# PHYS 1443 – Section 003
# Lecture #6

*Wednesday, Oct. 2, 2002*
*Dr. Jaehoon Yu*

1. Newton's laws and its use in uniform and non-uniform circular motion
2. Motion in Accelerated Frames
3. Motion in Resistive Forces
4. Numerical Modeling in Particle Dynamics (Euler Method)

Today's homework is homework #7, due 12:30pm, next Wednesday!!

# Announcements

- Due time for homework will be changed from <u>1am to 12pm</u> on the due day, if everyone prefers this…

- Term Exam
  - Exam grading not complete yet.  Will be done by next Monday
  - All scores are relative based on the curve
    - To take into account the varying difficulties of exams
    - This average will not be skewed by one or two outliers
  - Only two best of the three will be used for your final grading, after adjusting each exam scores to the overall average
  - Exam constitutes only 50% of the total
    - Do your homework well
    - Come to the class and do well with quizzes

# Newton's Second Law & Uniform Circular Motion
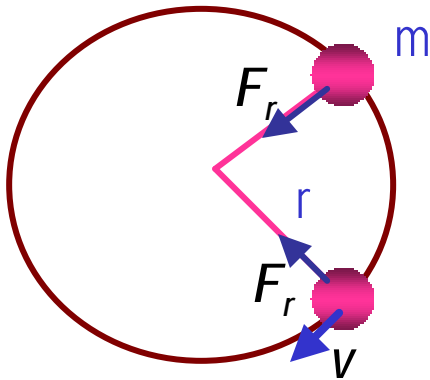


The centripetal acceleration is always perpendicular to velocity vector, **v**, for uniform circular motion.

$$a_r = \frac{v^2}{r}$$

*Are there forces in this motion? If so, what do they do?*

The force that causes the centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. This force is called **centripetal force.**

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

*What do you think will happen to the ball if the string that holds the ball breaks? Why?*

Based on Newton's 1st law, since the external force no longer exist, the ball will continue its motion without change and will fly away following the tangential direction to the circle.

# Example 6.2

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?
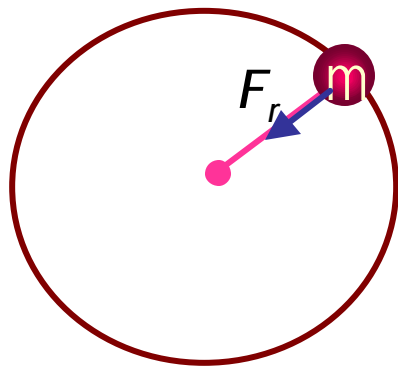


*Centripetal acceleration:*

$$a_r = \frac{v^2}{r}$$

*When does the string break?*

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

*When the centripetal force is greater than the sustainable tension.*
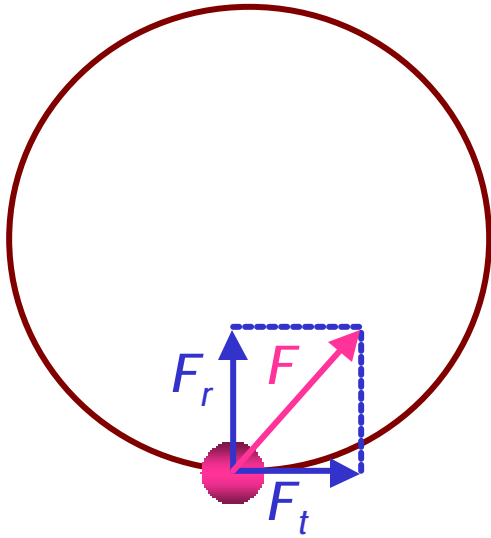
$$m\frac{v^2}{r} = T$$

$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

# Forces in Non-uniform Circular Motion

*The object has both tangential and radial accelerations.*

*What does this statement mean?*

*The object is moving under both tangential and radial forces.*

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

*These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.*

# Example 6.8

A ball of mass m is attached to the end of a cord of length R on Earth. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is $v$ and the cord makes an angle $\theta$ with vertical.

