PHYS 1443 – Section 003
Lecture #2

Wednesday, Sept. 9, 2002
Dr. Jaehoon Yu

1. One dimensional motion w/ constant acceleration
2. Kinetimatic equations of motion
3. Free fall
4. Coordinate systems
5. Vector; its properties, operations and components
6. Two dimensional motion
   - Displacement, Velocity, and Speed
   - Projectile Motion
   - Uniform Circular Motion

Today's homework is homework #3, due 1am, next Monday!!

Announcements

- Your e-mail account is automatically assigned by the university, according to the rule: fml####@exchange.uta.edu. Just subscribe to the PHYS1443-003-FALL02.
- e-mail:15 of you have subscribed so far.
  - This is the primary communication tool. So do it ASAP.
  - A test message will be sent this Wednesday.
- Homework registration: 44 of you have registered (I have 56 of you)
  - Roster will be locked at the end of the day Wednesday, Sept. 11

One Dimensional Motion

- Let's start with the simplest case: acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can draw equations of motion (description of motion, velocity and position as a function of time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xf} - v_{xi}}{t}$$

If $t_f=t$ and $t_i=0$

$$\boxed{v_{xf} = v_{xi} + a_x t}$$

For constant acceleration, simple numeric average

$$\boxed{\overline{v_x} = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t}$$

$$\overline{v_x} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t}$$

If $t_f=t$ and $t_i=0$

$$\boxed{x_f = x_i + \overline{v_x} t}$$

Resulting Equation of Motion becomes

$$\boxed{x_f = x_i + \overline{v_x} t = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\overline{v_x}t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t}$$

Displacement as a function of velocity and time

$$\boxed{x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you for specific physical problems!!

Ex02-08.ip

Example 2.8

A car traveling at constant speed of 45.0m/s (~162km/hr or ~100miles/hr), police starts chasing the car at the constant acceleration of 3.00m/s^2, one second after the car passes him. How long does it take for police to catch the violator?

- Let's call the time interval for police to catch the car; T
- Set up an equation:Police catches the violator when his final position is the same as the violator's.

$$x_f^{Police} = \frac{1}{2}aT^2 = \frac{1}{2}\times 3.00T^2$$

$$x_f^{Car} = v(T+1) = 45.0(T+1)$$

$$x_f^{Police} = x_f^{Car}; \quad \frac{1}{2}\times 3.00T^2 = 45.0(T+1)$$

Solutions for $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1.00T^2 - 30.0T - 30.0 = 0$$

$$T = 31.0 \text{ (possible)} \quad \text{or } T = -1.00 \text{ (not possible)}$$

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Ex02-12.ip

Example 2.12

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

1. Find the time the stone reaches at maximum height (*v=0*)
2. Find the maximum height
3. Find the time the stone reaches its original height
4. Find the velocity of the stone when it reaches its original height
5. Find the velocity and position of the stone at t=5.00s

$g=-9.80 m/s^2$

1

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00$$

$$t = \frac{20.0}{9.80} = 2.04s$$

2

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

3

$$t = 2.04 \times 2 = 4.08s$$

Other ways?

