

PHYS 1443 – Section 003

Lecture #4

Monday, Sept. 16, 2002

Dr. Jaehoon Yu

1. Uniform Circular Motion
2. Nonuniform Circular Motion
3. Relative Motion
4. Force
5. Newton's Laws of Motion

Today's homework is homework #5, due 1am, next Monday!!



Announcements

- Homework registration: 46 have registered
 - Roster has been locked at end of the day, Wednesday
 - Come see me if you need to register
 - You all have been doing very well (~80% average)!! Keep up the good work!!
- e-mail: 27 of you have subscribed so far.
 - A test message announcing the locking of the HW roster has been sent out.
 - 14 of you have replied for verification (Thank you!!)
 - This is the primary communication tool. So do it ASAP.
- Pre-lab paper??
 - URL included in your instruction sheets
 - Posted on my web page
- Posting of Lecture notes prior to the class?
 - I am sorry but the answer is NO!
 - I suggest you to read the book before the class
 - Concentrate on Example problems
 - Do it yourself without looking at the explanation to keep up in the class
- Remember the first term exam on **Sept. 30** in the class
- David Hunt, please come and see me after the class

Monday, Sept. 16, 2002



PHYS 1443-003, Fall 2002
Dr. Jaehoon Yu

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2} \bar{v}_x t = \frac{1}{2} (v_{xf} + v_{xi}) t$$

Displacement as a function of velocity and time

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you for specific physical problems!!



Displacement, Velocity, and Acceleration in 2-dim

- Displacement:

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

- Average Velocity:

$$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

- Instantaneous Velocity:

$$\vec{v} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Average Acceleration

$$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

- Instantaneous Acceleration:

$$\vec{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{r}}{dt} \right) = \frac{d^2 \vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?



Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 q_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^\circ$!!!

$$R = \left(\frac{v_i^2 \sin 2q_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^\circ$, i.e., $\theta_i = 45^\circ$!!!



Uniform Circular Motion

- A motion with a constant speed on a circular path.
 - The velocity of the object changes, because the direction changes
 - Therefore, there is an acceleration



The acceleration pulls the object inward: *Centripetal Acceleration*

Average
Acceleration

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta q = \frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r}$$

$$|\Delta \vec{v}| = v \frac{|\Delta \vec{r}|}{r}$$

$$\bar{a} = \frac{v}{\Delta t} \frac{|\Delta \vec{r}|}{r}$$

Instantaneous
Acceleration

$$a_r = \lim_{\Delta t \rightarrow 0} \bar{a} = \lim_{\Delta t \rightarrow 0} \frac{|\Delta \vec{r}|}{\Delta t} \frac{v}{r} = v \times \frac{v}{r} = \frac{v^2}{r}$$

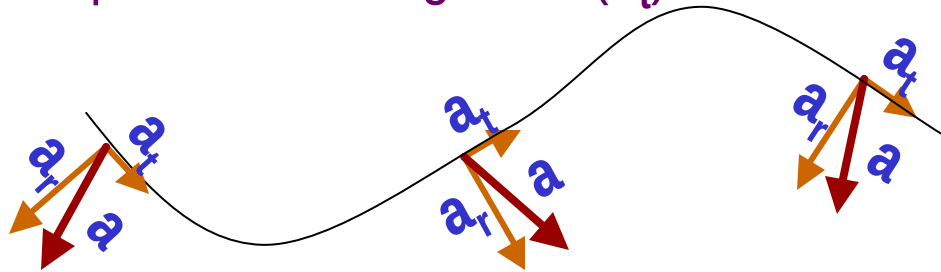
Is this correct in
dimension?

What story is this expression telling you?



