PHYS 3446 – Review

Thursday, Mar. 26, 2015
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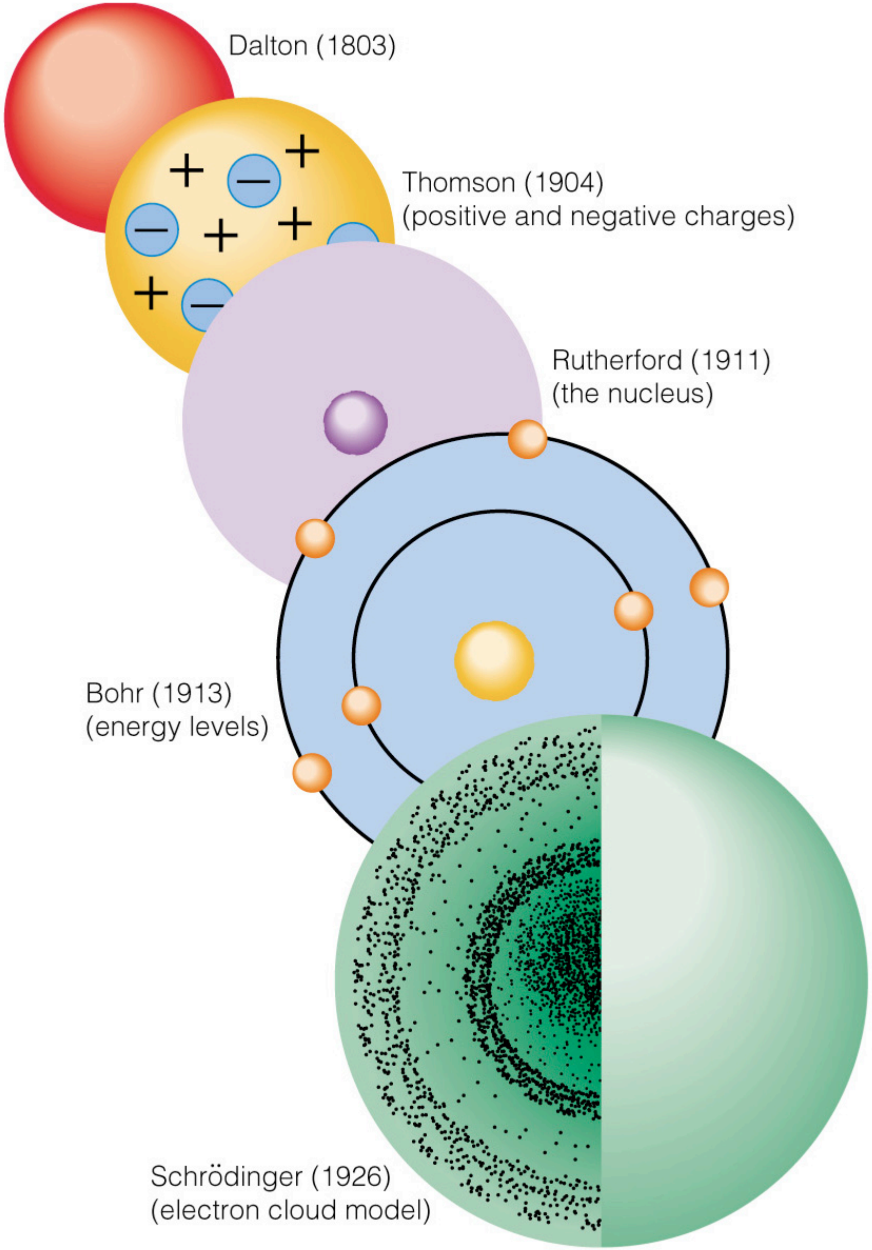
- Review
- Note Test deferred until Tues.