4

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

5-Velocity

$$v_{yf} = v_{yi} + a_y t$$

$$= 20.0 + (-9.80) \times 5.00$$

$$= -29.0(m/s)$$

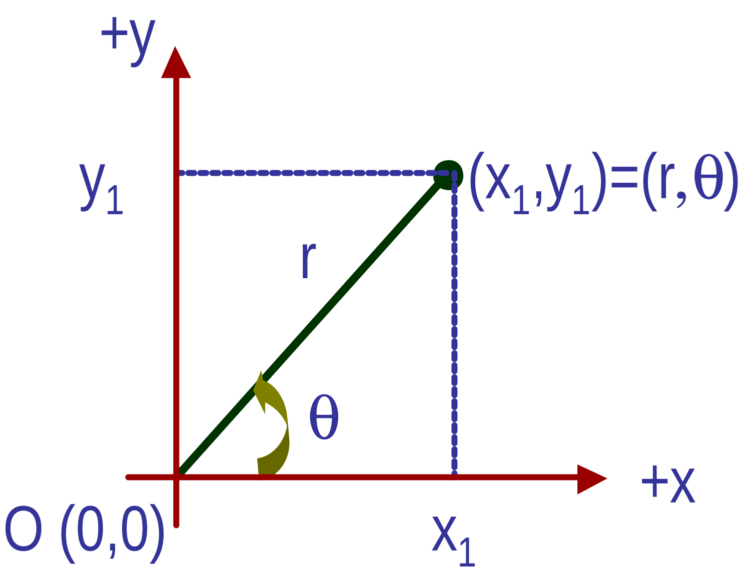
5-Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



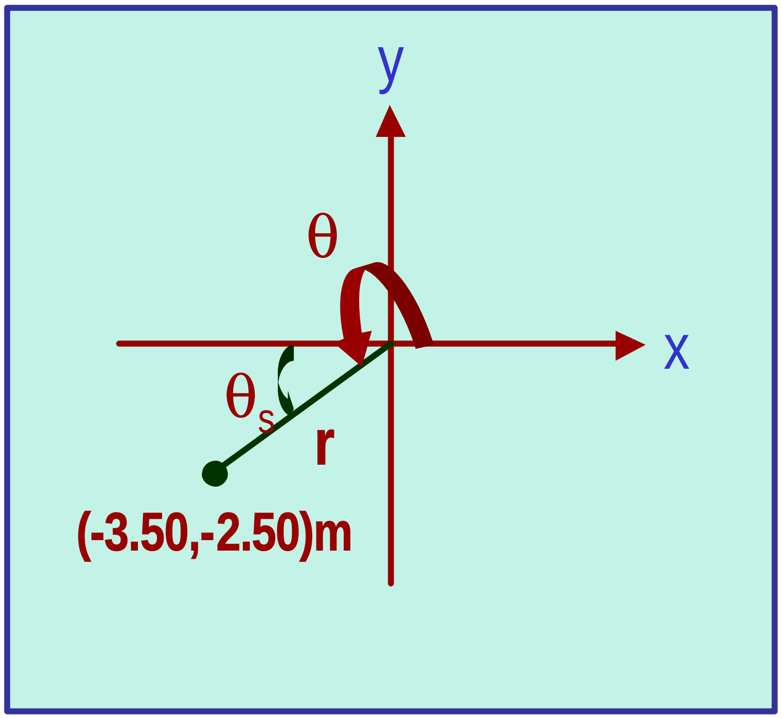
How are Cartesian and Polar coordinates related?

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$r = \sqrt{\left(x_1^2 + y_1^2\right)}$$
$$\tan\theta = \frac{y_1}{x_1}$$

Example 3.1

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$q = 180 + q_s$$

$$\tan q_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$q_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\boxed{\therefore q = 180 + q_s = 180^\circ + 35.5^\circ = 216^\circ}$$

Vector and Scalar

Vector quantities have both magnitude (size) and direction

Force, gravitational pull, momentum

Normally denoted in **BOLD** letters, ***F***, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, *F*, or absolute values: $\left|\vec{F}\right|$ or $|F|$

