PHYS 3446 – Lecture #8

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Nuclear Properties:

-Size

-Spin

-Magnetic dipole moment

-Stability

-Instability

Nuclear Properties: Binding Energy

- de Broglie's wavelength:
  – Where $\hbar$ is the Planck's constant
  – And $\lambdabar$ is the reduced wavelength

$$\lambdabar = \frac{\hbar}{p}$$

- Assuming 8 MeV energy was given to a nucleon (m_n~940MeV), its wavelength would be

$$\lambdabar = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mT}} = \frac{\hbar c}{\sqrt{2mc^2 T}} \approx \frac{197 Mev - fm}{\sqrt{2 \cdot 940 \cdot 8}} \approx 1.6\, fm$$

- Makes sense for nucleons to be inside a nucleus since the size is smaller than the nucleus.
- What about an electron with 8MeV? 25 fm—a lot bigger than nucleus? Wait a minute why 25fm? 197/sqrt(2x.511*8)=70 fm!
- Is 8 MeV electron still non-relativistic?
- What energy needed to for wavelength to be consistent with nucleus? (~120 GeV)

Nuclear Properties: Size

- Scatter very high E projectiles for head-on collisions
  - As E increases DCA becomes 0. $r_0^{min} = \frac{ZZ'e^2}{E}$
  - High E particles can probe deeper into nucleus
- Use electrons to probe the charge distribution (**form factor**) in a nucleus
  - What are the advantages of using electrons?
    - Electrons are fundamental particles ➔ No structure of their own
    - Electrons primarily interact through electromagnetic force
    - Electrons are not affected by the nuclear force
  - The radius of charge distribution can be regarded as an effective size of the nucleus

Nuclear Properties: Size

- At relativistic energies the magnetic moment of electron also contributes to the scattering
  - Neville Mott formulated Rutherford scattering in QM and included the spin effects
  - R. Hofstadter, *et al.*, discovered the effect of spin, nature of nuclear (& proton) form factor in late 1950s
- Mott scattering x-sec (scattering of a point particle) is related to Rutherford x-sec: $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$
- Deviation from the distribution expected for point-scattering provides a measure of size (structure)

Nuclear Properties: Size

- Another way to probe the nucleus is using strongly interacting particles (π mesons, protons, etc.)
  - What is the advantage of using these particles?
    - If the energy is high, Coulomb interaction can be neglected
    - These particles readily interact with nuclei, getting “absorbed” into the nucleus
    - Thus, probe strong interactions directly
- The size of a nucleus can be inferred from the diffraction pattern (analogous to light diffracted by a disk)

Nuclear Properties: Size

- All this phenomenological investigation resulted in a startlingly simple formula for the radius of the nucleus in terms of the number of nucleons or atomic number, A:

$$R = r_0 A^{1/3} \approx 1.2 \times 10^{-13} A^{1/3} cm = 1.2 A^{1/3} \mathrm{fm}$$

Does this formula make sense? why 1/3 power?

Consider a spherical nucleus

Nuclear Properties: Spin

- Both protons and neutrons are fermions with spin $1/2$
- Since nucleons inside a nucleus have spin they have orbital angular momentum
- In Quantum Mechanics orbital angular momenta are integers
- Thus the total angular momentum of a nucleus is
  - Integer: if an even number of nucleons in the nucleus
  - Half integer: if an odd number of nucleons in the nucleus
- Interesting facts are
  - All nuclei with even number of p and n are spin 0.
  - Large nuclei have very small spin in their ground state
- Hypothesis: Nucleon spins in the nucleus are very strongly paired to minimize their overall effect

Nuclear Properties: Magnetic Dipole Moment

- Every charged particle has a magnetic dipole moment associated with its spin $\vec{\mu} = g\frac{e}{2mc}\vec{S}$
- e, m and S are the charge, mass and the intrinsic spin of the charged particle
- The constant g is called Landé factor with a value:
  - $g = 2$: for a point like particle, such as the electron
  - $g \neq 2$: Particle possesses an anomalous magnetic moment, an indication of having a substructure (g-2 experiments)

Nuclear Properties: Magnetic Dipole Moment

- For electrons, $\mu_e \sim \mu_B$, where μ_B is Bohr Magneton

$$\mu_B = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11} \text{ MeV/T}$$

- For nucleons, magnetic dipole moment is measured as nuclear magneton, defined using proton mass

$$\mu_N = \frac{e\hbar}{2m_p c}$$

- Measured magnetic moments of proton and neutron:

