PHYS 1443 – Section 003
Lecture #5

Monday, Sept. 18, 2002
Dr. Jaehoon Yu

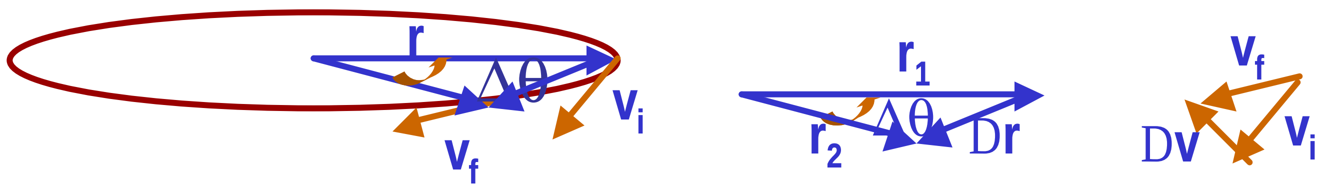
1. Newton's Laws of Motion
2. Application of Newton's Laws
3. Friction
4. Newton's laws and its use in uniform and non-uniform circular motion

Today's homework is homework #6, due 1am, next Wednesday!!

Remember the first term exam on Monday, Sept. 30!!

Uniform Circular Motion

- A motion with a constant speed on a circular path.
  - The velocity of the object changes, because the direction changes
  - Therefore, there is an acceleration



The acceleration pulls the object inward: *Centripetal Acceleration*

Average Acceleration

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta q = \frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r}$$

$$|\Delta \vec{v}| = v\frac{|\Delta \vec{r}|}{r}$$

$$\bar{a} = \frac{v}{\Delta t}\frac{|\Delta \vec{r}|}{r}$$

Instantaneous Acceleration

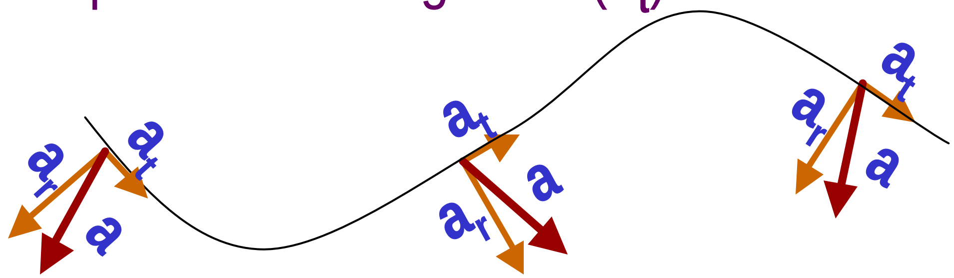
$$a_r = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t}\frac{v}{r} = v \times \frac{v}{r} = \frac{v^2}{r}$$

Is this correct in dimension?

What story is this expression telling you?

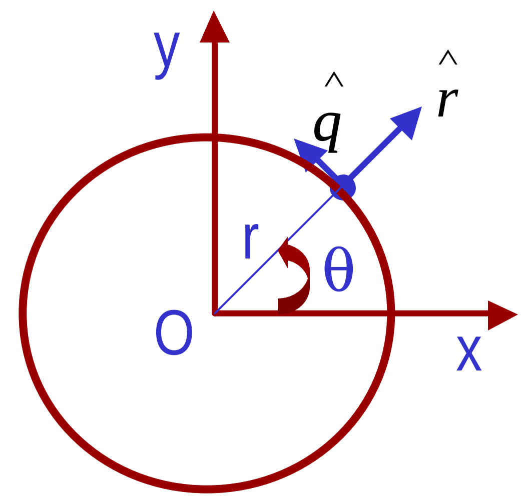
Non-uniform Circular Motion

- Motion not on a circle but through a curved path
  - Requires both tangential (**a_t**) and radial acceleration (**a_r**)



Tangential Acceleration: $$\boxed{a_t = \frac{d\left|\vec{v}\right|}{dt}}$$

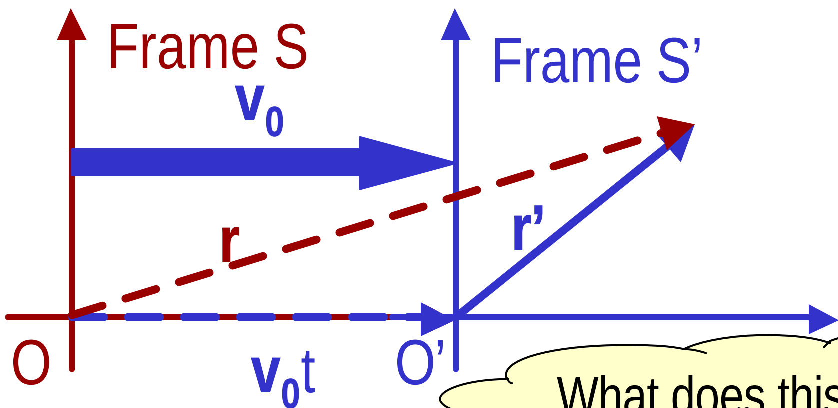
Radial Acceleration: $$\boxed{a_r = \frac{v^2}{r}}$$