**What are the forces involved in this motion?**
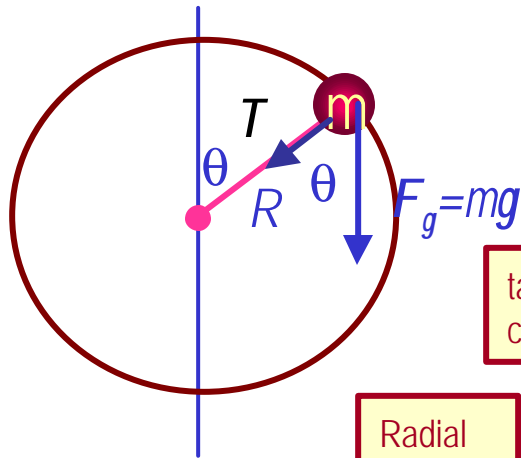
*The gravitational force $F_g$ and the radial force, $T$, providing tension.*

tangential comp.
$$\sum F_t = mg \sin \theta = ma_t \qquad a_t = g\sin\theta$$

Radial comp.
$$\sum F_r = T - mg\cos\theta = ma_r = m\frac{v^2}{R} \qquad T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

At what angles the tension becomes maximum and minimum. What are the tension?

# Motion in Accelerated Frames

Newton's laws are valid only when observations are made in an inertial frame of reference. What happens in a non-inertial frame?

*Fictitious forces are needed to apply Newton's second law in an accelerated frame.*

*This force does not exist when the observations are made in an inertial reference frame.*

Let's consider a free ball inside a box under uniform circular motion.

**What does this mean and why is this true?**
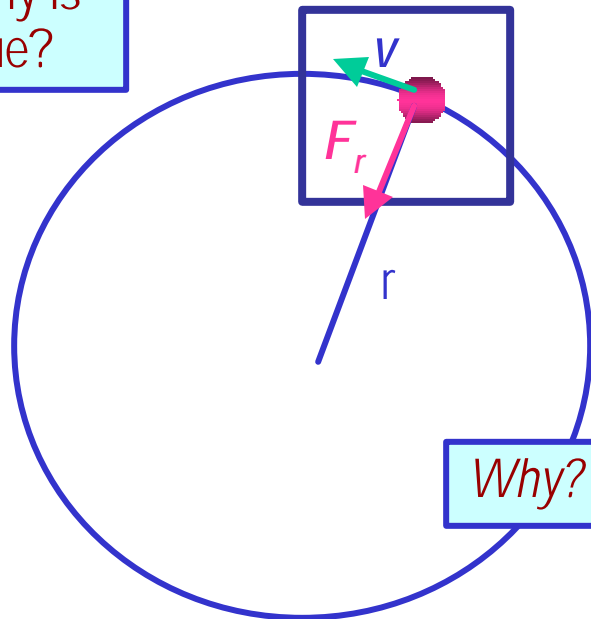


**How does this motion look like in an inertial frame (or frame outside a box)?**

*We see that the box has a radial force exerted on it but none on the ball directly, until…*

**How does this motion look like in the box?**

*The ball is tumbled over to the wall of the box and feels that it is getting force that pushes it toward the wall.*

*Why?*

According to Newton's first law, the ball wants to continue on its original movement tangentially but since the box is turning, the ball feels like it is being pushed toward the wall relative to everything else in the box.
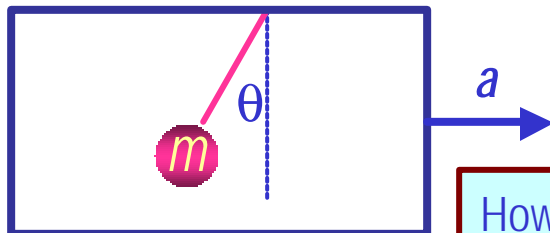
# Example 6.9

A ball of mass m is hung by a cord to the ceiling of a boxcar that is moving with an acceleration *a*. What do the inertial observer at rest and the non-inertial observer traveling inside the car conclude? How do they differ?
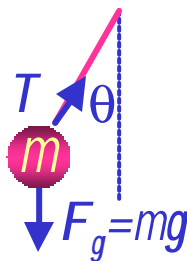
*This is how the ball looks like no matter which frame you are in.*

How do the free-body diagrams look for two frames?

How do the motions interpreted in these two frames? Any differences?

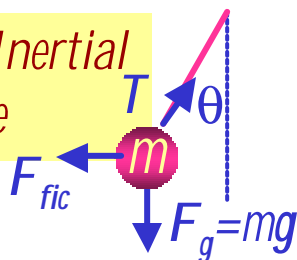**Inertial Frame**

$$\sum \vec{F} = \vec{F}_g + \vec{T}$$

$$\sum F_x = ma_x = ma_c = T \sin q$$

$$\sum F_y = T \cos q - mg = 0$$

$$T = \frac{mg}{\cos q} \qquad a_c = g \tan q$$

For an inertial frame observer, the forces being exerted on the ball are only **T** and **F**$_g$. The acceleration of the ball is the same as that of the box car and is provided by the x component of the tension force.

