PHYS 1443 – Section 003
Lecture #2

Wednesday, Aug. 27, 2003

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1. Dimensional Analysis
2. Fundamentals
3. One Dimensional Motion
   - Displacement
   - Velocity and Speed
   - Acceleration
   - Motion under constant acceleration

Announcements

- Homework: 14 of you have signed up (out of 35)
  - Roster will be locked at the end of the day Wednesday, Sept. 3
  - In order for you to obtain 100% on homework #1, you need to pickup the homework, attempt to solve it and submit it.
  - First real homework assignment will be issued next Wednesday.
  - Remember! Homework constitutes **15% of the total**….
- Your e-mail account is automatically assigned by the university, according to the rule: fml####@exchange.uta.edu. Just subscribe to the PHYS1443-003-FALL02.
- e-mail distribution list:1 of you have subscribed so far.
  - This is the primary communication tool. So subscribe to it ASAP.
  - Going to give **5, 3, and 1** extra credit points for those of you subscribe by today, tomorrow and Friday
  - A test message will be sent next Wednesday for verification purpose.
- First pop quiz will be next Wednesday. Will cover up to where we finish today.
- No class next Monday, Sept. 1, Labor day

Dimension and Dimensional Analysis

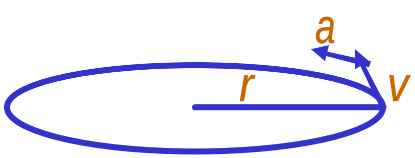
- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say “Dimension of Length, Mass or Time”
  - Dimensions are used as algebraic quantities: Can perform algebraic operations, addition, subtraction, multiplication or division

Dimension and Dimensional Analysis

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
    - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^nT^m] =[L]\{T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

- Show that the expression *[v] = [at]* is dimensionally correct
  - Speed: *[v]* =L/T
  - Acceleration: *[a]* =L/T^2
  - Thus, *[at]* = (L/T^2)xT=L$T^{(-2+1)}$ =LT^{-1} =L/T= *[v]*
- Suppose the acceleration *a* of a circularly moving particle with speed *v* and radius *r* is proportional to r^n and v^m. What are *n* and *m*?



$$a = \underbrace{k}_{\text{Dimensionless constant}}\underbrace{r^n}_{\text{Length}}\underbrace{v^m}_{\text{Speed}}$$

$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$-m = -2 \Rightarrow m = 2$$

$$n + m = n + 2 = 1 \Rightarrow n = -1$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Some Fundamentals

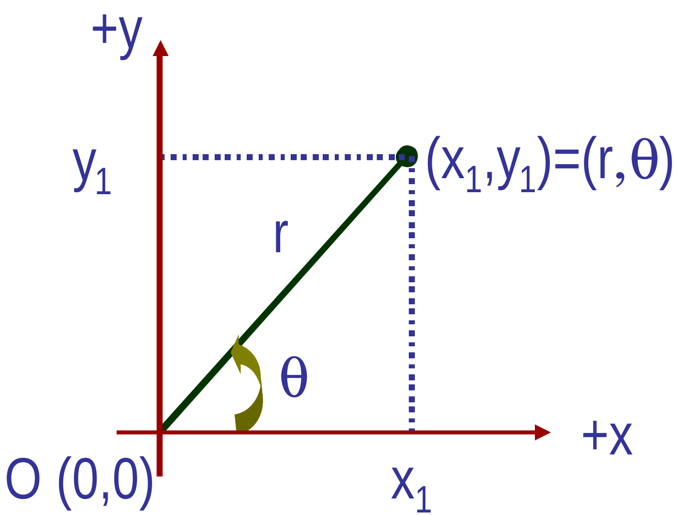
- Kinematics: Description of Motion without understanding the cause of the motion
- Dynamics: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - Scalar: Physical quantities that require magnitude but no direction
    - Speed, length, mass, height, volume, area, magnitude of a vector quantity, etc
  - Vector: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration, Force, Momentum
    - It does not make sense to say "I ran with velocity of 10miles/hour."
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle)in celestial problems
    - Simplification of the problem (The first step in setting up to solve a problem…)
  - Any other examples?