Total Acceleration: $$\boxed{\vec{a} = \vec{a}_r + \vec{a}_t = \frac{d\left|\vec{v}\right|}{dt}\hat{q} - \frac{v^2}{r}\hat{r}}$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\vec{v}' = \vec{v} - \vec{v}_0$$

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$$\vec{a}' = \vec{a}, \text{ when } \quad \vec{v}_0 \text{ is constant}$$

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Newton's First Law and Inertial Frames

Galileo's statement on natural states of matter:

Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!

This statement is formulated by Newton into the **1st law of motion (Law of Inertia)**:

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

What does this statement tell us?

1. *When no force is exerted on an object, the acceleration of the object is 0.*
2. *Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.*
3. *Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.*

A frame of reference that is moving at constant velocity is called an *Inertial Frame*

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is

$$[m][a] = [M][LT^{-2}]$$

The unit of force in SI is

$$[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$$

$$1\ N \equiv 1\ kg \cdot m/s^2 \approx \frac{1}{4}\ lbs$$

See Table 5.1 for lbs to kgm/s^2 conversion.

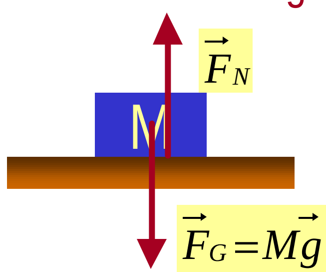
Free Body Diagrams

- Diagrams of vector forces acting on an object

⇒ A great tool to solve a problem using forces or using dynamics

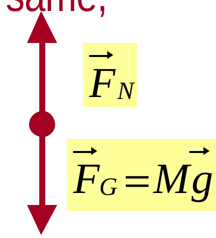
1. Select a point on an object and w/ information given
2. Identify all the forces acting only on the selected object
3. Define a reference frame with positive and negative axes specified
4. Draw arrows to represent the force vectors on the selected point
5. Write down net force vector equation
6. Write down the forces in components to solve the problems

⇒ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion



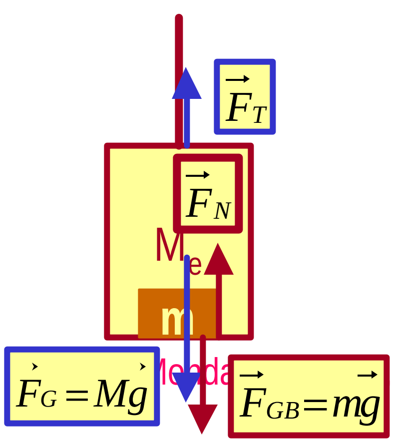
Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?



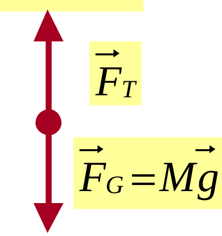
Gravitational force

A force supporting the object exerted by the floor



Which one would you like to select to draw FBD?

What do you think are the forces acting on this elevator?



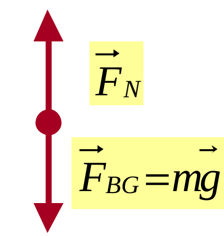
Gravitational force

The force pulling the elevator (Tension)

What about the box in the elevator?

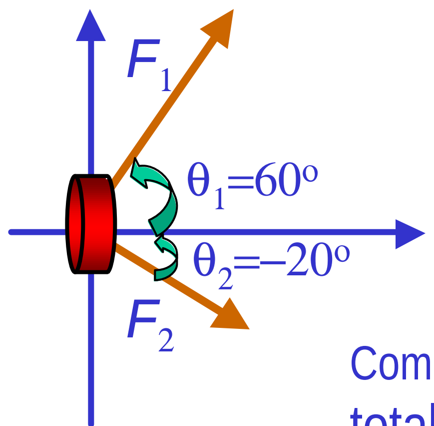
Gravitational force

Normal force



Example 5.1

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of F_1

$$F_{1x} = \left|\vec{F_1}\right| \cos q_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0\,N$$

$$F_{1y} = \left|\vec{F_1}\right| \sin q_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9\,N$$

