PHYS 1443 – Section 003
Lecture #19

Monday, Nov. 20, 2002

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1. Energy of the Simple Harmonic Oscillator
2. Simple Pendulum
3. Other Types of Pendulum
4. Damped Oscillation

Today's homework is homework #19 due 12:00pm, Wednesday, Nov. 27!!

Announcements

- Evaluation today
- We have classes next week, both Monday and Wednesday
- Remember the Term Exam on Monday, Dec. 9 in the class

Equation of Simple Harmonic Motion

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The solution for the 2nd order differential equation

$$x = A\cos(\omega t + \phi)$$

Amplitude

Phase

Angular Frequency

Phase constant

Generalized expression of a simple harmonic motion

Let's think about the meaning of this equation of motion

What happens when t=0 and ϕ=0?

$$x = A\cos(0+0) = A$$

What is ϕ if x is not A at t=0?

$$x = A\cos(\phi) = x'$$

$$\phi = \cos^{-1}(x')$$

What are the maximum/minimum possible values of x?

A/-A

An oscillation is fully characterized by its:

- Amplitude
- Period or frequency
- Phase constant

More on Equation of Simple Harmonic Motion

What is the time for full cycle of oscillation?

Since after a full cycle the position must be the same

$$x = A\cos\left(w\left(t+T\right)+f\right) = A\cos\left(wt+2p+f\right)$$

The period $T = \frac{2p}{w}$

One of the properties of an oscillatory motion

How many full cycles of oscillation does this undergo per unit time?

$$f = \frac{1}{T} = \frac{w}{2p}$$

Frequency

What is the unit?

1/s=Hz

Let’s now think about the object’s speed and acceleration.

$$x = A\cos\left(wt+f\right)$$

Speed at any given time $v = \frac{dx}{dt} = -wA\sin\left(wt+f\right)$

Max speed $v_{max} = wA$

Acceleration at any given time $a = \frac{dv}{dt} = -w^2 A\cos\left(wt+f\right) = -w^2 x$

Max acceleration $a_{max} = w^2 A$

What do we learn about acceleration?

Acceleration is reverse direction to displacement

Acceleration and speed are $\pi/2$ off phase:

When v is maximum, a is at its minimum

Energy of the Simple Harmonic Oscillator

How do you think the mechanical energy of the harmonic oscillator look without friction?

Kinetic energy of a harmonic oscillator is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mw^2 A^2 \sin^2(wt+f)$$

The elastic potential energy stored in the spring

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(wt+f)$$

Therefore the total mechanical energy of the harmonic oscillator is

$$E = KE + PE = \frac{1}{2}\left[mw^2 A^2 \sin^2(wt+f) + kA^2 \cos^2(wt+f)\right]$$

Since $w = \sqrt{k/m}$

$$E = KE + PE = \frac{1}{2}\left[kA^2 \sin^2(wt+f) + kA^2 \cos^2(wt+f)\right] = \frac{1}{2}kA^2$$

Total mechanical energy of a simple harmonic oscillator is a constant of a motion and is proportional to the square of the amplitude

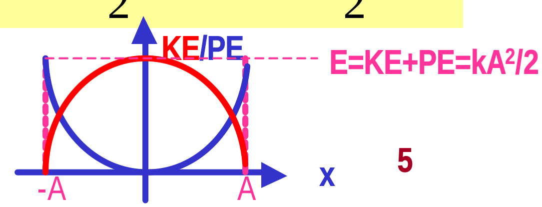
Maximum KE is when PE=0

$$KE_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}mw^2 A^2 \sin^2(wt+f) = \frac{1}{2}mw^2 A^2 = \frac{1}{2}kA^2$$

One can obtain speed

$$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = +\sqrt{k/m\left(A^2 - x^2\right)} = +w\sqrt{A^2 - x^2}$$



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Example 13.4

A 0.500kg cube connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless track. a) Calculate the total energy of the system and the maximum speed of the cube if the amplitude of the motion is 3.00 cm.

From the problem statement, A and k are

$$k = 20.0N/m$$

$$A = 3.00cm = 0.03m$$

The total energy of the cube is

$$E = KE + PE = \frac{1}{2}kA^2 = \frac{1}{2}(20.0)\times(0.03)^2 = 9.00\times10^{-3}J$$

Maximum speed occurs when kinetic energy is the same as the total energy

$$KE_{\max} = \frac{1}{2}mv_{\max}^2 = E = \frac{1}{2}kA^2$$

$$v_{\max} = A\sqrt{\frac{k}{m}} = 0.03\sqrt{\frac{20.0}{0.500}} = 0.190m/s$$

b) What is the velocity of the cube when the displacement is 2.00 cm.

velocity at any given displacement is

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = \sqrt{20.0\cdot\left(0.03^2 - 0.02^2\right)/0.500} = 0.141m/s$$

c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm.

Kinetic energy, KE

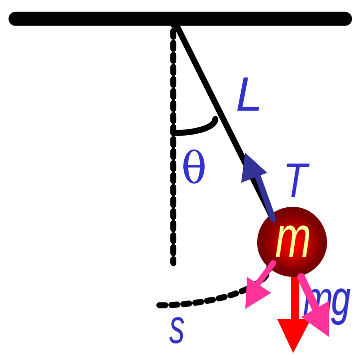
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}0.500\times(0.141)^2 = 4.97\times10^{-3}J$$

Potential energy, PE

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}20.0\times(0.02)^2 = 4.00\times10^{-3}J$$

The Pendulum

A simple pendulum also performs periodic motion.



