cross sectional area of a typical nucleus!

picobarn

femtobarn

Total Cross Section

- Total cross section is the integration of the differential cross section over the entire solid angle, Ω:

$$\sigma_{Total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}(\theta,\phi)\, d\Omega = 2\pi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}(\theta)$$

- Total cross section represents the effective size of the scattering center integrated over all possible impact parameters (and consequently all possible scattering angles)

Cross Section of Rutherford Scattering

- The impact parameter in Rutherford scattering is

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- Thus,

$$\frac{db}{d\theta} = -\frac{1}{2}\frac{ZZ'e^2}{2E}\mathrm{cosec}^2\frac{\theta}{2}$$

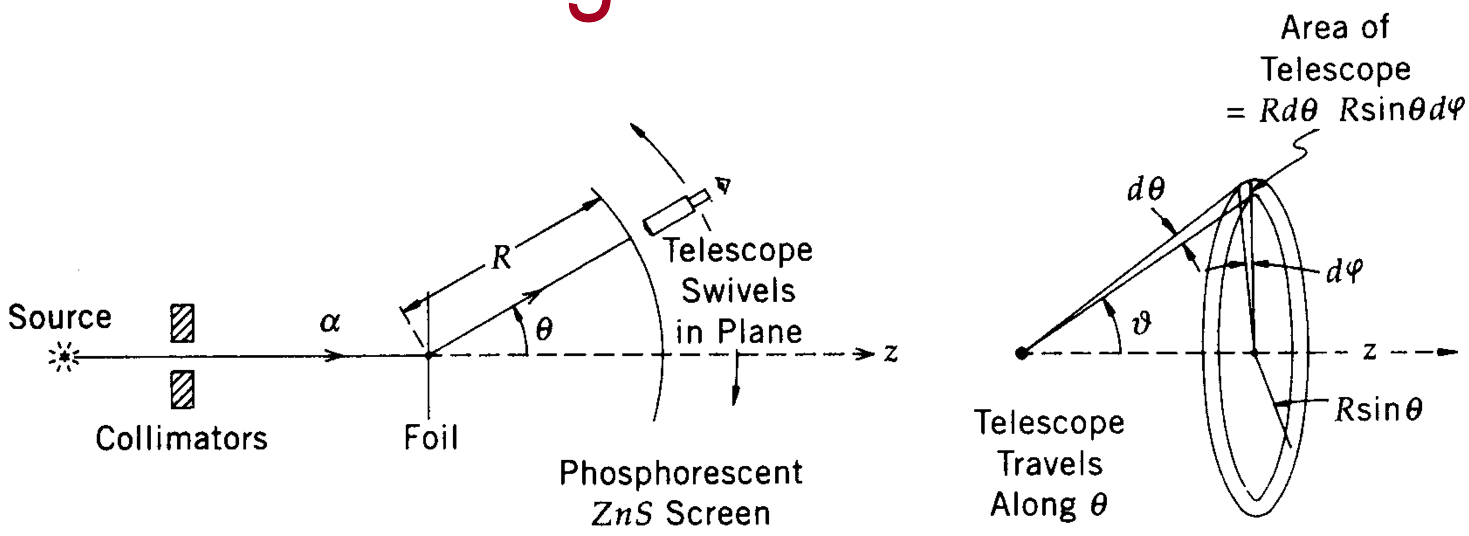
- Differential cross section of Rutherford scattering is

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta} = \left(\frac{ZZ'e^2}{4E}\right)^2 \mathrm{cosec}^4\frac{\theta}{2} = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

what happened? can you say “trig identity?” sin(2x)=2sin(x) cos(x)

plot/applets

Measuring Cross Sections



- Rutherford scattering experiment
  - Used a collimated beam of α particles emitted from Radon
  - A thin Au foil target
  - A scintillating glass screen with ZnS phosphor deposit
  - Telescope to view limited area of solid angle
  - Telescope only needs to move along θ not ϕ. Why?
    - Due to the spherical symmetry, scattering only depends on θ not ϕ.

Total X-Section of Rutherford Scattering

- To obtain the total cross section of Rutherford scattering, one integrates the differential cross section over all θ:

$$\sigma_{Total} = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta) \sin\theta d\theta = 8\pi \left(\frac{ZZ'e^2}{4E} \right)^2 \int_0^1 d\left(\sin\frac{\theta}{2} \right) \frac{1}{\sin^3\frac{\theta}{2}}$$

- What is the result of this integration?
  - Infinity!!
- Does this make sense?
  - Yes
- Why?
  - Since the Coulomb force's range is infinite (particle with very large impact parameter still contributes to integral through very small scattering angle)
- What would be the sensible thing to do?
  - Integrate to a cut-off angle since after certain distance the force is too weak to impact the scattering. ($\theta=\theta_0>0$); note this is sensible since alpha particles far away don't even see charge of nucleus due to screening effects.