Components of F_2

$$F_{2x} = \left|\vec{F_2}\right| \cos q_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7\,N$$

$$F_{2y} = \left|\vec{F_2}\right| \sin q_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7\,N$$

Components of total force F

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7\,N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2\,N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29\,m/s^2$$

$$a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17\,m/s^2$$

$$\left|\vec{a}\right| = \sqrt{(29)^2 + (17)^2} = 34\,m/s^2$$

$$q = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right) m/s^2$$

Gravitational Force and Weight

Gravitational Force, F_g

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

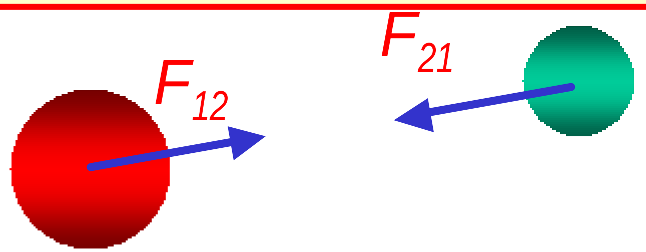
Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using spring scale.

Actual unit of weight is in the unit of force but the unit of mass is commonly used in place of force.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force, F_{12}, exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force, F_{21}, exerted on object 1 by object 2.



$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act on different objects.

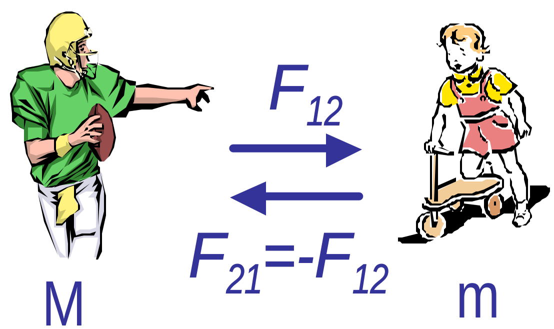
What is the reaction force to the force of a free fall object?

The force exerted by the ground when it completed the motion.

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example 5.3

A large man and a small boy stand facing each other on **frictionless ice**. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_{b} \qquad F_{12x} = ma_{bx} \qquad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_{M} \qquad F_{21x} = Ma_{Mx} \qquad F_{21y} = Ma_{My} = 0$$

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F \qquad a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

$$\therefore\ v_{bxf} \rangle v_{Mxf} \quad \text{if} \quad M \rangle m \quad \text{by the ratio of the masses}$$

b) Who moves farther while their hands are in contact?

$$x_b = v_{bxf}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{m}v_{Mxf}t + \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(v_{Mxf}t + \frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external forces are applied.

Normal Force, *n:*

Reaction force that balances gravitational force, keeping objects stationary.

Tension, *T:*

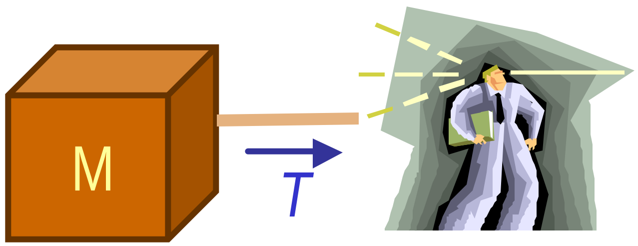
Magnitude of the force exerted on an object by a string or a rope.

Free-body diagram

*A graphical tool which is a **diagram of external forces on an object** and is extremely useful analyzing forces and motion!! Drawn only on the object.*

Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



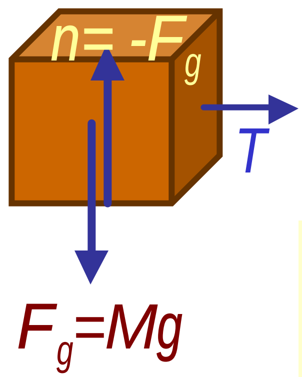
What are the forces being exerted on the box?