Non-uniform Circular Motion

- Motion not on a circle but through a curved path
 - Requires both tangential ($\mathbf{a}_t$) and radial acceleration ($\mathbf{a}_r$)



Tangential Acceleration:

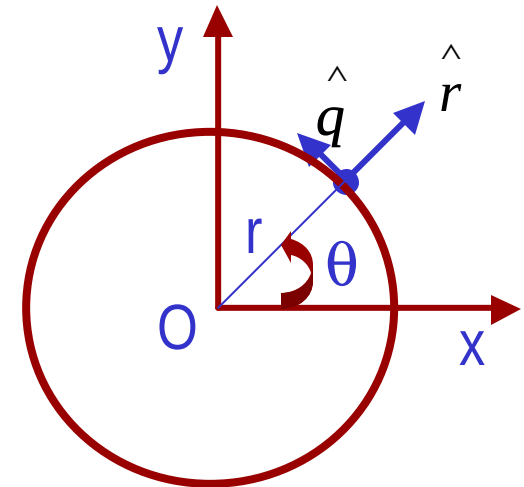
$$a_t = \frac{d|\vec{v}|}{dt}$$

Radial Acceleration:

$$a_r = \frac{v^2}{r}$$

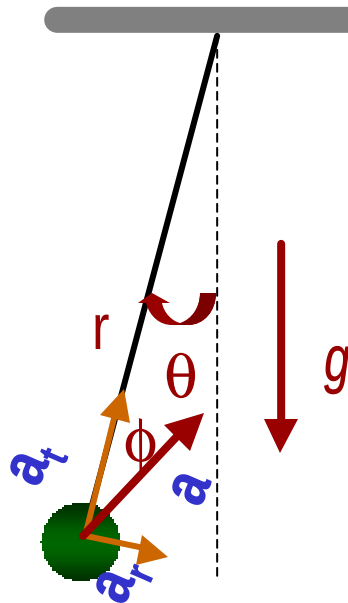
Total Acceleration:

$$\vec{a} = \vec{a}_r + \vec{a}_t = \frac{d|\vec{v}|}{dt} \hat{q} - \frac{v^2}{r} \hat{r}$$



Example 4.8

A ball tied to the end of a string of length 0.5m swings in a vertical circle under the influence of gravity, $-g$. When the string makes an angle $\theta=20^\circ$ wrt vertical axis the ball has a speed of 1.5m/s. Find the **magnitude of the radial component of acceleration** at this time.



$$a_r = \frac{v^2}{r} = \frac{(1.5)^2}{0.5} = 4.5 (m / s^2)$$

What is the magnitude of tangential acceleration when $\theta=20^\circ$?

$$a_t = g \sin \theta = g \sin(20^\circ) = 3.4 m / s^2$$

Find the magnitude and direction of the total acceleration a at $\theta=20^\circ$.

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} = 5.6 (m / s^2)$$

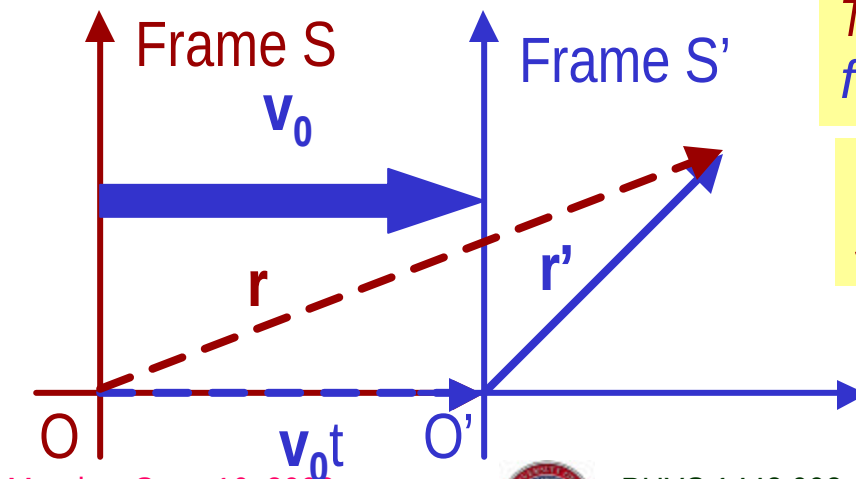
$$f = \tan^{-1} \left(\frac{a_t}{a_r} \right) = \tan^{-1} \left(\frac{3.4}{4.5} \right) = 37^\circ$$

Observations in Different Reference Frames

Results of Physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you, the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



The position vector r' is still r' in the moving frame S' . no matter how much time has passed!!