History of Atomic Models cnt'd

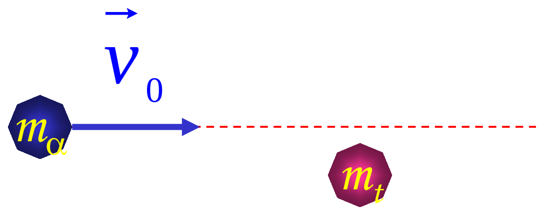
Lec.2

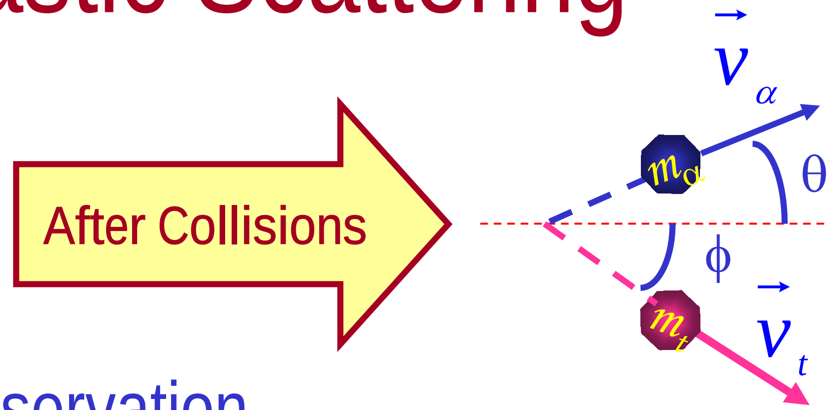
- atomic models
- elastic scattering
- Rutherford scattering



Elastic Scattering

Lec.3





- From momentum conservation

$$\vec{v}_0 = \frac{m_\alpha \vec{v}_\alpha + m_t \vec{v}_t}{m_\alpha} = \vec{v}_\alpha + \frac{m_t}{m_\alpha}\vec{v}_t$$

- From kinetic energy conservation (Elastic only!)

$$v_0^2 = v_\alpha^2 + \frac{m_t}{m_\alpha} v_t^2 \qquad \text{Eq. 1.2}$$

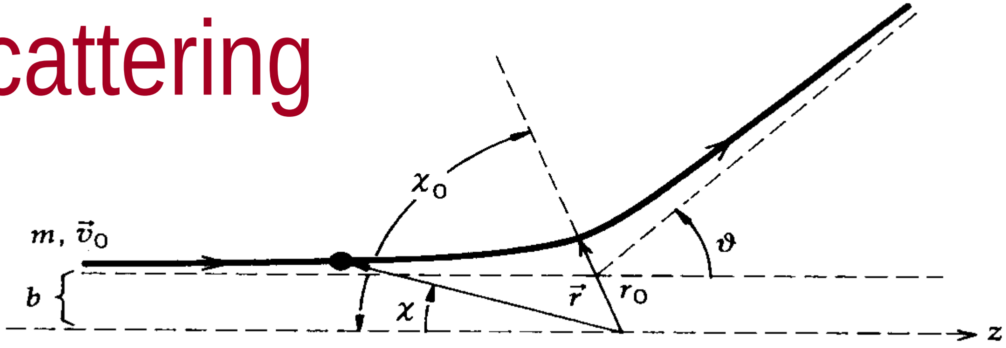
- From these, we obtain

$$v_t^2\left(1 - \frac{m_t}{m_\alpha}\right) = 2\vec{v}_\alpha \cdot \vec{v}_t \qquad \text{Eq. 1.3}$$

look at limiting cases

Rutherford Scattering

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$



- From the solution for b, we can learn the following
  1. For fixed b, E and Z'
     - The scattering is larger for a larger value of Z.
       - Since Coulomb potential is stronger with larger Z.
  2. For fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - Since the speed of the low energy particle is smaller
       - The particle spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Since the closer the incident particle is to the nucleus, the stronger Coulomb force it feels

