PHYS 3446 – Lecture #7

Thursday, Feb. 12 2015 Dr. ***Brandt***

Nuclear Properties:

-Nucleus Notation

-Binding Energy

-Size

-Spin

-Magnetic dipole moment

-Stability

-Instability

Nuclear Phenomenology

- What did Rutherford scattering experiment do?
  - Demonstrated the existence of a positively charged central core in an atom
  - The formula did not quite work for high energy α particles (E>25MeV), especially for low Z target nuclei.
- In 1920's, James Chadwick found
  - Serious discrepancies between Coulomb scattering expectation and the elastic scattering of α particle on He.
  - None of the known effects, including quantum effects, described the discrepancy.
- Clear indication of something more than Coulomb force involved in the interactions
- Chadwick's discovered neutron in 1932 → Nuclei consist of nucleons, protons and neutrons

Nucleus Labeling

- What are good quantities to label nuclei of an atom X?
  - Electrical Charge or atomic number Z (number of protons)
    - Most chemical properties depend on charge
  - Total number of nucleons A ($=N_p+N_n$)

$$ {}^{A}X^{N_p} = {}^{A}X^{Z} $$

  - Examples

Hydrogen ${}^{1}H^{1}$ Helium ${}^{4}He^{2}$ Carbon ${}^{12}C^{6}$

Nitrogen ${}^{14}N^{7}$ Oxygen ${}^{16}O^{8}$ Fluorine ${}^{19}F^{9}$

Types of Nuclei

- **Isotopes**: Nuclei with the same Z but different A
  - Same number of protons but different number of neutrons
  - Have similar chemical properties

  $^{12}C^{6}$ $^{13}C^{6}$ $^{238}U^{92}$ $^{235}U^{92}$

- **Isobars**: Nuclei with same A but different Z
  - Same number of nucleons but different number of protons
  - Different Chemical properties

  $^{238}U^{92}$ $^{238}Pu^{94}$

- **Isomers** or **resonances** of the ground state: Nucleus excited to a higher energy level

Nuclear Properties: Masses of Nuclei

- How many protons and neutrons does nucleus ${}^{A}X^{Z}$ have?
  - $N_p = Z$ and $N_n = A - Z$
- So what would be the expectation for its mass?

$$M\left({}^{A}X^{Z}\right) = \quad M\left(A, Z\right) = \quad Zm_p + \left(A - Z\right)m_n$$

  - Where $m_p = 938.27 MeV/c^2$ and $m_n = 939.56 MeV/c^2$
- However the measured mass turns out to be

$$M\left(A, Z\right) \quad < Zm_p + \left(A - Z\right)m_n$$

  - The energy difference is termed "binding energy", and acts to keep the the nucleus together
  - Energetically favorable for nucleus to remain intact

Nuclear Properties: Binding Energy

- The mass deficit

$$\Delta M\left(A,Z\right)=M\left(A,Z\right)-Zm_{p}-\left(A-Z\right)m_{n}$$

- Is always negative and is proportional to the nuclear binding energy
- How are the B.E. and mass deficit related?

$$B.E.=\Delta M\left(A,Z\right)c^{2}$$

- What is the physical meaning of B.E.?
  - A minimum energy required to release *<u>all</u>* nucleons from a nucleus
  - Typically more interested in B.E./nucleon

