PHYS 1443 – Section 003
Lecture #8

Monday, Oct. 9, 2002

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1. Power
2. Potential Energy
   - Gravitational Potential Energy
   - Elastic Potential Energy
3. Conservative Forces and Mechanical Energy Conservation

Today's homework is homework #9, due 12:00pm, next Monday!!

Announcement

- If your term exam score is less than 40, come talk to me before next exam

Work and Kinetic Energy

Work in physics is done only when a sum of forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work.

Mathematically, work is written in scalar product of force vector and the displacement vector

$$W = \sum \vec{F}_i \cdot \vec{d} = Fd\cos\theta$$

*Kinetic Energy is the energy associated with motion and capacity to perform work. Work requires change of energy after the completion ← **Work-Kinetic energy theorem***

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule

Power

- Rate at which work is done
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill? ➔ 8 cylinder car climbs up faster

Is the amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $$\overline{P} = \frac{W}{\Delta t}$$

Instantaneous power $$P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d}{dt}\left(\vec{s}\right) = \vec{F} \cdot \vec{v} = Fv \cos q$$

Unit? $$J/s = Watts$$ $$1HP = 746\,Watts$$

What do power companies sell? $$1kWH = 1000\,Watts \times 3600\,s = 3.6 \times 10^{6}\,J$$

Energy

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg, \quad Weight = mg = 14200N$$

$$mn = mmg = 227N$$

Coefficient of Rolling Friction; μ=0.016

Air Drag

$$f_a = \frac{1}{2} D r A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2 v^2 = 0.647 v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Example 7.14

A compact car has a mass of 800kg, and its efficiency is rated at 18%. Find the amount of gasoline used to accelerate the car from rest to 27m/s (~60mi/h). Use the fact that the energy equivalent of 1gal of gasoline is 1.3×10^{8}J.

First let's compute what the kinetic energy needed to accelerate the car from rest to a speed *v*.

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\times 800\times(27)^2 = 2.9\times10^5\,J$$

Since the engine is only 18% efficient we must divide the necessary kinetic energy with this efficiency in order to figure out what the total energy needed is.

$$W_E = \frac{K_f}{e} = \frac{1}{2e}mv^2 = \frac{2.9\times10^5\,J}{0.18} = 16\times10^5\,J$$

Then using the fact that 1gal of gasoline can putout 1.3×10^{8}J, we can compute the total volume of gasoline needed to accelerate the car to 60 mi/h.

$$V_{gas} = \frac{W_E}{1.3\times10^8\,J/gal} = \frac{16\times10^5\,J}{1.3\times10^8\,J/gal} = 0.012gal$$

Kinetic Energy at High Speed

The laws of Newtonian mechanics is no longer valid for object moving at the speed close to that of light, c. It must be more generalized for these special cases.➔ Theory of relativity.

The kinetic energy must be modified to reflect the fact that the object is moving very high speed.

$$K = mc^2\left(\frac{1}{\sqrt{1-\left(v/c\right)^2}}-1\right)$$

What does this expression tell you?

The speed of an object cannot be faster than light in vacuum.
🡐 Have not seen any particle that runs faster than light, yet.

However this equation must satisfy the Newtonian expression!!

$$K = mc^2\left(\frac{1}{\sqrt{1-\left(v/c\right)^2}}-1\right) = mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2+\frac{3}{8}\left(\frac{v}{c}\right)^4+\ldots-1\right)$$

$$K = mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2-1\right) = \frac{1}{2}mc^2\times\left(\frac{v}{c}\right)^2 = \frac{1}{2}mv^2$$

Potential Energy

Energy associated with a system of objects → Stored energy which has Potential or possibility to work or to convert to kinetic energy

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called, conservative forces which results in principle of conservation of mechanical energy.

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What other forms of energies in the universe?

Mechanical Energy

Chemical Energy

Biological Energy

Electromagnetic Energy

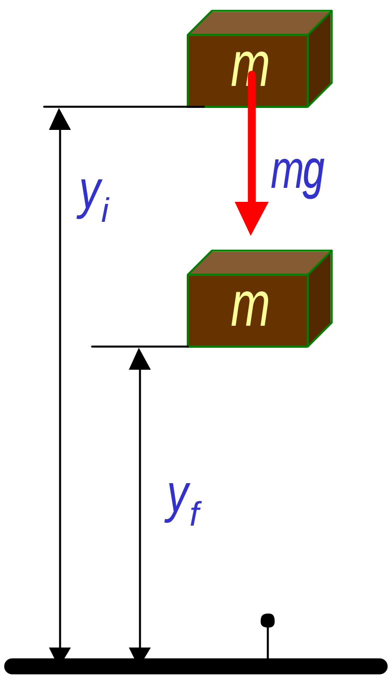
Nuclear Energy

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to the other.