Total Cross Section

lec. 4 diff +total xsec

- Total cross section is the integration of the differential cross section over the entire solid angle, Ω:

$$\sigma_{Total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}(\theta,\phi)\, d\Omega = 2\pi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}(\theta)$$

- Total cross section represents the effective size of the scattering center integrated over all possible impact parameters (and consequently all possible scattering angles)

$$1\ \text{barn} = 10^{-24}\text{cm}^2$$

Total X-Section of Rutherford Scattering

- To obtain the total cross section of Rutherford scattering, one integrates the differential cross section over all θ:

$$\sigma_{Total} = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta) \sin\theta d\theta = 8\pi \left(\frac{ZZ'e^2}{4E} \right)^2 \int_0^1 d\left(\sin\frac{\theta}{2} \right) \frac{1}{\sin^3 \frac{\theta}{2}}$$

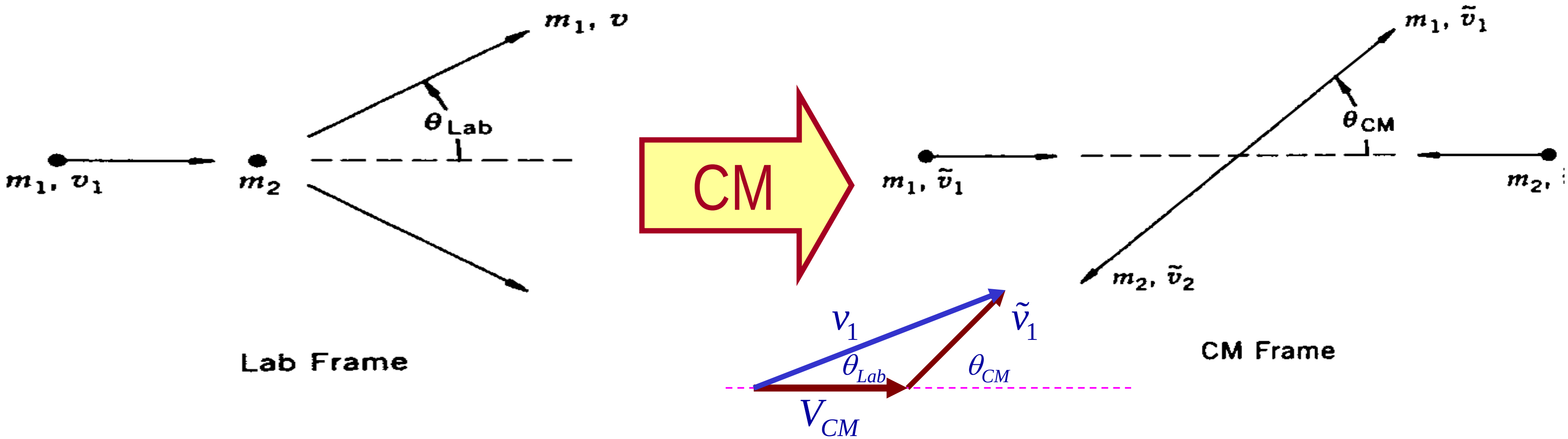
- What is the result of this integration?
  - Infinity!!
- Does this make sense?
  - Yes
- Why?
  - Since the Coulomb force's range is infinite (particle with very large impact parameter still contributes to integral through very small scattering angle)
- What would be the sensible thing to do?
  - Integrate to a cut-off angle since after certain distance the force is too weak to impact the scattering. ($\theta = \theta_0 > 0$); note this is sensible since alpha particles far away don't even see charge of nucleus due to screening effects.

lec 4

Lab Frame and CM Frame

- The CM is moving at a constant velocity in the lab frame independent of the form of the central potential
- The motion is that of a fictitious particle with mass μ (the reduced mass) and coordinate **r**.
- In the frame where the CM is stationary, the dynamics becomes equivalent to that of a single particle of mass μ scattering off of a fixed scattering center.
- Frequently we define the Center of Mass frame as the frame where the sum of the momenta of all the interacting particles is 0.