**Non-Inertial Frame**

$$\sum \vec{F} = \vec{F}_g + \vec{T} + \vec{F}_{fic}$$

$$\sum F_x = T \sin q - F_{fic} = 0 \qquad F_{fic} = ma_{fic} = T \sin q$$

$$\sum F_y = T \cos q - mg = 0$$

$$T = \frac{mg}{\cos q} \qquad a_{fic} = g \tan q$$

In the non-inertial frame observer, the forces being exerted on the ball are **T**, **F**$_g$, and **F**$_{fic}$.

For some reason the ball is under a force, **F**$_{fic}$, that provides acceleration to the ball.

While the mathematical expression of the acceleration of the ball is identical to that of inertial frame observer's, the cause of the force, or physical law is dramatically different.

# Motion in Resistive Forces

*Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.*

*Some examples?*     *Air resistance, viscous force of liquid, etc*

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed.   They almost always increase with increasing speed.
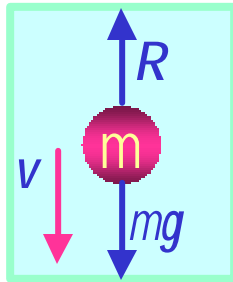
Two different cases of proportionality:
1.    Forces linearly proportional to speed: Slowly moving or very small objects
2.    Forces proportional to square of speed: Large objects w/ reasonable speed

# Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R} \qquad \sum F_x = 0$$

This equation also tells you that

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt} \qquad \frac{dv}{dt} = g - \frac{b}{m}v \qquad \frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (**terminal speed**) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, $\tau = m/b$.

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0, \quad v_t = \frac{mg}{b}$$

# Example 6.11

A small ball of mass 2.00g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The ball reaches a terminal speed of 5.00 cm/s. Determine the time constant $\tau$ and the time it takes the ball to reach 90% of its terminal speed.
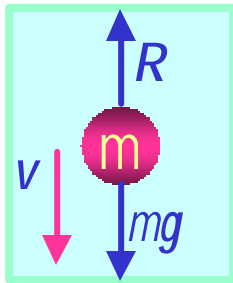


$$v_t = \frac{mg}{b}$$

Determine the time constant $\tau$.

$$\therefore b = \frac{mg}{v_t} = \frac{2.00\times10^{-3}kg \cdot 9.80m/s^2}{5.00\times10^{-2}m/s} = 0.392\,kg/s$$

$$t = \frac{m}{b} = \frac{2.00\times10^{-3}kg}{0.392\,kg/s} = 5.10\times10^{-3}s$$

Determine the time it takes the ball to reach 90% of its terminal speed.

$$v = \frac{mg}{b}\left(1-e^{-t/t}\right) = v_t\left(1-e^{-t/t}\right) \qquad 0.9v_t = v_t\left(1-e^{-t/t}\right)$$

$$\left(1-e^{-t/t}\right) = 0.9; \quad e^{-t/t} = 0.1$$

$$t = -t \cdot \ln 0.1 = 2.30t = 2.30\cdot 5.10\times10^{-3} = 11.7\,(ms)$$

# Numerical Modeling in Particle Dynamics

- The method we have been using to solve for particle dynamics is called **Analytical Method** ➜ Solve motions using differential equations
    - Use Newton's second law for net force in the motion
    - Use net force to determine acceleration, a=$\Sigma$F/m
    - Use the acceleration to determine velocity, dv/dt=a
    - Use the velocity to determine position, dx/dt=v
- Some motions are too complicated to solve analytically ➜ Numeric method or Euler method to describe the motion
    - Differential equations are divided in small increments of time or position
    - Acceleration is determined from net force: $a(x,v,t) = \dfrac{\sum F(x,v,t)}{m}$
    - Determine velocity using a(x,v,t): $v(x,t) = v(x,t-\Delta t) + a(x,v,t)\Delta t$
    - Determine position using a and v: $x(t) = x(t-\Delta t) + v(x,t)\Delta t$

Compute the quantities at every small increments of time $\Delta t$ and plot position, velocity, or acceleration as a function of time to describe the motion.