

PHYS 1443 – Section 003

Lecture #7

Monday, Oct. 7, 2002

Dr. Jaehoon Yu

1. Numerical Modeling in Particle Dynamics (Euler Method)
2. Work & Scalar product of vectors
3. Kinetic Energy
4. Power
5. Potential Energy
 - Gravitational Potential Energy
 - Elastic Potential Energy
6. Conservative Forces and Mechanical Energy Conservation

Today's homework is homework #8, due 12:00pm, next Monday!!

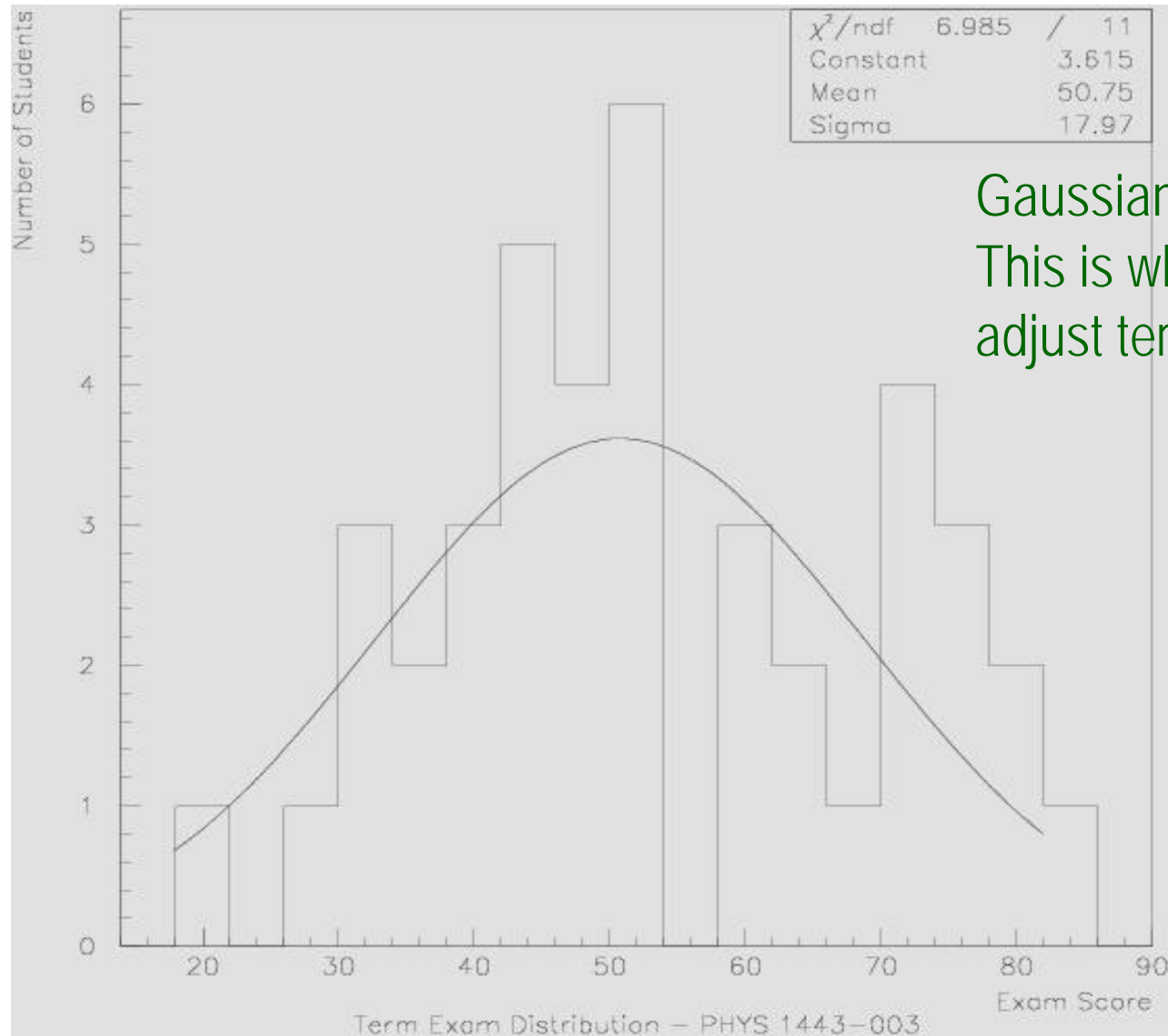


Announcements

- Term Exam
 - Grading is completed
 - Maximum Score: 85
 - Numerical Average: 51.7 → Very good!!!
 - One person missed the exam without a prior approval
 - Can look at your exam after the class
 - All scores are relative based on the curve
 - One worst after the adjustment will be dropped
 - Exam constitutes only 50% of the total
 - Do your homework well
 - Come to the class and do well with quizzes