Relationship of variables in Lab and CM



- The speed of CM is

$$v_{CM} = \dot{R}_{CM} = \frac{m_1 v_1}{m_1 + m_2}$$

- Speeds of the particles in CM frame are

$$\tilde{v}_1 = v_1 - v_{CM} = \frac{m_2 v_1}{m_1 + m_2} \quad \text{and} \quad \tilde{v}_2 = v_{CM} = \frac{m_1 v_1}{m_1 + m_2}$$

- The momenta of the two particles are equal and opposite!!

Quantities in Special Relativity

- Fractional velocity $\vec{\beta} = \vec{v}/c$
- Lorentz γ factor $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$
- Relative momentum and the total energy of the particle moving at a velocity $\vec{v} = \vec{\beta}c$ is

$$\vec{P} = \gamma M\vec{v} = \gamma M\vec{\beta}c$$

$$E = \sqrt{T^2 + E_{Rest}^2} = \sqrt{P^2c^2 + M^2c^4} = \gamma Mc^2$$

- Square of four momentum P=(E,**pc**), rest mass E

$$P^2 = Mc^2 = E^2 - p^2c^2$$

Relativistic Variables

- The invariant scalar, s, is defined as:

$$s = (P_1 + P_2)^2 = (E_1 + E_2)^2 - \left(\vec{P}_1 + \vec{P}_2\right)^2 c^2$$
$$= m_1^2 c^4 + m_2^2 c^4 + 2E_1 m_2 c^2$$

- So what is this in the CM frame?

$$s = (P_1 + P_2)^2 =$$
$$= (E_{1CM} + E_{2CM})^2 - \cancelto{0}{\left(\vec{P}_{1CM} + \vec{P}_{2CM}\right)^2} c^2$$
$$= (E_{1CM} + E_{2CM})^2 = \left(E_{ToT}^{CM}\right)^2$$

- Thus, $\sqrt{s}$ represents the total available energy in the CM; At the LHC, eventually $\sqrt{s} = 14\,\mathrm{TeV}$

Problem 1.1 *Using Eq.* (1.38) *calculate the approximate total cross sections for Rutherford scattering of a* 10 MeV *α-particle from a lead nucleus for impact parameters b less than* 10^{-12}, 10^{-10} *and* 10^{-8} cm. *How well do these agree with the values of πb^2?*

There are various ways of doing this problem. We will list below two very simple methods.

Method I. In general, the total cross section for Rutherford scattering is given by (see Eq. (1.38) in the text)

$$\sigma_{\mathrm{TOT}} = 8\pi \left(\frac{ZZ'e^2}{4E}\right)^2 \int_0^1 \frac{\mathrm{d}(\sin\frac{\theta}{2})}{(\sin\frac{\theta}{2})^3}. \tag{1.1}$$

However, if the impact parameter is restricted to a finite range, say $b \leq b_0$, then we can write the total cross section as

$$\sigma_{\mathrm{TOT}}(b_0) = 8\pi \left(\frac{ZZ'e^2}{4E}\right)^2 \int_{\theta_{b_0}}^1 \frac{\mathrm{d}(\sin\frac{\theta}{2})}{(\sin\frac{\theta}{2})^3}, \tag{1.2}$$

where θ_{b_0} is the scattering angle corresponding to the ir eter b_0 and is given by (see Eq. (1.32) of the text)

$$b_0 = \frac{ZZ'e^2}{2E} \cot\frac{\theta_{b_0}}{2}.$$

Carrying out the integration in (1.2), we obtain

$$\begin{aligned}\sigma_{\mathrm{TOT}}(b_0) &= 8\pi \left(\frac{ZZ'e^2}{4E}\right)^2 \left(-\frac{1}{2}\right)\left(1 - \mathrm{cosec}^2\frac{\theta_{b_0}}{2}\right)\\ &= 4\pi \left(\frac{ZZ'e^2}{4E}\right)^2 \cot^2\frac{\theta_{b_0}}{2}\\ &= \pi\left(\frac{ZZ'e^2}{2E}\cot\frac{\theta_{b_0}}{2}\right)^2 = \pi b_0^2,\end{aligned} \tag{1.4}$$