Measuring Cross Sections

- With the flux of N_0 per unit area per second
- Any α particles in range b to b+db will be scattered into θ to θ-dθ
- The telescope aperture limits the measurable area to

$$A_{Tele} = Rd\theta \cdot R\sin\theta d\phi = R^2 d\Omega$$

- How could they have increased the rate of measurement?
  - By constructing an annular telescope
  - By how much would it increase? $2\pi/d\phi$

Measuring Cross Sections

- Fraction of incident particles (N_0) approaching the target in the small area $\Delta\sigma = bd\phi db$ at impact parameter b is dn/N_0.
  - so dn particles scatter into $R^2 d\Omega$, the aperture of the telescope
- This fraction is the same as
  - The sum of $\Delta\sigma$ over all N nuclear centers throughout the foil divided by the total area (S) of the foil.
  - Or, in other words, the probability for incident particles to enter within the N areas divided by the probability of hitting the foil. This ratio can be expressed as

$$-\frac{dn}{N_0} = \frac{N}{S}\Delta\sigma(\theta,\phi) = \frac{Nbd\phi d\theta}{S} \qquad \text{Eq. 1.39}$$

Measuring Cross Sections

- For a foil with thickness t, mass density ρ, atomic weight A:

$$N = \frac{\rho t S}{A} A_0$$

A_0: Avogadro's number of atoms per mole

- Since from what we have learned previously

$$-\frac{dn}{N_0} = \frac{N b d\phi d\theta}{S}$$

- The number of α scattered into the detector angle (θ,ϕ) is

$$dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$

Eq. 1.40

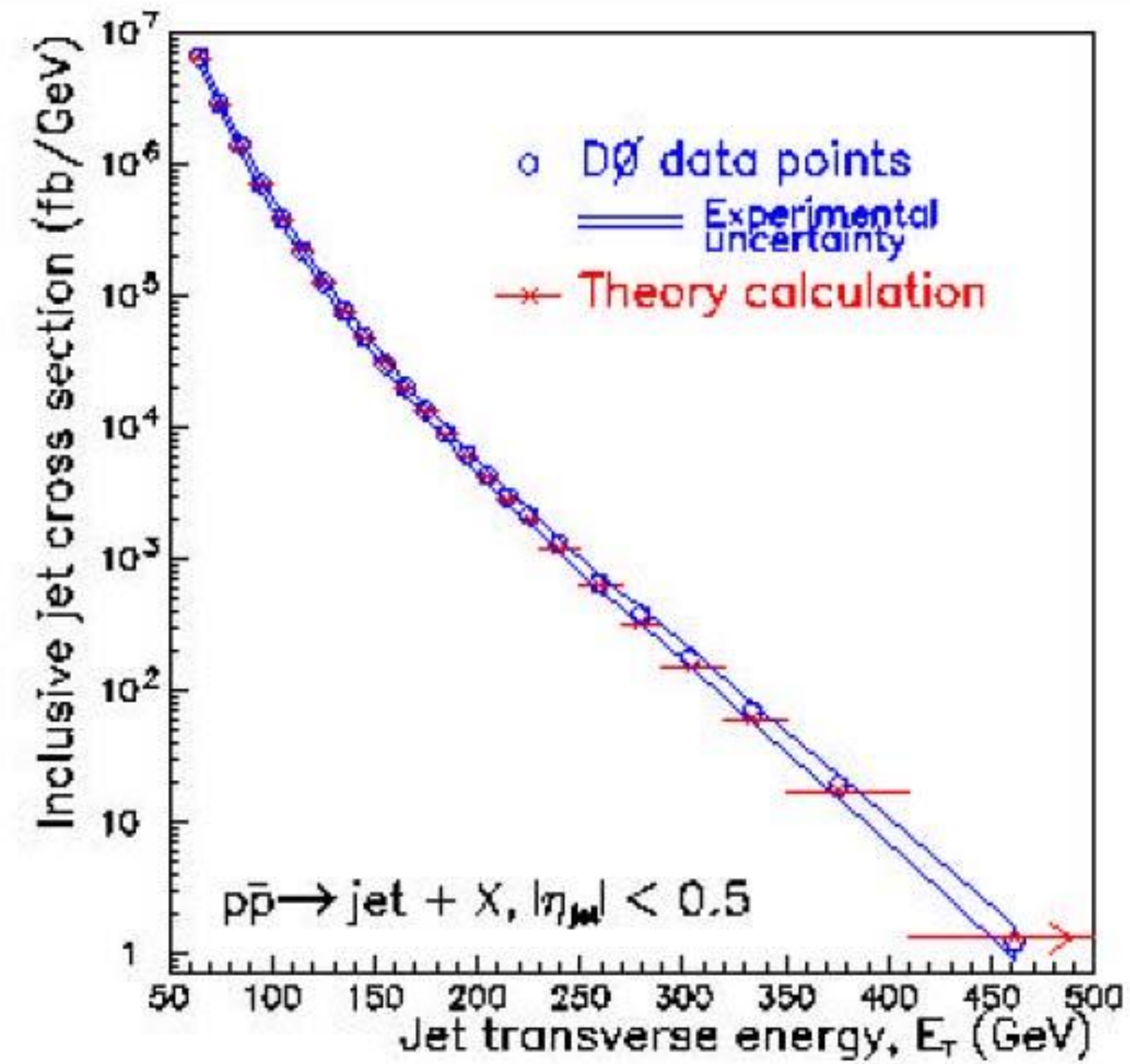
Measuring Cross Sections

$$dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$

- dn: Number of detected particles/sec
- N_0: Projectile particle flux
- $\frac{N}{S}$: Density of the target particles
- $\frac{d\sigma(\theta,\phi)}{d\Omega}$: Scattering cross section
- $d\Omega$: Detector acceptance

- This is a general expression for any scattering process, independent of the theory
- This gives an observed counts per second

Example Cross Section: Jet +X



- Inclusive jet production cross section as a function of transverse energy

triumph of QCD

what if there were an excess at high energy?

HW #2 Due Thurs. Feb. 5

1. Plot the differential cross section of the Rutherford scattering as a function of the scattering angle θ for three sensible choices of the lower limit of the angle.

   (use Z_{Au}=79, Z_{he}=2, E=10keV).
2. Compute the total cross section of the Rutherford scattering in unit of barns for your cut-off angles.
3. Find a plot of a cross section from a current HEP experiment, and write a few sentences about what is being measured.
4. Book problem 1.10
5. Book problem 1.11 *OR* 1.12