First Term Exam Distributions

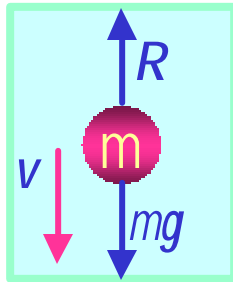


Gaussian Mean: 51
This is what I will use to
adjust term exam scores



Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write $R=bv$



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R}$$

$$\sum F_x = 0$$

This equation also tells you that

$$\sum F_y = mg - bv = ma = m \frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

$$\frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (**terminal speed**) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

How do the speed and acceleration depend on time?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0, \quad v_t = \frac{mg}{b}$$

$$v = \frac{mg}{b} \left(1 - e^{-bt/m} \right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b} \frac{b}{m} e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

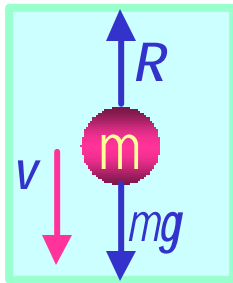
$$\frac{dv}{dt} = \frac{mg}{b} \frac{b}{m} e^{-t/\tau} = \frac{mg}{b} \frac{b}{m} \left(1 - 1 + e^{-t/\tau} \right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, $\tau = m/b$.



Example 6.11

A small ball of mass 2.00g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The ball reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the ball to reach 90% of its terminal speed.



Determine the time constant τ .

$$v_t = \frac{mg}{b}$$

$$\therefore b = \frac{mg}{v_t} = \frac{2.00 \times 10^{-3} \text{ kg} \cdot 9.80 \text{ m/s}^2}{5.00 \times 10^{-2} \text{ m/s}} = 0.392 \text{ kg/s}$$

$$\tau = \frac{m}{b} = \frac{2.00 \times 10^{-3} \text{ kg}}{0.392 \text{ kg/s}} = 5.10 \times 10^{-3} \text{ s}$$

Determine the time it takes the ball to reach 90% of its terminal speed.

$$v = \frac{mg}{b} \left(1 - e^{-t/\tau} \right) = v_t \left(1 - e^{-t/\tau} \right)$$

$$0.9v_t = v_t \left(1 - e^{-t/\tau} \right)$$

$$\left(1 - e^{-t/\tau} \right) = 0.9; \quad e^{-t/\tau} = 0.1$$

$$t = -\tau \cdot \ln 0.1 = 2.30\tau = 2.30 \cdot 5.10 \times 10^{-3} = 11.7 \text{ (ms)}$$



Numerical Modeling in Particle Dynamics

- The method we have been using to solve for particle dynamics is called **Analytical Method** → Solve motions using differential equations
 - Use Newton's second law for net force in the motion
 - Use net force to determine acceleration, $a = \Sigma F/m$
 - Use the acceleration to determine velocity, $dv/dt = a$
 - Use the velocity to determine position, $dx/dt = v$
- Some motions are too complicated to solve analytically → Numeric method or Euler method to describe the motion
 - Differential equations are divided in small increments of time or position
 - Acceleration is determined from net force:
$$a(x, v, t) = \frac{\sum F(x, v, t)}{m}$$
 - Determine velocity using $a(x, v, t)$:
$$v(x, t) = v(x, t - \Delta t) + a(x, v, t) \Delta t$$
 - Determine position using a and v :
$$x(t) = x(t - \Delta t) + v(x, t) \Delta t$$

Compute the quantities at every small increments of time Δt and plot position, velocity, or acceleration as a function of time to describe the motion.



Work Done by a Constant Force

Work in physics is done only when a sum of forces exerted on an object made a motion to the object.



Which force did the work? Force \vec{F}

How much work did it do?

$$W = \sum \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Unit?

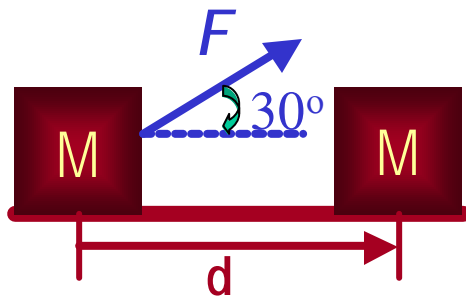
$$\begin{aligned} &N \cdot m \\ &= J \text{ (for Joule)} \end{aligned}$$

What does this mean?