where we have used the identification in (1.3). It follows, therefore, that

b_0 (cm)	$\sigma_{\mathrm{TOT}}(b_0) = \pi b_0^2$ (cm^2)
10^{-12}	3.2×10^{-24}
10^{-10}	3.2×10^{-20}
10^{-8}	3.2×10^{-16}

Problem 1.10 *What is the minimum impact parameter needed to deflect 7.7 MeV α-particles from gold nuclei by at least 1°? What about by at least 30°? What is the ratio of probabilities for deflections of $\theta > 1°$ relative to $\theta > 30°$? (See the CRC Handbook for the density of gold.)*

For the scattering of a 7.7 MeV α-particle from gold, we have

$$Z = 2, \quad Z' = 79, \quad E = 7.7\,\mathrm{MeV}, \tag{1.74}$$

so that we obtain

$$\begin{aligned} \frac{ZZ'e^2}{2E} &= ZZ' \times \frac{\hbar c}{2E} \times \frac{e^2}{\hbar c} \\ &= 2 \times 79 \times \frac{197\,\mathrm{MeV-F}}{2 \times 7.7\,\mathrm{MeV}} \times \frac{1}{137} \\ &\approx 14.5 \times 10^{-13}\,\mathrm{cm} \approx 1.4 \times 10^{-12}\,\mathrm{cm}. \end{aligned} \tag{1.75}$$

We know from Eq. (1.32) of the text that

$$b = \frac{ZZ'e^2}{2E} \cot\frac{\theta}{2}, \tag{1.76}$$

which leads to

$$b(\theta) \approx 1.4 \times 10^{-12} \cot\frac{\theta}{2}\,\mathrm{cm}. \tag{1.77}$$

We note that for

$$\theta = 1° = \frac{\pi}{180} \approx \frac{1}{60} \ll 1, \tag{1.78}$$

we have

$$\tan\frac{\theta}{2} \approx \frac{\theta}{2} \approx \frac{1}{120}, \quad \cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}} \approx 120. \tag{1.79}$$

Similarly, for

$$\theta = 30° = \frac{\pi}{6} \approx \frac{1}{2} \tag{1.80}$$

we have

$$\tan\frac{\theta}{2} \approx \frac{\theta}{2} \approx \frac{1}{4}, \quad \cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}} \approx 4. \tag{1.81}$$

Using these values in (1.77), we obtain

$$\begin{aligned} b(\theta = 1°) &\approx 1.4 \times 10^{-12} \times 120\,\mathrm{cm} = 1.7 \times 10^{-10}\,\mathrm{cm}, \\ b(\theta = 30°) &\approx 1.4 \times 10^{-12} \times 4\,\mathrm{cm} = 5.6 \times 10^{-12}\,\mathrm{cm}. \end{aligned} \tag{1.82}$$

As we have already seen in Problem 1.1, the probability of scattering for angles greater than θ_b goes as the area πb^2. Therefore, using (1.82) we have

$$\begin{aligned} \frac{\sigma(\theta > 1°)}{\sigma(\theta > 30°)} &= \frac{b^2(\theta = 1°)}{b^2(\theta = 30°)} \\ &\approx \left(\frac{1.7 \times 10^{-10}}{5.6 \times 10^{-12}}\right)^2 \approx 900. \end{aligned} \tag{1.83}$$

In other words, there will be approximately 900 more particle collisions for $\theta > 1°$ than for $\theta > 30°$.