Nuclear Properties: Binding Energy

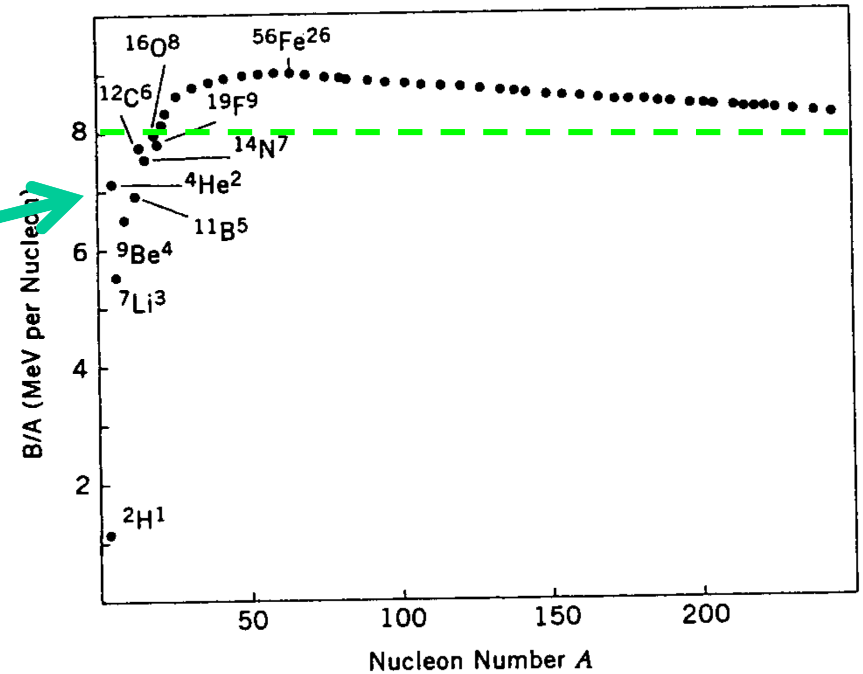
- BE per nucleon is

$$\frac{-BE}{A}$$

$$= \frac{-\Delta M(A,Z)c^2}{A}$$

$$= \frac{\left(Zm_p + (A-Z)m_n - M(A,Z)\right)c^2}{A}$$

He



- Rapidly increase with A till A~60 at which point BE~9 MeV.
- A>60, the B.E gradually decreases ➔ For most of the large A nucleus, BE~8 MeV.

Nuclear Properties: Binding Energy

- de Broglie's wavelength: $\lambda\!\!\!^{-} = \frac{\hbar}{p}$
  - Where $\hbar$ is the Planck's constant
  - And $\lambda\!\!\!^{-}$ is the reduced wavelength
- Assuming 8 MeV energy was given to a nucleon (m_n~940MeV), its wavelength would be

$$\lambda\!\!\!^{-} = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mT}} = \frac{\hbar c}{\sqrt{2mc^2T}} \approx \frac{197 Mev-fm}{\sqrt{2\cdot 940\cdot 8}} \approx 1.6\,fm$$

- Makes sense for nucleons to be inside a nucleus since the size is smaller than the nucleus.
- What about an electron with 8MeV? 25 fm—a lot bigger than nucleus? What energy needed to for wavelength to be consistent with nucleus? (~120 GeV)

Nuclear Properties: Size

- Sizes of subatomic particles are not as clearly defined as normal matter
  - Must be treated quantum mechanically via
    - probability distributions or expectation values
    - Atomic size is the average coordinate of the outermost electron and calculable via QM using Coulomb potential
    - Not calculable for nucleus since the potential is not known
      - Must rely on experimental measurements
- For Rutherford scattering of low E projectile $r_0^{min} = \frac{ZZ'e^2}{E}$
  - DCA provides an upper bound on the size of a nucleus
  - These result in $R_{Au} < 3.2 \times 10^{-12}$cm or $R_{Ag} < 2 \times 10^{-12}$cm

Nuclear Properties: Size

- Scatter very high E projectiles for head-on collisions
  - As E increases DCA becomes 0. $r_0^{min} = \frac{ZZ'e^2}{E}$
  - High E particles can probe deeper into nucleus
- Use electrons to probe the charge distribution (**form factor**) in a nucleus
  - What are the advantages of using electrons?
    - Electrons are fundamental particles ➔ No structure of their own
    - Electrons primarily interact through electromagnetic force
    - Electrons are not affected by the nuclear force
  - The radius of charge distribution can be regarded as an effective size of the nucleus

Nuclear Properties: Size

- At relativistic energies the magnetic moment of electron also contributes to the scattering
  - Neville Mott formulated Rutherford scattering in QM and included the spin effects
  - R. Hofstadter, *et al.*, discovered the effect of spin, nature of nuclear (& proton) form factor in late 1950s
- Mott scattering x-sec (scattering of a point particle) is related to Rutherford x-sec: $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$
- Deviation from the distribution expected for point-scattering provides a measure of size (structure)

Nuclear Properties: Size

- Another way to probe the nucleus is using strongly interacting particles (π mesons, protons, etc.)
  - What is the advantage of using these particles?
    - If the energy is high, Coulomb interaction can be neglected
    - These particles readily interact with nuclei, getting "absorbed" into the nucleus
    - Thus, probe strong interactions directly
- **The size of a nucleus can be inferred from the diffraction pattern (analogous to light diffracted by a disk)**

Nuclear Properties: Size

- All this phenomenological investigation resulted in a startlingly simple formula for the radius of the nucleus in terms of the number of nucleons or atomic number, A:

$$R = r_0 A^{1/3} \approx 1.2\times 10^{-13} A^{1/3} cm = 1.2 A^{1/3}\mathrm{fm}$$

Does this formula make sense? why 1/3 power?

Consider a spherical nucleus