PHYS 1443 – Section 003
Lecture #3

Wednesday, Sept. 11, 2002
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1. 2-D Displacement, Velocity and Speed
2. Projectile Motion
3. Uniform Circular Motion
4. Nonuniform Circular Motion
5. Relative Motion

Today's homework is homework #4, due 1am, next Wednesday!!

Announcements

- e-mail:24 of you have subscribed so far.
  - This is the primary communication tool. So do it ASAP.
  - A test message will be sent this Wednesday.
- Homework registration: 45 of you have registered (I have 56 of you)
  - Roster will be locked at the end of the day today (5:30pm), Sept. 11

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\overline{v_x}t = \frac{1}{2}(v_{xf} + v_{xi})t$$

Displacement as a function of velocity and time

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you for specific physical problems!!

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is <u>**ALWAYS**</u> toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable “y”
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
- Average Velocity: $\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
- Instantaneous Velocity: $\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
- Average Acceleration: $\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
- Instantaneous Acceleration: $\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$

How is each of these quantities defined in 1-D?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane: $\vec{r}_i = x_i\vec{i} + y_i\vec{j}$ $\vec{r}_f = x_f\vec{i} + y_f\vec{j}$
- Velocity vectors in x-y plane: $\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j}$ $\vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$

Velocity vectors in terms of acceleration vector

$$v_{xf} = v_{xi} + a_x t \qquad v_{yf} = v_{yi} + a_y t$$

$$\vec{v}_f = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j} = \vec{v}_i + \vec{a}t$$

- How are the position vectors written in acceleration vectors?

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{r}_f = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

$$\vec{r}_f = (x_i\vec{i} + y_i\vec{j}) + (v_{xi}\vec{i} + v_{yi}\vec{j})t + \frac{1}{2}(a_x\vec{i} + a_y\vec{j})t^2 = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Ex04-01.ip

Example 4.1

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\,(m/s)$$

$$v_{yf}=v_{yi}+a_y t=-15\,(m/s)$$

$$\vec{v}(t)=\left\{(20+4.0t)\vec{i}-15\vec{j}\right\}m/s$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}=\left\{(20+4.0\times 5.0)\vec{i}-15\vec{j}\right\}m/s=\left(40\vec{i}-15\vec{j}\right)m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(40)^2+(-15)^2}=43\ m/s$$

$$\theta=\tan^{-1}\left(\frac{-15}{40}\right)=\tan^{-1}\left(\frac{-3}{8}\right)=-21^{\circ}$$

Determine the *x* and *y* components of the particle at t=5.0 s.

$$x_f=v_{xi}t+\frac{1}{2}a_x t^2=20\times 5+\frac{1}{2}\times 4\times 5^2=150\,(m)$$

$$y_f=v_{yi}t=-15\times 5=-75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f=x_f\vec{i}+y_f\vec{j}=\left(150\vec{i}-75\vec{j}\right)m$$

Fig04-06.ip

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the assumptions
  – Free fall acceleration, -*g*, is constant over the range of the motion
  – Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  – Horizontal motion with constant velocity and
  – Vertical motion under constant acceleration

Show that a projectile motion is a parabola!!!

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$$v_{xi} = v_i \cos\theta_i; \quad v_{yi} = v_i \sin\theta_i$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} = -g\vec{j}$$

$a_x=0$ →

$$x_f = v_{xi} t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi} t + \frac{1}{2}(-g)t^2 = v_i \sin\theta_i t - \frac{1}{2} g t^2$$

Plug in the t above →

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2} g \left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2 v_i^2 \cos^2\theta_i}\right) x_f^2$$

What kind of parabola is this?

Example 4.2

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by *y* component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\text{ sec}$$

Distance is determined by *x* component in 2-dim, because the ball is at *y=0* position when it completed it's flight.

$$x_f = v_{xi}t$$

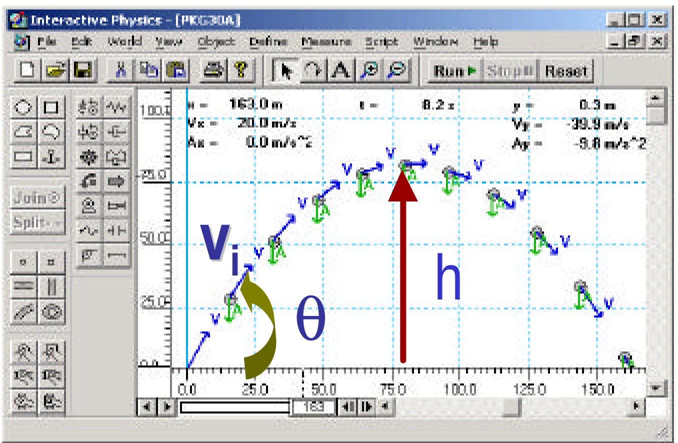
$$= 20 \times 8 = 160\ (m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta_i - g t_A = 0$$

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

$$y_f = h = v_{yi} t + \frac{1}{2}(-g)t^2$$

$$y_f = v_i \sin\theta_i \left(\frac{v_i \sin\theta_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$y_f = \left(\frac{v_i^2 \sin^2\theta_i}{2g}\right)$$

Since no acceleration in x, it still flies even if $v_y=0$

$$R = v_{xi} 2 t_A = 2 v_i \cos\theta_i \left(\frac{v_i \sin\theta_i}{g}\right)$$

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g}\right)$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when θ_i=90°!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i$=90°, i.e., θ_i=45°!!!

Ex04-05a.ip

Example 4.5

- A stone was thrown upward from the top of a building at an angle of 30º to horizontal with initial speed of 20.0m/s. If the height of the building is 45.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos q_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yi} = v_i \sin q_i = 20.0 \times \sin 30^\circ = 10.0 m/s$$

$$y_f = -45.0 = v_{yi} t - \frac{1}{2} g t^2$$

$$gt^2 - 20.0t - 90.0 = 9.80t^2 - 20.0t - 90.0 = 0$$

$$t = \frac{20.0 \pm \sqrt{(-20)^2 - 4 \times 9.80 \times (-90)}}{2 \times 9.80}$$

$$t = -2.18 s \quad \text{or} \quad t = 4.22 s$$

$$t = 4.22 s$$

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos q_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin q_i - gt = 10.0 - 9.80 \times 4.22 = -31.4 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.3^2 + (-31.4)^2} = 35.9 m/s$$