PHYS 1443 – Section 003
Lecture #22

Monday, Dec. 2, 2002
Dr. Jaehoon Yu

1. Absolute and Relative Pressure
2. Buoyant Force and Archimedes' Principle
3. Traveling Waves: Superposition and Interference
4. Speed of Waves on Strings
5. Reflection and Transmission
6. Sinusoidal Waves

No Homework Today!!

Announcements

- Homework Due date extensions
  - Homework # 20: Due noon, tomorrow, Dec. 3
  - Homework # 21: Due 6pm, Friday, Dec. 6
- Final Term Exam
  - Monday, Dec. 9, between 12:00pm – 1:30pm for 1.5 hours in the class room
  - Covers chapters 11 – 15
  - Review Wednesday, Dec. 4

Fluid and Pressure

What are the three states of matter?

Solid, Liquid, and Gas

How do you distinguish them?

By the time it takes for a particular substance to change its shape in reaction to external forces.

What is a fluid?

A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

We will first learn about mechanics of fluid at rest, *fluid statics*.

In what way do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the forces perpendicular to the surfaces of the object.

This force by the fluid on an object usually is expressed in the form of the force on a unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and dimension of pressure?

Unit: N/m^2

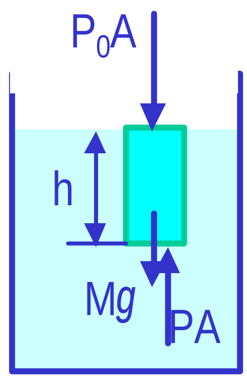
Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude.   Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's consider a liquid contained in a cylinder with height h and cross sectional area A immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho Ah$

Since the system is in its equilibrium

$$PA - P_0A - Mg = PA - P_0A - \rho Ahg = 0$$

Therefore, we obtain $P = P_0 + \rho gh$

The pressure at the depth *h* below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by ρ*gh*.

Atmospheric pressure P_0 is

$$1.00atm = 1.013 \times 10^5 Pa$$

What else can you learn from this?

Pascal's Law and Hydraulics

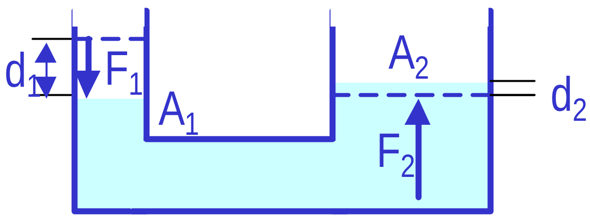
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + rgh$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied on to the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force get multiplied by the ratio of the areas A_2/A_1 is transmitted to the F_2 on an area.

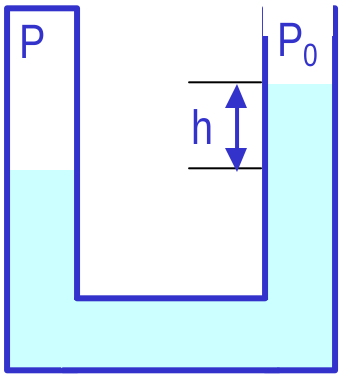
This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Absolute and Relative Pressure

How can one measure pressure?



One can measure pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure P_0.

The measured pressure of the system is $P = P_0 + rgh$

This is called the absolute pressure, because it is the actual value of the system's pressure.

In many cases we measure pressure difference with respect to atmospheric pressure due to changes in P_0 depending on the environment. This is called gauge or relative pressure.

$$P - P_0 = rgh$$

The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm is

$$P_0 = rgh = (13.595 \times 10^3 kg/m^3)(9.80665\ m/s^2)(0.7600\ m)$$

$$= 1.013 \times 10^5 Pa = 1 atm$$

Buoyant Forces and Archimedes' Principle

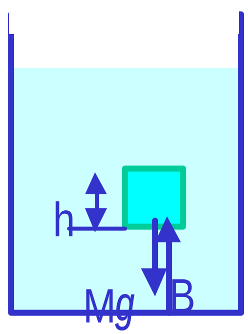
Why is it so hard to put a beach ball under water while a piece of small steel sinks in the water?

The water exerts force on an object immersed in the water.
This force is called Buoyant force.

How does the Buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called, Archimedes' principle. What does this mean?



Let's consider a cube whose height is h and is filled with fluid and at its equilibrium. Then the weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

And the pressure at the bottom of the cube is larger than the top by ρgh.

Therefore,

$$\Delta P = B / A = rgh$$

$$B = \Delta PA = rghA = rVg$$

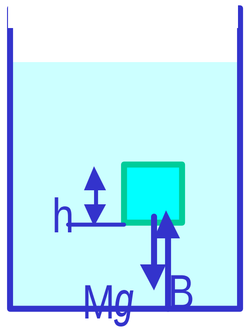
$$B = F_g = rVg = Mg$$

Where Mg is the weight of the fluid.

More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object



Let's consider an object of mass M, with density ρ_0, is immersed in the fluid with density ρ_f .

The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

Therefore total force of the system is $F = B - F_g = (\rho_f - \rho_0) V g$

What does this tell you?