The net force exerted on the bob is

$$\sum F_r = T - mg\cos\theta_A = 0$$

$$\sum F_t = -mg\sin\theta_A = ma = m\frac{d^2s}{dt^2}$$

Since the arc length, s, is $s = L\theta_A$

$$\frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2} = -g\sin\theta \quad \text{results} \Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Again became a second degree differential equation, satisfying conditions for simple harmonic motion

If θ is very small, sinθ~θ $\quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta = -\omega^2\theta$ giving angular frequency $\omega = \sqrt{\frac{g}{L}}$

The period for this motion is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

The period only depends on the length of the string and the gravitational acceleration

Example 13.5

Christian Huygens (1629-1695), the greatest clock maker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1s. How much shorter would out length unit be had this suggestion been followed?

Since the period of a simple pendulum motion is

$$T = \frac{2p}{w} = 2p\sqrt{\frac{L}{g}}$$

The length of the pendulum in terms of T is

$$L = \frac{T^2 g}{4p^2}$$

Thus the length of the pendulum when T=1s is

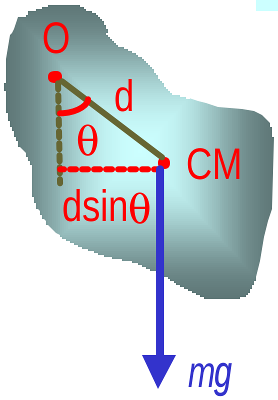
$$L = \frac{T^2 g}{4p^2} = \frac{1 \times 9.8}{4p^2} = 0.248\ m$$

Therefore the difference in length with respect to the current definition of 1m is

$$\Delta L = 1 - L = 1 - 0.248 = 0.752\ m$$

Physical Pendulum

Physical pendulum is an object that oscillates about a fixed axis which does not go through the object's center of mass.



Consider a rigid body pivoted at a point O that is a distance d from the CM.

The magnitude of the net torque provided by the gravity is

$$\sum \tau = -mgd\sin\theta$$

Then $$\sum \tau = I\alpha = I\frac{d^2\theta}{dt^2} = -mgd\sin\theta$$

Therefore, one can rewrite $$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\sin\theta \approx -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

Thus, the angular frequency ω is $$\omega = \sqrt{\frac{mgd}{I}}$$

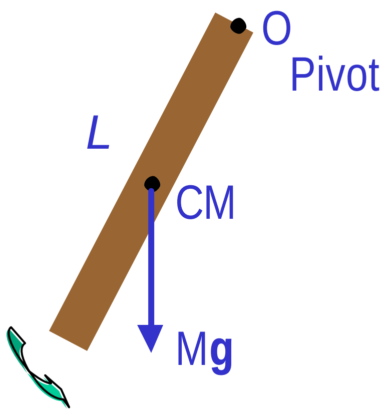
And the period for this motion is $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

By measuring the period of physical pendulum, one can measure moment of inertia.

Does this work for simple pendulum?

Example 13.6

A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane. Find the period of oscillation if the amplitude of the motion is small.



Moment of inertia of a uniform rod, rotating about the axis at one end is $I = \frac{1}{3}ML^2$

The distance d from the pivot to the CM is L/2, therefore the period of this physical pendulum is

$$T = \frac{2p}{w} = 2p\sqrt{\frac{I}{Mgd}} = 2p\sqrt{\frac{2ML^2}{3MgL}} = 2p\sqrt{\frac{2L}{3g}}$$

Calculate the period of a meter stick that is pivot about one end and is oscillating in a vertical plane.

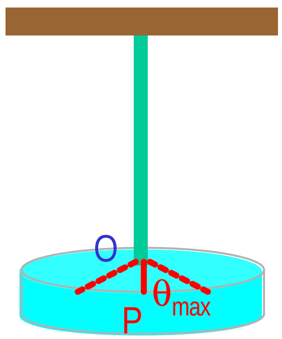
Since L=1m, the period is

$$T = 2p\sqrt{\frac{2L}{3g}} = 2p\sqrt{\frac{2}{3\cdot 9.8}} = 1.64s$$

So the frequency is

$$f = \frac{1}{T} = 0.61s^{-1}$$

Torsional Pendulum



When a rigid body is suspended by a wire to a fixed support at the top and the body is twisted through some small angle θ, the twisted wire can exert a restoring torque on the body that is proportional to the angular displacement.

The torque acting on the body due to the wire is

$$\tau = -\kappa\theta$$

κ is the torsion constant of the wire

Applying the Newton's second law of rotational motion

$$\sum \tau = I\alpha = I\frac{d^2\theta}{dt^2} = -\kappa\theta$$

Then, again the equation becomes

$$\frac{d^2\theta}{dt^2} = -\left(\frac{\kappa}{I}\right)\theta = -\omega^2\theta$$

Thus, the angular frequency ω is

$$\omega = \sqrt{\frac{\kappa}{I}}$$

And the period for this motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$

This result works as long as the elastic limit of the wire is not exceeded