Physical work is done only by the component of the force along the movement of the object.

Example 7.1

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F=50.0\text{N}$ at an angle of 30.0° with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \sum \vec{F} \cdot \vec{d} = |\vec{F}| |\vec{d}| \cos q$$

$$W = 50.0 \times 3.00 \times \cos 30^\circ = 130\text{J}$$

Does work depend on mass of the object being worked on?

Yes

Why don't I see the mass term in the work at all then?

It is reflected in the force. If the object has smaller mass, it would take less force to move it the same distance as the heavier object. So it would take less work. Which makes perfect sense, doesn't it?

Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them

$$\vec{A} \cdot \vec{B} \equiv |\vec{A}| |\vec{B}| \cos q$$

- Operation is commutative

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos q = |\vec{B}| |\vec{A}| \cos q = \vec{B} \cdot \vec{A}$$

- Operation follows distribution law of multiplication

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

- Scalar products of Unit Vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

- How does scalar product look in terms of components?

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{A} \cdot \vec{B} = (A_x B_x \hat{i} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

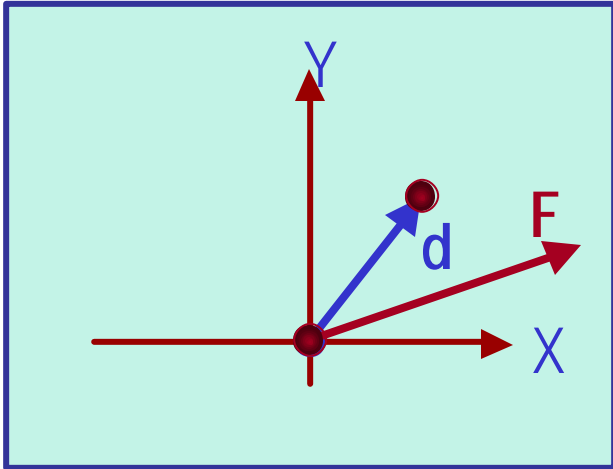
Angle between
the vectors

$$\cos q = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| |\vec{B}|}$$



Example 7.3

A particle moving in the xy plane undergoes a displacement $\mathbf{d}=(2.0\mathbf{i}+3.0\mathbf{j})\text{m}$ as a constant force $\mathbf{F}=(5.0\mathbf{i}+2.0\mathbf{j})\text{ N}$ acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$|\vec{d}| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6\text{m}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4\text{N}$$

b) Calculate the work done by the force \mathbf{F} .

$$W = \vec{F} \cdot \vec{d} = (2.0\hat{i} + 3.0\hat{j}) \cdot (5.0\hat{i} + 2.0\hat{j}) = 2.0 \times 5.0 \hat{i} \cdot \hat{i} + 3.0 \times 2.0 \hat{j} \cdot \hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between \mathbf{d} and \mathbf{F} ?

$$W = \vec{F} \cdot \vec{d} = |\vec{F}| |\vec{d}| \cos \theta$$

Work Done by Varying Force

- If the force depends on position of particle during the motion
 - one must consider work segment in small segment of the position where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

- Then add them all up throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

- If more than one force is acting, the net work is done by the net force

$$W(\text{net}) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

One of the forces depends on position is force by a spring

$$F_s = -kx$$

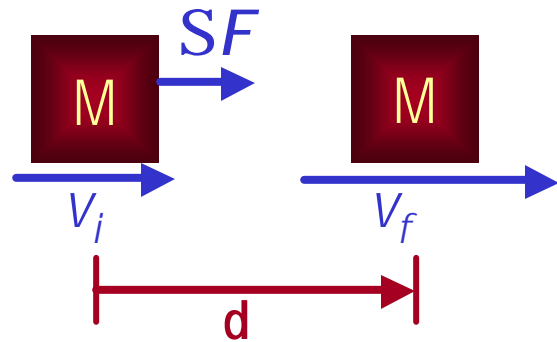
The work done by the spring force is

$$W = \int_{-x_{\max}}^0 F_s dx = \int_{-x_{\max}}^0 (-kx) dx = \frac{1}{2} kx^2$$



Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
 - If forces exerting on the object during the motion are so complicated
 - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement d to increase its speed from v_i to v_f .