$$\mu_p \approx 2.79\mu_N \qquad \mu_n \approx -1.91\mu_N$$

Nuclear Properties: Magnetic Dipole Moment

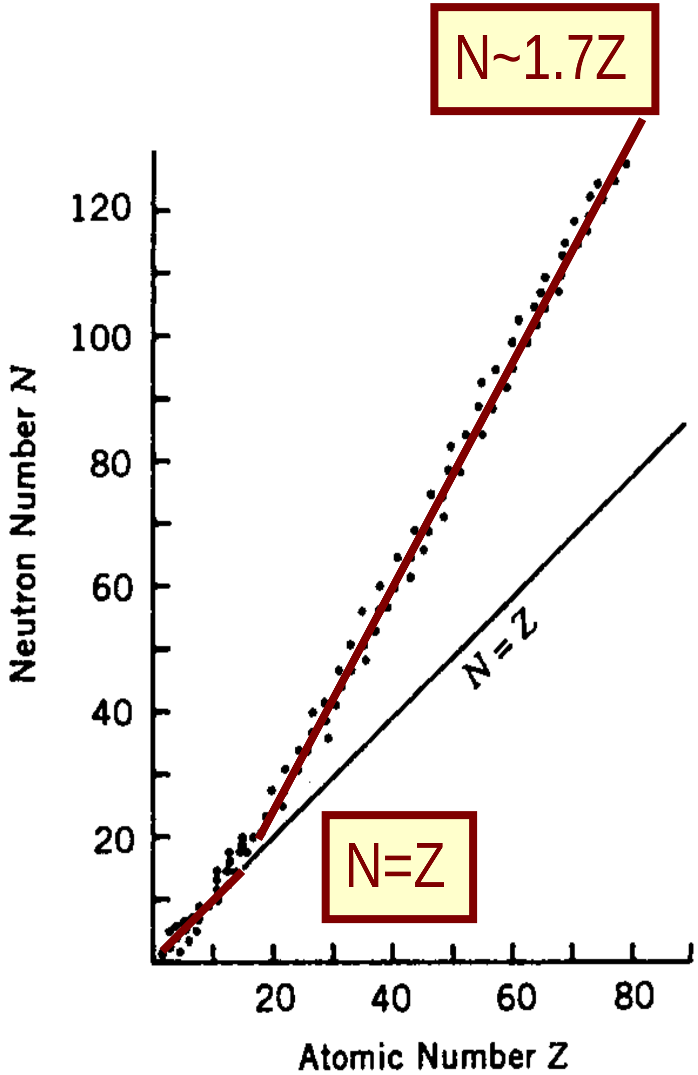
- What important information do you get from these magnetic moment measurments?
  - The Landé factors of the nucleons deviate significantly from 2.
    - Indication of substructure
  - An electrically neutral neutron has a significant magnetic moment
    - Must have extended charge distribution
- Measurements show that magnetic moment of nuclei are between $-3\mu_N$ and $10\mu_N$
  - Indication of strong pairing
  - Electrons cannot reside in nucleus

Nuclear Properties: Stability

- The number of protons and neutrons inside stable nuclei are
  - A<40: Equal (N=Z)
  - A>40: N~1.7Z
  - Neutrons outnumber protons
  - Most are even-p + even–n

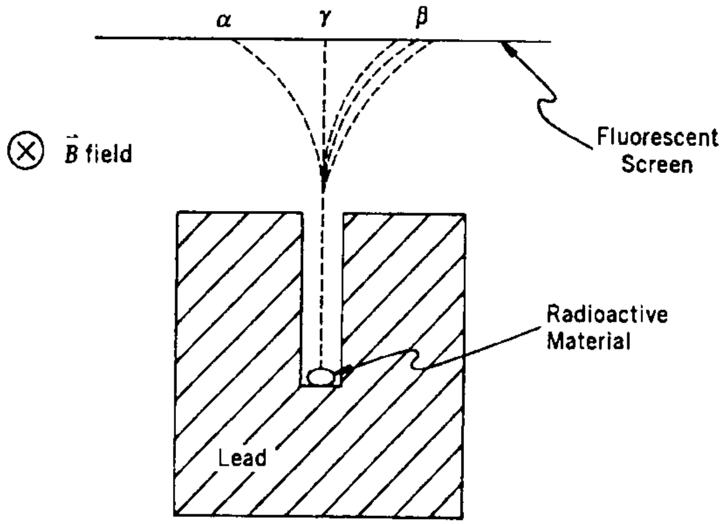
N	Z	N_{nucl}
Even	Even	156
Even	Odd	48
Odd	Even	50
Odd	Odd	5

    - See table 2.1
  - Supports strong pairing



Nuclear Properties: Instability

- In 1896 H. Becquerel accidentally discovered natural radioactivity
  - Study of fluorescent properties of Uranium salts
- Nuclear radioactivity involves emission of three types of radiation: α, β, and γ
- These can be characterized using the device on the right
  - α: Nucleus of He
  - β: electrons
  - γ: photons



- What do you see from the behavior in the magnetic field?
  - α and β are charged particles while γ is neutral.
  - α is mono-energetic
  - β has broad spectrum

Nature of the Nuclear Force

- Scattering experiments help to
  - Determine the properties of nuclei
  - Provide more details on the characteristics of the nuclear force
- From what we have learned, it is clear that there is no classical analog to nuclear force
  - Gravitational force is too weak to provide the binding
  - Can't have an electromagnetic origin
    - Deuteron nucleus has one neutron and one proton
    - Coulomb force destabilizes the nucleus

Short-range Nature of the Nuclear Force

- Atomic structure/periodic table is well explained by the electromagnetic interaction
  - Implies that the range of nuclear force cannot be much greater than the radius of the nucleus
  - Nuclear force should be felt on the few fm scale
- Binding energy is ~constant per nucleon (observed to be about 8 MeV), essentially independent of the size of the nucleus
  - If the nuclear force were a long-range force, like the Coulomb force , then for A nucleons, there would be ½ A(A-1) pair-wise interactions
  - Thus, the BE per neucleon, which reflects all possible interactions among the nucleons, would grow as a function of A

$$B \propto A(A-1)$$

For large A ⇒

$$BE = \frac{B}{A} \propto A \neq \text{constant}$$

XXXXX

Short-range Nature of the Nuclear Force

- Long-range nature of nuclear force is contradicted by the experimental measurement that the BE/nucleon stays constant
  - Nuclear force must saturate
  - Any given nucleon can only interact with a finite number of nucleons in its vicinity
- What is the effect of adding more nucleons to a nucleus?
  - Only increases the size of the nucleus
    - Recall that $R \sim A^{1/3}$
  - The size of a nucleus grows slowly with A and keeps the nuclear density constant

$\Rightarrow$Further supports short-range nature of nuclear force