The position vector r is no longer r in the stationary frame S when time t has passed.

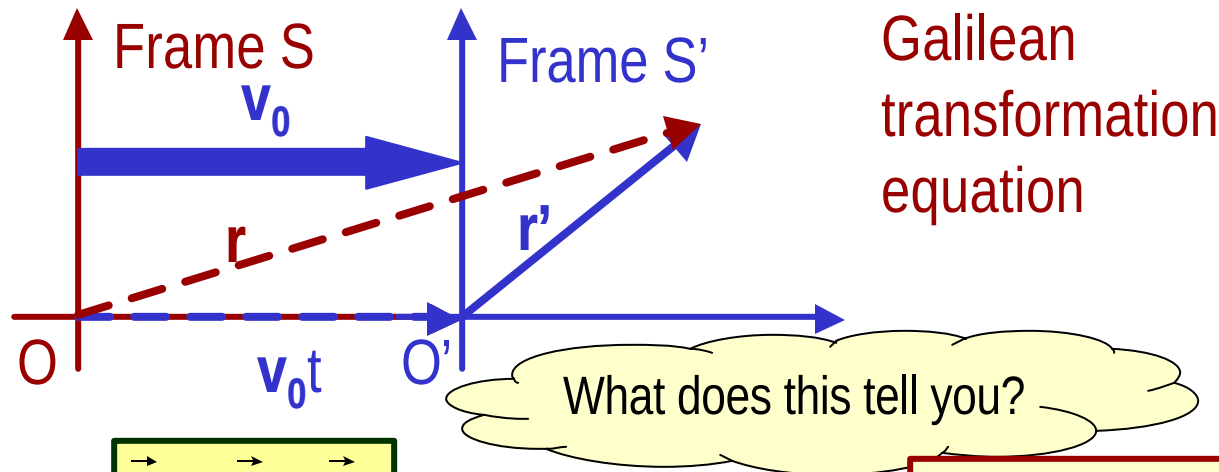
How are these position vectors related to each other?

$$\vec{r}(t) = \vec{r}' + \vec{v}_0 t$$



Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$$\vec{a}' = \vec{a}, \text{ when } \vec{v}_0 \text{ is constant}$$

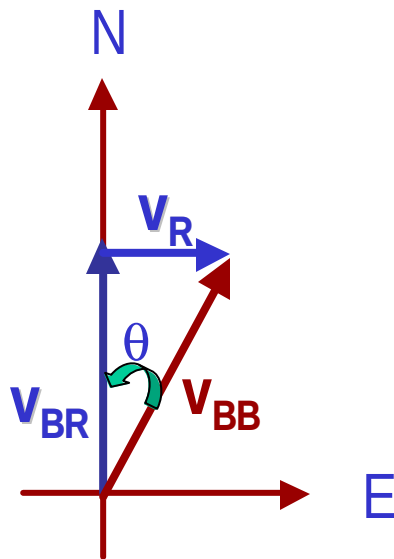
The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.