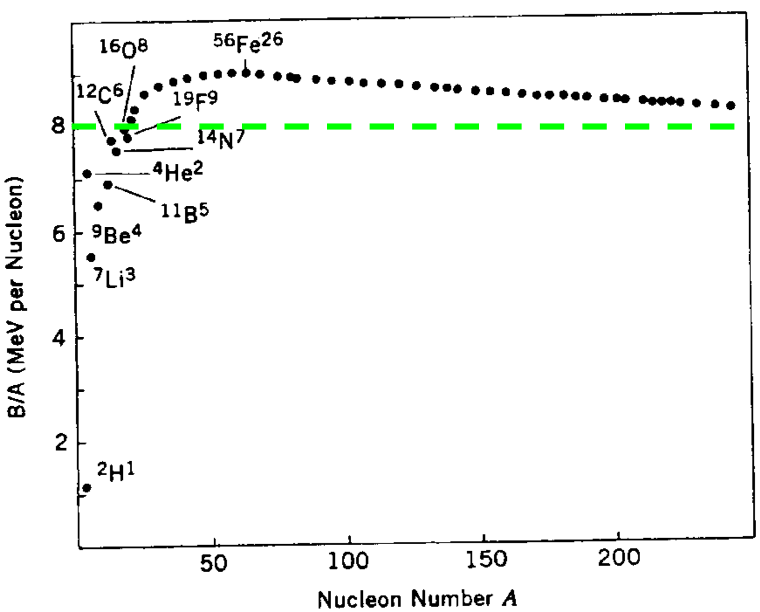
lec. 7 Nuclear Properties: Binding Energy

- The mass deficit

$$\Delta M(A,Z) = M(A,Z) - Zm_p - (A-Z)m_n$$

is always negative and is proportional to the nuclear binding energy $B.E = \Delta M(A,Z)c^2$

- What is the physical meaning of BE?
  - A minimum energy required to release all nucleons from a nucleus



- Rapidly increase with A till A~60 at which point BE~9 MeV.
- A>60, the B.E gradually decrease ➔ For most of the large A nucleus, BE~8 MeV.

Nuclear Properties: Sizes

- The size of a nucleus can be inferred from the diffraction pattern
- All this phenomenological investigation resulted in a startlingly simple formula for the radius of the nucleus in terms of the number of nucleons or atomic number, A:

$$R = r_0 A^{1/3} \approx 1.2 \times 10^{-13} A^{1/3} cm = 1.2 A^{1/3} \mathrm{fm}$$

Nuclear Properties: Magnetic Dipole Moments

- For electrons, $\mu_e \sim \mu_B$, where μ_B is Bohr Magneton

$$\mu_B = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11}\ \text{MeV/T}$$

- For nucleons, magnetic dipole moment is measured in nuclear magneton, defined using proton mass

$$\mu_N = \frac{e\hbar}{2m_p c}$$

- Measured magnetic moments of proton and neutron:

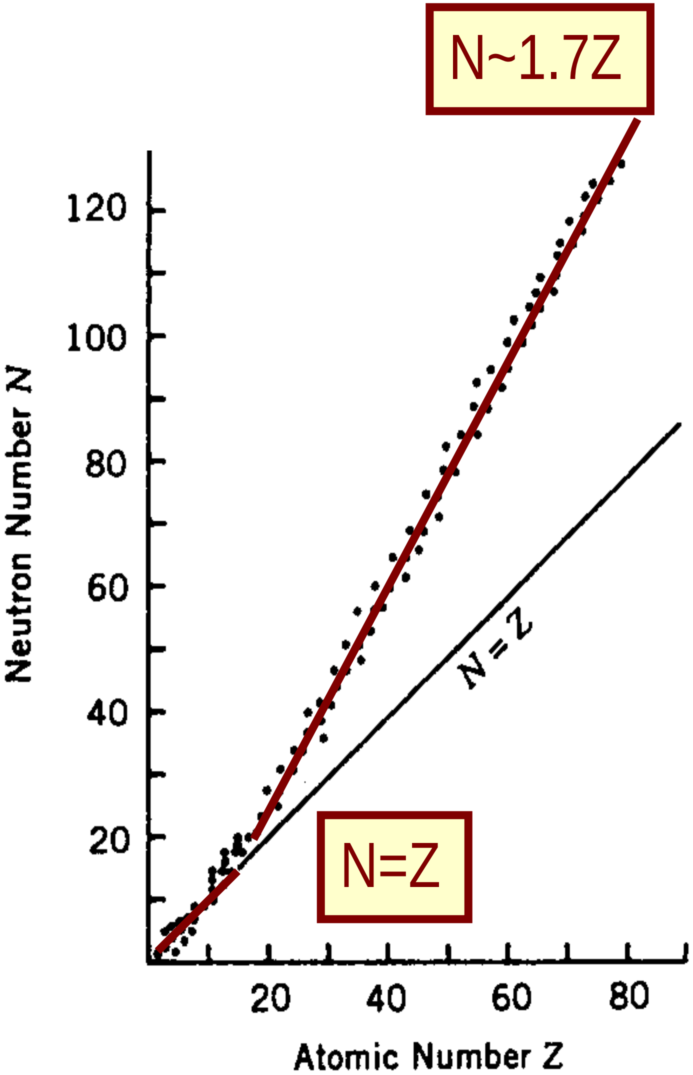
$$\mu_p \approx 2.79\mu_N \qquad \mu_n \approx -1.91\mu_N$$

Lec.8 Nuclear Properties: Stability

- The number of protons and neutrons inside stable nuclei are
  - A<40: Equal (N=Z)
  - A>40: N~1.7Z
  - Neutrons outnumber protons
  - Most are even-p + even–n

N	Z	N_{nucl}
Even	Even	156
Even	Odd	48
Odd	Even	50
Odd	Odd	5

    - See table 2.1
  - Supports strong pairing



Nuclear Potential

- A square well nuclear potential → provides the basis of quantum theory with discrete energy levels and corresponding bound state just like in atoms
  - Presence of nuclear quantum states have been confirmed through
    - Scattering experiments
    - Studies of the energies emitted in nuclear radiation
- Studies of mirror nuclei and the scatterings of protons and neutrons demonstrate
  - Aside from the Coulomb effects, the forces between two neutrons, two protons or a proton and a neutron are the same
    - Nuclear force has nothing to do with electrical charge
    - Protons and neutrons behave the same under the nuclear force
  - Inferred as <u>charge independence</u> of nuclear force.

Nuclear Radiation: Alpha Decay

- Represents the disintegration of a parent nucleus to a daughter through emission of a He nucleus
- Reaction equation is ${}^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}He^{2}$

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha$$

$$T_D + T_\alpha = \left(M_P - M_D - M_\alpha\right)c^2 = \Delta M c^2 = Q > 0$$

How does this compare to binding energy?

$$\Delta M\left(A,Z\right) = M\left(A,Z\right) - Zm_p - \left(A-Z\right)m_n < 0$$