The work on the object by the net force ΣF is

$$W = \left(\sum \vec{F} \right) \cdot \vec{d} = (ma)d \cos 0 = (ma)d$$

Displacement

$$d = \frac{1}{2} (v_f + v_i) t$$

Acceleration

$$a = \frac{v_f - v_i}{t}$$

Work

$$W = (ma)d = \left[m \left(\frac{v_f - v_i}{t} \right) \right] \frac{1}{2} (v_f + v_i) t = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Kinetic Energy

$$KE = \frac{1}{2} m v^2$$

Work

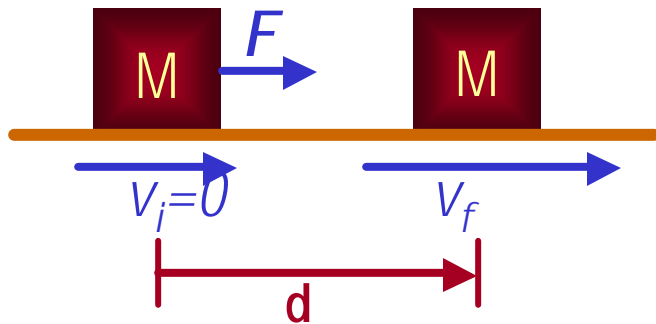
$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = KE_f - KE_i = \Delta KE$$

The work done by the net force caused change of object's kinetic energy.



Example 7.7

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force F is

$$W = \vec{F} \cdot \vec{d} = |\vec{F}| |\vec{d}| \cos \theta = 12 \times 3.0 \cos 0 = 36(J)$$

From the work-kinetic energy theorem, we know

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since initial speed is 0, the above equation becomes

$$W = \frac{1}{2}mv_f^2$$

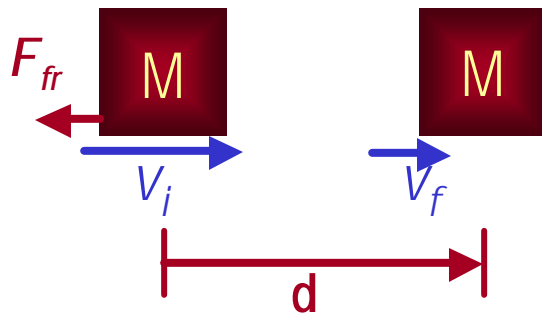
Solving the equation for v_f we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5 m/s$$



Work and Energy Involving Kinetic Friction

- Some How do you think the work looks like if there is friction?
 - Why doesn't static friction matter?



Friction force F_{fr} works on the object to slow down

The work on the object by the friction F_{fr} is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d$$

$$\Delta KE = -F_{fr} d$$

The final kinetic energy of an object with initial kinetic energy, friction force and other source of work is

$$KE_f = KE_i + \sum W - F_{fr} d$$



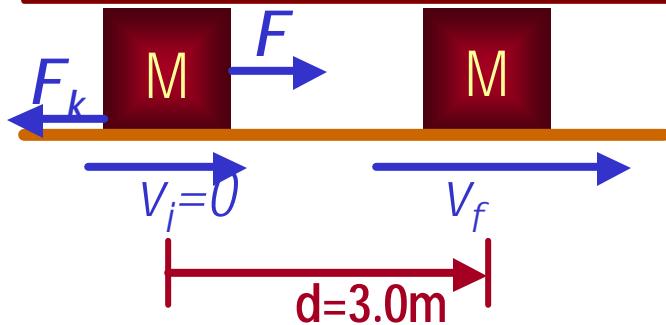
$t=0, KE_i$

Friction
Engine work

$t=T, KE_f$

Example 7.8

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction $\mu_k=0.15$ by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force F is

$$W_F = \vec{F} \cdot \vec{d} = |\vec{F}| |\vec{d}| \cos \mathbf{q} = 12 \times 3.0 \cos 0 = 36 (J)$$

$$W_k = \vec{F}_k \cdot \vec{d} = |\mathbf{m}_k mg| |\vec{d}| \cos \mathbf{q}$$

$$= 0.15 \times 6.0 \times 9.8 \times 3.0 \cos 180 = -26 (J)$$

Work done by friction F_k is

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2} m v_f^2$$

Solving the equation
for v_f , we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 10}{6.0}} = 1.8 m/s$$