Scalar quantities have magnitude only

Can be completely specified with a value and its unit

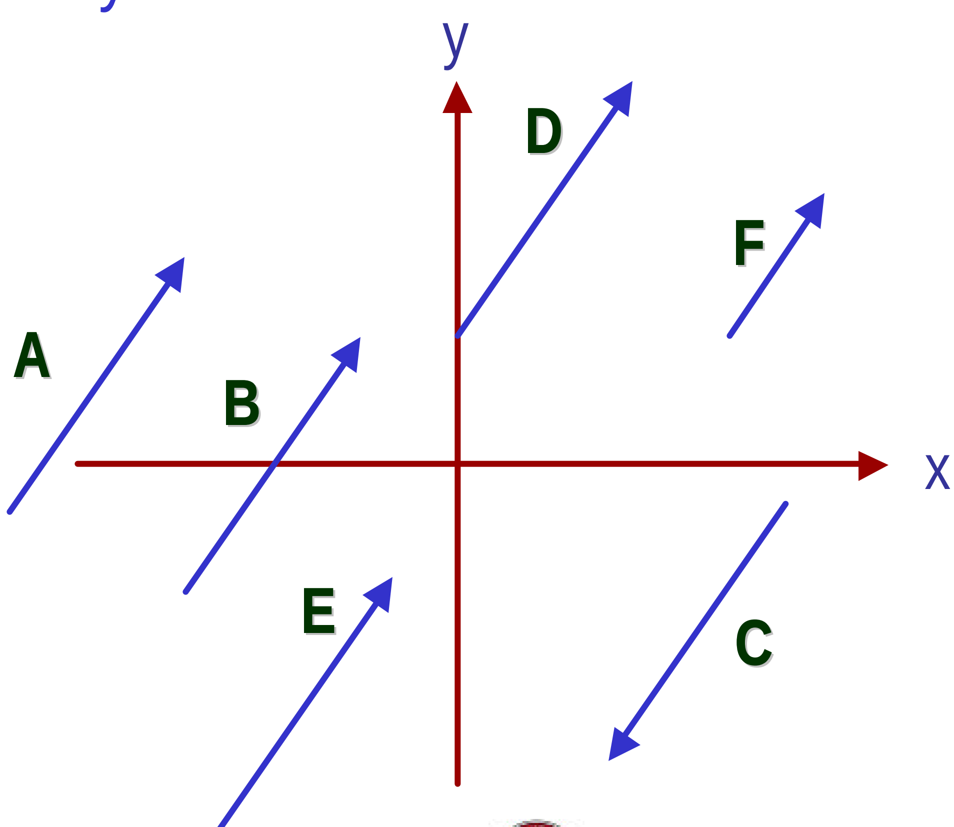
Energy, heat, mass, weight

Normally denoted in normal letters, *E*

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system.



Which ones are the same vectors?

A=B=E=D

Why aren't the others?

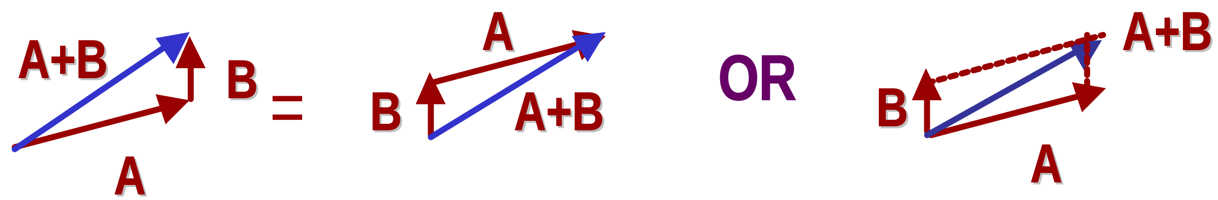
C: The same magnitude but opposite direction: **C=-A:** A negative vector

