PHYS 1443 – Section 003
Lecture #21

Wednesday, Nov. 27, 2002
Dr. Jaehoon Yu

1. Gravitational Field
2. Energy in Planetary and Satellite Motions
3. Escape Speed
4. Fluid and Pressure
5. Variation of Pressure and Depth
6. Absolute and Relative Pressure

Today's homework is homework #21 due 6:00pm, Friday, Dec. 6!!

Announcements

- Remember the Term Exam on Monday, Dec. 9 in the class
  - Covers all material in chapters 11 – 15
  - Review on Wednesday, Dec. 4
- Happy Thanksgiving!!!

The Gravitational Field

The gravitational force is a field force. The force exists every point in the space.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will fill the gravitational force, $\vec{F}_g = m\vec{g}$, exerted by M.

Therefore the gravitational field g is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

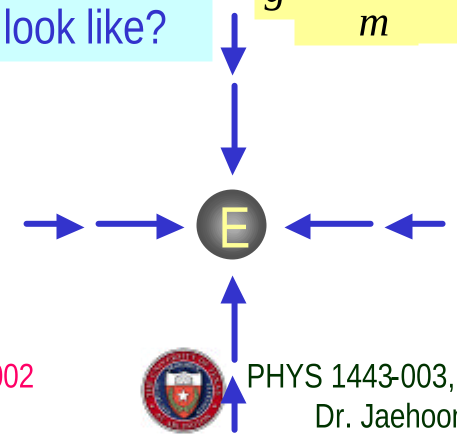
In other words, the gravitational field at a point in space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

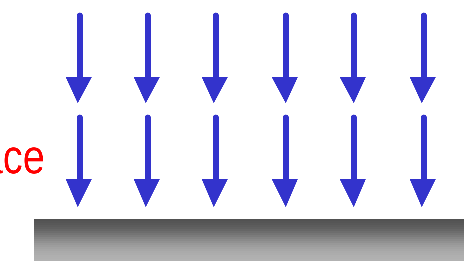
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

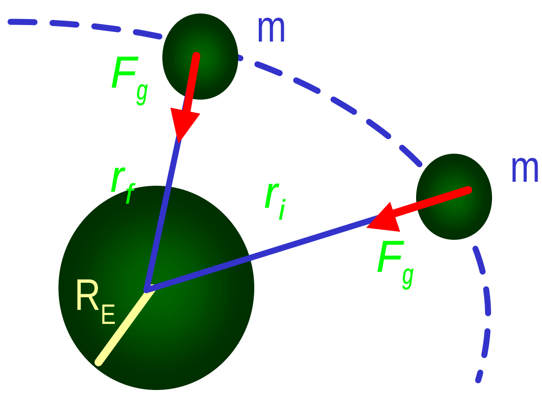
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Because gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be looked at as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it only performed work while the path was radial direction only. Therefore, the work performed by the gravitational force that depends on the position becomes

$$dW = \vec{F}\cdot d\vec{r} = F(r)dr \xrightarrow{\text{For the whole path}} W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of work in the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is

$$F(r) = -\frac{GM_E m}{r^2}$$

So the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since potential energy only matters for differences, by taking the infinite distance as the initial point of the potential energy, we get

$$U = -\frac{GM_E m}{r}$$

For any two particles?

$$U = -\frac{Gm_1 m_2}{r}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$U = \sum_{i,j} U_{i,j}$$

Example 14.6

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

The above formula becomes

$$\Delta U = -GM_E m\frac{(r_f - r_i)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$$r_i \approx R_E \quad \text{and} \quad r_f \approx R_E$$

Therefore, ΔU becomes

$$\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$$

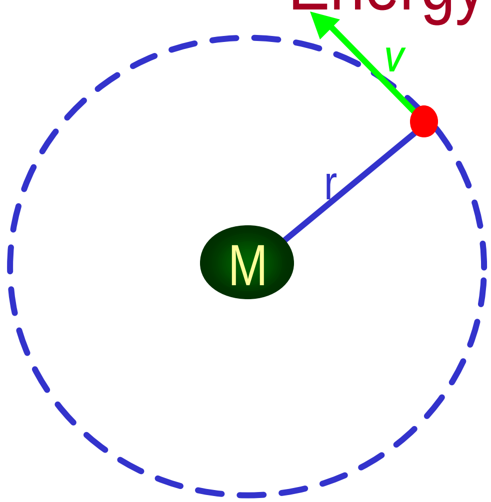
Since on the surface of the Earth the gravitational field is

$$g = \frac{GM_E}{R_E^2}$$

The potential energy becomes

$$\Delta U = -mg\Delta y$$

Energy in Planetary and Satellite Motions



Consider an object of mass m moving at a speed *v* near a massive object of mass M (M>>m).

What's the total energy?

$$E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Systems like the Sun and the Earth or the Earth and the Moon whose motions are contained within a closed orbit is called *Bound Systems.*

For a system to be bound, the total energy must be negative.

Assuming a circular orbit, in order for the object to be kept in the orbit the gravitational force must provide the radial acceleration. Therefore from Newton's second law of motion

$$\frac{GM_E m}{r^2} = ma = m\frac{v^2}{r}$$

The kinetic energy for this system is

$$\frac{1}{2}mv^2 = \frac{GM_E m}{2r}$$

Therefore the total mechanical energy of the system is

$$E = K + U = -\frac{GMm}{2r}$$

Since the gravitational force is conservative, the total mechanical energy of the system is conserved.

Example 14.7

The space shuttle releases a 470kg communication satellite while in an orbit that is 280km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit, which is an orbit in which the satellite stays directly over a single location on the Earth, How much energy did the engine have to provide?