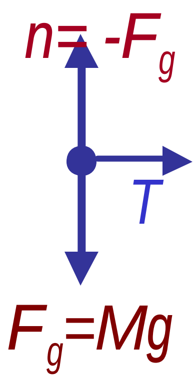
Gravitational force: F_g

Normal force: n

Tension force: T

Free-body diagram





Total force:
$F=F_g+n+T=T$

$$\sum F_x = T = Ma_x \qquad a_x = \frac{T}{M}$$

$$\sum F_y = -F_g + n = Ma_y = 0 \qquad a_y = 0$$

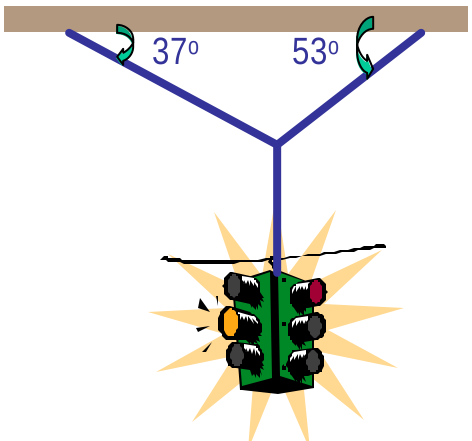
If T is a constant force, a_x, is constant

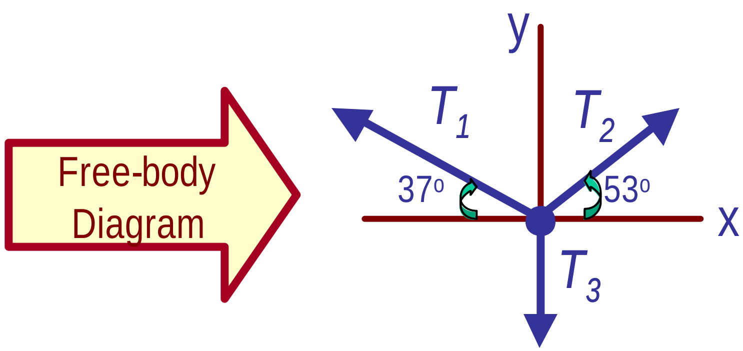
$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

Example 5.4

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0^o and 53.0^o with the horizontal. Find the tension in the three cables.





$$\vec{F} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3$$

Newton's 2nd law

x-comp. of net force

$$F_x = \sum_{i=1}^{i=3} T_{ix} = 0$$

$$-T_1 \cos(37^\circ) + T_2 \cos(53^\circ) = 0$$

$$\therefore T_1 = \frac{\cos(53^\circ)}{\cos(37^\circ)} T_2 = 0.754\, T_2$$

y-comp. of net force

$$F_y = \sum_{i=1}^{i=3} T_{iy} = 0$$

$$T_1 \sin(37^\circ) + T_2 \sin(53^\circ) - mg = 0$$

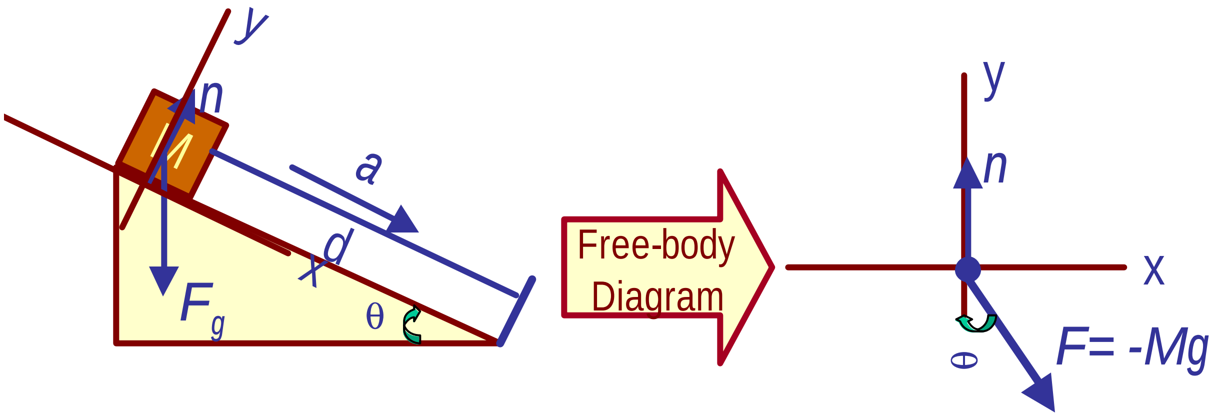
$$T_2 [\sin(53^\circ) + 0.754 \times \sin(37^\circ)] = 1.25 T_2 = 125\, N$$

$$T_2 = 100\, N; \quad T_1 = 0.754\, T_2 = 75.4\, N$$

Example 5.6

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.



$$\vec{F} = \vec{F_g} + \vec{n} = m\vec{a}$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$a_x = g\sin\theta$$

$$F_y = Ma_y = n - F_{gy} = n - mg\cos\theta = 0$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta\, t^2$$

$$\therefore t = \sqrt{\frac{2d}{g\sin\theta}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$