Nuclear Radiation: Alpha Decay

lec 10-11

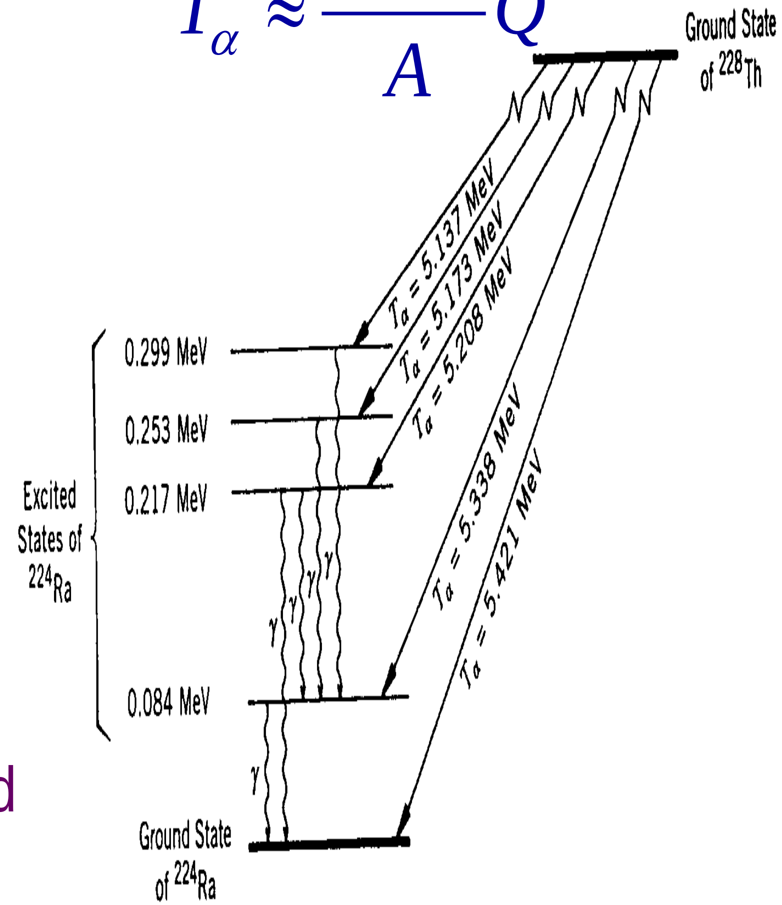
- Most energetic α-particles produced alone
  - Parent nucleus decays to the ground state of a daughter and produces an α-particle whose KE is $T_\alpha \approx \frac{A-4}{A} Q$
- Less energetic ones accompanied by delayed photons
  - Indicates quantum energy levels
  - Parent decays to an excited state of the daughter after emitting an α

    $$^{A}X^{Z} \rightarrow {}^{A-4}Y^{*Z-2} + {}^{4}He^{2}$$

  - Daughter then subsequently de-excite by emitting a photon

    $$^{A-4}Y^{*Z-2} \rightarrow {}^{A-4}Y^{Z-2} + \gamma$$

  - Difference in the two Q values correspond to photon energy



Nuclear Radiation: α-Decay Example

- ^{240}Pu94 decay reaction is

$$^{240}Pu^{94} \rightarrow {}^{236}U^{92} + {}^{4}He^{2}$$

- α particles observed with 5.17MeV and 5.12 MeV
- Since $Q = \frac{A}{A-4} T_{\alpha}$
- We obtain the two Q-values

$$Q_1 \approx \frac{240}{236} 5.17 MeV = 5.26 MeV \qquad Q_2 \approx \frac{240}{236} 5.12 MeV = 5.21 MeV$$

- Which yields a photon energy of $E_{\gamma} = \Delta Q = Q_1 - Q_2 = 0.05 MeV$
- Consistent with experimental measurement, 45KeV
- Indicates the energy level spacing of order 100KeV for nuclei
  - Compares to order 1eV spacing in atomic levels

β–Decay Reaction Equations with Neutrinos

lec. 12

- Electron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}_{e}$$

- Positron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu_{e}$$

- Electron capture

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu_{e}$$

Particle Numbers

- Baryon numbers: A quantum number assigned to baryons (protons, neutrons…)
  - Baryons +1
  - Anti-baryons: -1
  - Protons and neutrons are baryons with baryon number +1 each
- Experimentally observed that baryon number is conserved.
- Hadrons are strongly interacting particles (all baryons are hadrons, but not vice-versa)
- Baryons consist of three quarks

weak

- Mesons consist of a quark and an anti-quark

Lepton Number

- Three charged leptons exist in nature with their own associated neutrinos

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- These three types of neutrinos are distinct from each other
  - muon neutrinos never produce leptons other than muons or anti-muons

$$\nu_\mu + {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + \mu^-$$

$$\nu_\mu + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + e^-$$

$$\nu_\mu + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \tau^-$$