F: The same direction but different magnitude

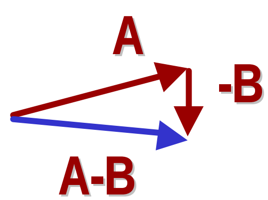
Fig03-07.ip

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



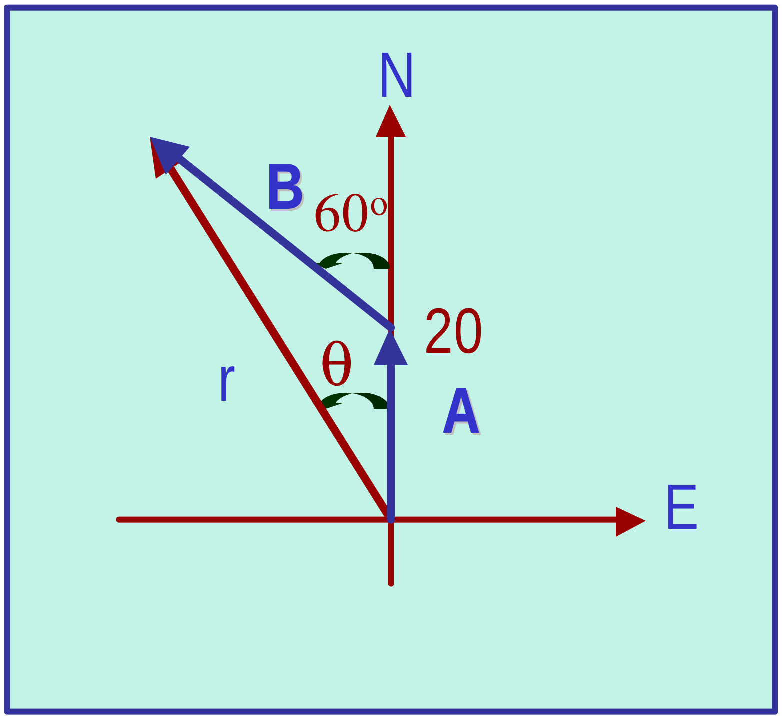
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A, B**=2**A**

$$\boxed{|B| = 2|A|}$$

Example 3.2

A car travels 20.0km due north followed by 35.0km in a direction 60.0º west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{|\vec{B}|\sin 60}{|\vec{A}| + |\vec{B}|\cos 60}$$

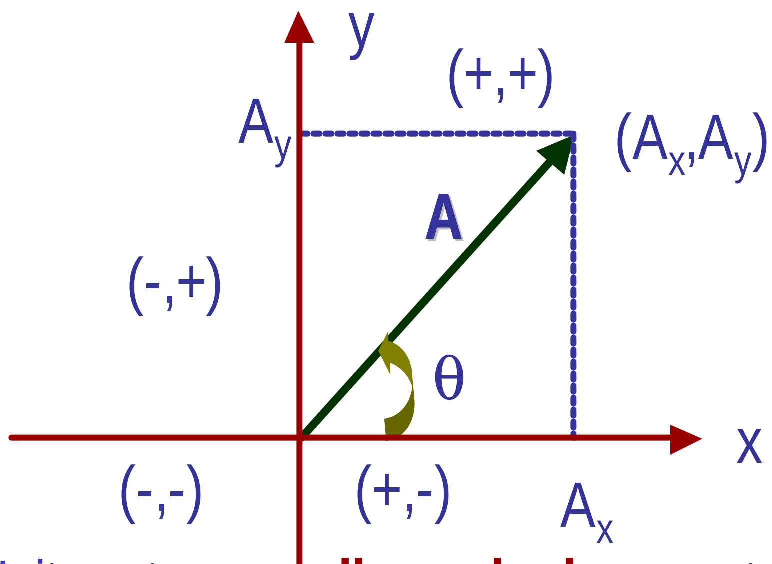
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

- Coordinate systems are useful in expressing vectors in their components



$$\left.\begin{aligned} A_x &= \left|\vec{A}\right| \cos\theta \\ A_y &= \left|\vec{A}\right| \sin\theta \end{aligned}\right\} \text{Components}$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$
$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

- Unit vectors are <u>**dimensionless**</u> vectors whose <u>**magnitude are exactly 1**</u>
  - Unit vectors are usually expressed in **i,j,k** or $\vec{i}\,,\ \vec{j}\,,\ \vec{k}$
  - Vectors can be expressed using components and unit vectors

So the above vector **A** can be written as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples 3.3 & 3.4

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$
$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = \left(4.0\vec{i} - 2.0\vec{j}\right)m$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2} = \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$q = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements:
d$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_1$=(-13**i**+15**j**)cm

$$\vec{D} = \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k}$$
$$= 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(cm)$$

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$
$$= \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$\left|\vec{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65\,(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?