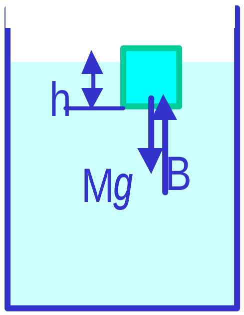
The total force applies to different directions, depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger that the fluid's, the object will sink to the bottom of the fluid.

More Archimedes' Principle

Case 2: Floating object

Let's consider an object of mass M, with density ρ_0, is in static equilibrium floating on the surface of the fluid with density ρ_f , and the volume submerged in the fluid is V_f.



The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating its density is always smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

Example 15.5

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = r_w V_w g = r_w V_c g = 0.98N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98N}{r_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$r_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg/m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is either not made of pure gold or hollow.

Example 15.6

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is V_i. Then the weight of the iceberg F_{gi} is

$$F_{gi} = r_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is V_w. The buoyant force B caused by the displaced water becomes

$$B = r_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$r_i V_i g = r_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{r_i}{r_w} = \frac{917\,kg/m^3}{1030\,kg/m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

Superposition and Interference

If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves.

Superposition Principle

The waves that follow this principle are called ***linear waves*** which in general have small amplitudes. The ones that don't are ***nonlinear waves*** with larger amplitudes.

Thus, one can write the resultant wave function as

$$y = y_1 + y_2 + \cdots + y_n = \sum_{i=1}^{n} y_i$$

Two traveling linear waves can pass through each other without being destroyed or altered.

What do you think will happen to the water waves when you throw two stones in the pond?

They will pass right through each other.

What happens to the waves at the point where they meet?

The shape of wave will change → Interference

Constructive interference: The amplitude increases when the waves meet

Destructive interference: The amplitude decreases when the waves meet

Speed of Waves on Strings

How do we determine the speed of a transverse pulse traveling on a string?

If a string under tension is pulled sideways and released, the tension is responsible for accelerating a particular segment of the string back to the equilibrium position.

So what happens when the tension increases?

The acceleration of the particular segment increases

Which means?

The speed of the wave increases.

Now what happens when the mass per unit length of the string increases?

For the given tension, acceleration decreases, so the wave speed decreases.

Which law does this hypothesis based on?

Newton's second law of motion

Based on the hypothesis we have laid out above, we can construct a hypothetical formula for the speed of wave

$$v = \sqrt{\frac{T}{m}}$$

T: Tension on the string

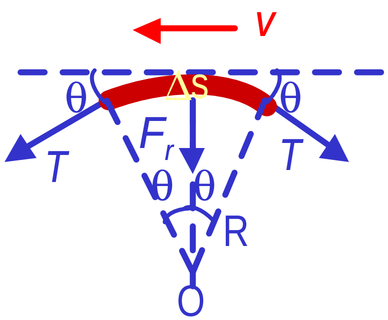
μ: Unit mass per length

Is the above expression dimensionally sound?

$T=[MLT^{-2}]$, $\mu=[ML^{-1}]$

$(T/\mu)^{1/2}=[L^{2}T^{-2}]^{1/2}=[LT^{-1}]$

Speed of Waves on Strings cont'd



Let's consider a pulse moving right and look at it in the frame that moves along with the the pulse.

Since in the reference frame moves with the pulse, the segment is moving to the left with the speed *v*, and the centripetal acceleration of the segment is

$$a_r = \frac{v^2}{R}$$

Now what do the force components look in this motion when θ is small?

$$\sum F_t = T\cos\theta - T\cos\theta = 0$$

$$\sum F_r = 2T\sin\theta \approx 2T\theta$$

What is the mass of the segment when the line density of the string is μ?

$$m = \mu\Delta s = \mu R 2\theta = 2\mu R\theta$$

Using the radial force component

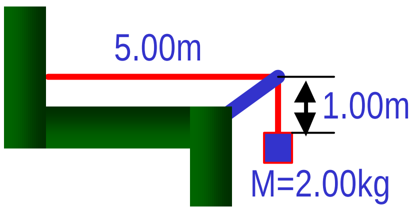
$$\sum F_r = ma = m\frac{v^2}{R} = 2\mu R\theta\frac{v^2}{R} = 2T\theta$$

Therefore the speed of the pulse is

$$v = \sqrt{\frac{T}{\mu}}$$

Example 16.2

A uniform cord has a mass of 0.300kg and a length of 6.00m. The cord passes over a pulley and supports a 2.00kg object. Find the speed of a pulse traveling along this cord.



Since the speed of wave on a string with line density μ and under the tension T is

$$v = \sqrt{\frac{T}{m}}$$

The line density μ is

$$m = \frac{0.300\,kg}{6.00\,m} = 5.00 \times 10^{-2}\,kg/m$$

The tension on the string is provided by the weight of the object. Therefore

$$T = Mg = 2.00 \times 9.80 = 19.6\,kg \cdot m/s^2$$

Thus the speed of the wave is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{19.6}{5.00 \times 10^{-2}}} = 19.8\,m/s$$