Gravitational Potential Energy

Potential energy given to an object by gravitational field in the system of Earth due to its height from the surface



When an object is falling, gravitational force, Mg, performs work on the object, increasing its kinetic energy. The potential energy of an object at a height y which is the potential to work is expressed as

$$U_g = \vec{F}_g \cdot \vec{y} = mg\left(-\vec{j}\right)\cdot y\left(-\vec{j}\right) = mgy$$

$$U_g \equiv mgy$$

Work performed on the object by the gravitational force as the brick goes from y_i to y_f is:

$$W_g = U_i - U_f$$

$$= mgy_i - mgy_f = -\Delta U_g$$

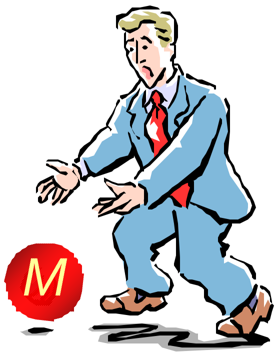
What does this mean?

Work by the gravitational force as the brick goes from y_i to y_f is negative of the change in the system's potential energy

➔ Potential energy was lost in order for gravitational force to increase the brick's kinetic energy.

Example 8.1

A bowler drops bowling ball of mass 7kg on his toe. Choosing floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3J$$

$$U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06J$$

$$\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of ball at the hand and of the toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7\times 9.8\times(-1.3) = -89.2J$$

$$U_f = mgy_f = 7\times 9.8\times(-1.77) = -121.4J$$

$$\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Elastic Potential Energy

Potential energy given to an object by a spring or an object with elasticity in the system consists of the object and the spring without friction.

The force spring exerts on an object when it is distorted from its equilibrium by a distance x is

$$F_s = -kx$$

The work performed on the object by the spring is

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

The potential energy of this system is

$$U_s \equiv \frac{1}{2}kx^2$$

What do you see from the above equations?

The work done on the object by the spring depends only on the initial and final position of the distorted spring.

Where else did you see this trend?

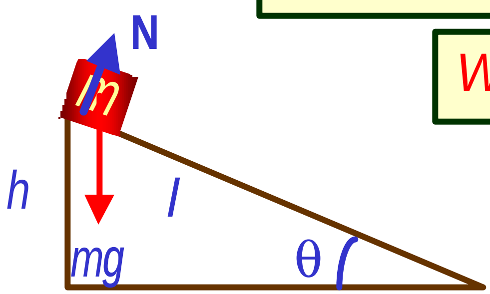
The gravitational potential energy, U_g

So what does this tell you about the elastic force?

A conservative force!!!

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path.



When directly falls, the work done on the object is $W_g = mgh$

When sliding down the hill of length l, the work is $W_g = F_{g-incline} \times l$ $= mg \sin q \times l$

$W_g = mg\left(l \sin q\right)$ $= mgh$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺ $W_g = mgh$

So the work done by the gravitational force on an object is independent on the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

The forces like gravitational or elastic forces are called conservative forces

1. *If the work performed by the force does not depend on the path*
2. *If the work performed on a closed path is 0.*

Total mechanical energy is conserved!! $E_M \equiv KE_i + PE_i = KE_f + PE_f$

More Conservative and Non-conservative Forces

A potential energy can be associated with a conservative force

A work done on a object by a conservative force is the same as the potential energy change between initial and final states

$$W_c = U_i - U_f = -\Delta U$$

So what is a conservative force?

The force that conserves mechanical energy.

OK. Then what are non-conservative forces?

The force that does not conserve mechanical energy.

The work by these forces depends on the path.

Can you tell me an example?

Friction

Why is it a non-conservative force?

Because the longer the path of an object's movement, the more work the friction forces perform on it.

What happens to the mechanical energy?

Kinetic energy converts to thermal energy and is not reversible.

Total mechanical energy is not conserved but the total energy is still conserved. It just exists in a different form.

$$E_T \equiv E_M + E_{Other}$$

$$KE_i + PE_i = KE_f + PE_f + W_{Frictio}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

Potential energy function

What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.