Example 4.9

A boat heading due north with a speed 10.0km/h is crossing the river whose stream has a uniform speed of 5.00km/h due east. Determine the velocity of the boat seen by the observer on the bank.



$$\vec{v}_{BB} = \vec{v}_{BR} + \vec{v}_R$$

$$|\vec{v}_{BB}| = \sqrt{|\vec{v}_{BR}|^2 + |\vec{v}_R|^2} = \sqrt{(10.0)^2 + (5.00)^2} = 11.2 \text{ km/h}$$

$$\therefore \vec{v}_{BR} = 10.0 \hat{j} \text{ and } \vec{v}_R = 5.00 \hat{i}$$

$$\vec{v}_{BB} = 5.00 \hat{i} + 10.0 \hat{j}$$

$$q = \tan^{-1} \left(\frac{v_{BBY}}{v_{BBX}} \right) = \tan^{-1} \left(\frac{5.00}{10.0} \right) = 26.6^\circ$$

How long would it take for the boat to cross the river if the width is 3.0km?

$$v_{BB} \cos q \cdot t = 3.0 \text{ km}$$

$$t = \frac{3.0}{v_{BB} \cos q} = \frac{3.0}{11.2 \times \cos(26.6^\circ)} = 0.30 \text{ hrs} = 18 \text{ min}$$



Force

We've been learning kinematics; describing motion without understanding what the cause of the motion was. Now we are going to learn dynamics!!

~~FORCES~~ are what cause an object to move

Can someone tell me what **FORCE** is?

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

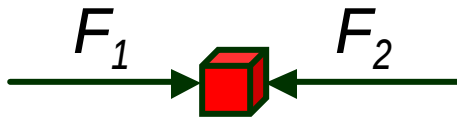
FORCES are what cause any change in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Forces cause acceleration.

What happens there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the **NET FORCE**, determines the motion of the object.



NET FORCE,
 $F = F_1 + F_2$

When net force on an object is 0, it has constant velocity and is at its equilibrium!!



More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.



Newton's First Law and Inertial Frames

Galileo's statement on natural states of matter:

Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!

This statement is formulated by Newton into the **1st law of motion (Law of Inertia)**:

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

What does this statement tell us?

1. *When no force is exerted on an object, the acceleration of the object is 0.*
2. *Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.*
3. *Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the **Inertia**.*

A frame of reference that is moving at constant velocity is called an **Inertial Frame**



Mass

Mass: An inherent property of an object

1. Independent of the object's surroundings: The same no matter where you go.
2. Independent of method of measurement: The same no matter how you measure it

The heavier an object gets the bigger the inertia!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.



Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x$$

$$\sum_i F_{iy} = ma_y$$

$$\sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is

$$[m][a] = [M][LT^{-2}]$$

The unit of force in SI is

$$[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m / s^2$$

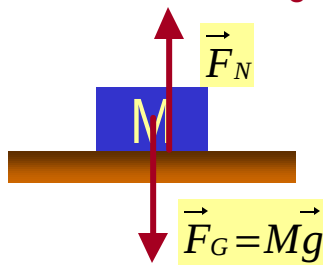
See Table 5.1 for lbs to kgm/s² conversion.

$$1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m} / \text{s}^2 \approx \frac{1}{4} \text{ lbs}$$



Free Body Diagrams

- Diagrams of vector forces acting on an object
- ⇒ A great tool to solve a problem using forces or using dynamics
- 1. Select a point on an object (preferably the one with mass) and w/ information given
- 2. Identify all the forces acting only on the selected object
- 3. Define a reference frame with positive and negative axes specified
- 4. Draw arrows to represent the force vectors on the selected point
- 5. Write down net force vector equation
- 6. Write down the forces in components to solve the problems
- ⇒ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion

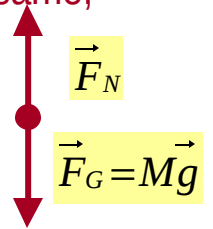


Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?

Gravitational force

A force supporting the object exerted by the floor

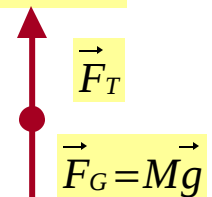


Which one would you like to select to draw FBD?

What do you think are the forces acting on this elevator?

Gravitational force

The force pulling the elevator (Tension)



What about the box in the elevator?

Gravitational force

Normal force