What is the radius of the geosynchronous orbit?

$$T = 1day = 8.64\times10^{4}\,s$$

From Kepler's 3rd law $T^{2} = K_{E} r_{GS}^{3}$ Where K_E is $K_{E} = \frac{4p^{2}}{GM_{E}} = 9.89\times10^{-14}\,s^{2}/m^{3}$

Therefore the geosynchronous radius is

$$r_{GS} = \sqrt[3]{\frac{T^{2}}{K_{E}}} = \sqrt[3]{\frac{\left(8.64\times10^{4}\right)^{2}}{9.89\times10^{-14}}} = \sqrt[3]{\frac{\left(8.64\times10^{4}\right)^{2}}{9.89\times10^{-14}}} = 4.23\times10^{7}\,m$$

Because the initial position before the boost is 280km

$$r_{i} = R_{E} + 2.80\times10^{5}\,m = 6.65\times10^{6}\,m$$

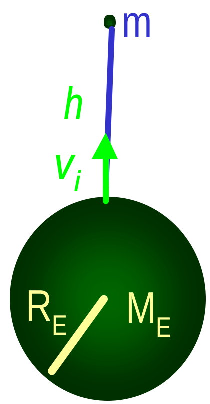
The total energy needed to boost the satellite at the geosynchronous radius is the difference of the total energy before and after the boost

$$\Delta E = -\frac{GM_{E}m_{s}}{2}\left(\frac{1}{r_{GS}} - \frac{1}{r_{i}}\right)$$

$$= -\frac{6.67\times10^{-11}\times5.98\times10^{24}\times470}{2}\left(\frac{1}{4.23\times10^{7}} - \frac{1}{6.65\times10^{6}}\right) = 1.19\times10^{10}\,J$$

Escape Speed

v_f=0 at h=r_{max}



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

Because the total energy is conserved

$$E = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height h of the object.

$$h = r_{max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for the object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.37\times 10^{6}}}$$

$$= 1.12\times 10^{4}\, m/s = 11.2\, km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object

Fluid and Pressure

What are the three states of matter?

Solid, Liquid, and Gas

How do you distinguish them?

By the time it takes for a particular substance to change its shape in reaction to external forces.

What is a fluid?

A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

We will first learn about mechanics of fluid at rest, *fluid statics*.

In what way do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the forces perpendicular to the surfaces of the object.

This force by the fluid on an object usually is expressed in the form of the force on a unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and dimension of pressure?

Unit: N/m^2

Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example 15.1

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep.
a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0ºC and 1 atm) is 1000kg/m^3. So the total mass of the water in the mattress is

$$m = r_W V_M = 1000 \times 2.00 \times 2.00 \times 0.300 = 1.20 \times 10^3 kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20 \times 10^3 \times 9.8 = 1.18 \times 10^4 N$$

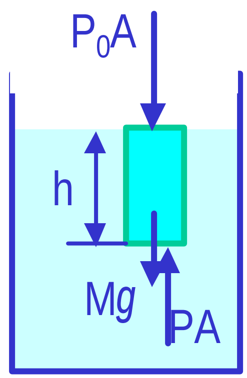
b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is 4.00 m^2, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18 \times 10^4}{4.00} = 2.95 \times 10^3$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's consider a liquid contained in a cylinder with height h and cross sectional area A immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho Ah$

Since the system is in its equilibrium

$$PA - P_0A - Mg = PA - P_0A - \rho Ahg = 0$$

Therefore, we obtain $P = P_0 + \rho gh$

The pressure at the depth *h* below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by ρgh.

Atmospheric pressure P_0 is

$$1.00 atm = 1.013 \times 10^5 Pa$$

What else can you learn from this?

Pascal's Law and Hydraulics

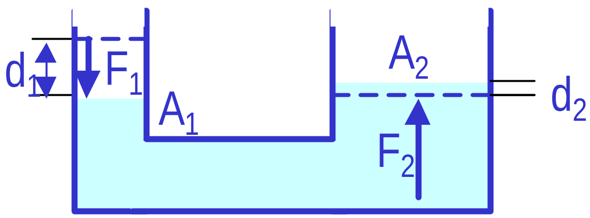
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + rgh$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied on to the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force get multiplied by the ratio of the areas A_2/A_1 is transmitted to the F_2 on an area.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Example 15.2

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's law, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_2}{A_1} F_2 = \frac{p\,(0.15)^2}{p\,(0.05)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3\,N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{p\,(0.05)^2} = 1.88 \times 10^5\,Pa$$

Example 15.3

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

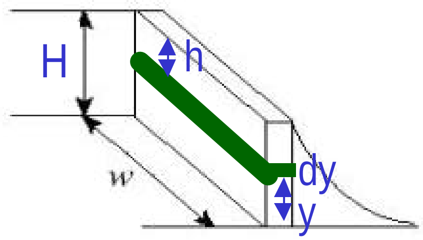
$$P - P_0 = r_W gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4\, Pa$$

Estimating the surface area of the eardrum at $1.0cm^2 = 1.0x10^{-4}\ m^2$, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9\,N$$

Example 15.4

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.



Since the water pressure varies as a function of depth, we will have to do some calculus to figure out the total force.

The pressure at the depth h is

$$P = rgh = rg(H - y)$$

The infinitesimal force dF exerting on a small strip of dam dy is

$$dF = PdA = rg(H - y)wdy$$

Therefore the total force exerted by the water on the dam is

$$F = \int_{y=0}^{y=H} rg(H - y)wdy = rg\left[Hy - \frac{1}{2}y^2\right]_{y=0}^{y=H} = \frac{1}{2}rgH^2$$