Some More Fundamentals

- Motions:Can be described as long as the position is known at any time (or position is expressed as a function of time)
  - Translation: Linear motion along a line
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ➔ Motion in one-dimension is a motion on a line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

$$x = r\cos q$$
$$y = r\sin q$$

$$r = \sqrt{\left(x_1^2 + y_1^2\right)}$$
$$\tan q = \frac{y_1}{x_1}$$

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final potions of motion and is a vector quantity. How is this different than distance?

Average velocity is defined as:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Displacement per unit time in the period throughout the motion

Average speed is defined as:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

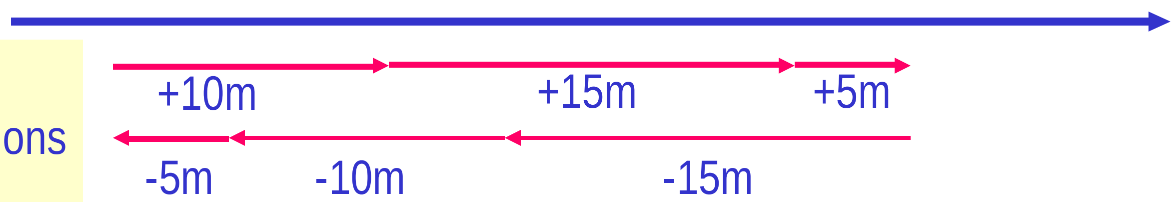
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0$

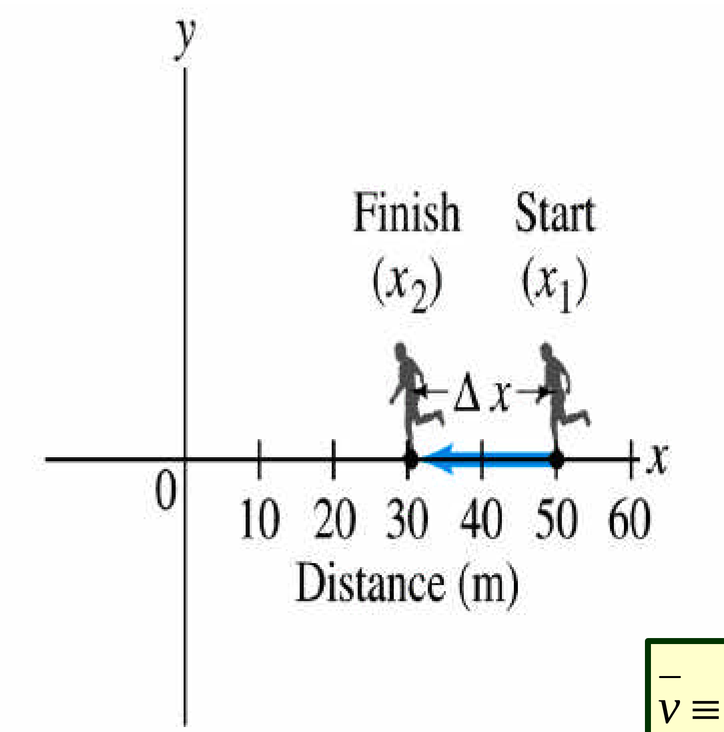
Average Velocity: $\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)}$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}} = \frac{60}{20} = 3(m/s)}$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00 s time interval, the runners position changes from x_1=50.0m to x_2=30.5m, as shown in the figure. Find the displacement, distance, average velocity, and average speed.



- Displacement:

$$\boxed{\Delta x \equiv x_2 - x_1 = 30.5 - 50.0 = -19.5(m)}$$

- Distance:

$$\boxed{|\Delta x| \equiv |x_2 - x_1| = |30.5 - 50.0| = 19.5(m)}$$

- Average Velocity:

$$\boxed{\bar{v}_x \equiv \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)}$$

- Average Speed:

$$\boxed{\bar{v} \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{19.5}{3.00} = 6.50(m/s)}$$

*Magnitude of Vectors are Expressed in absolute values

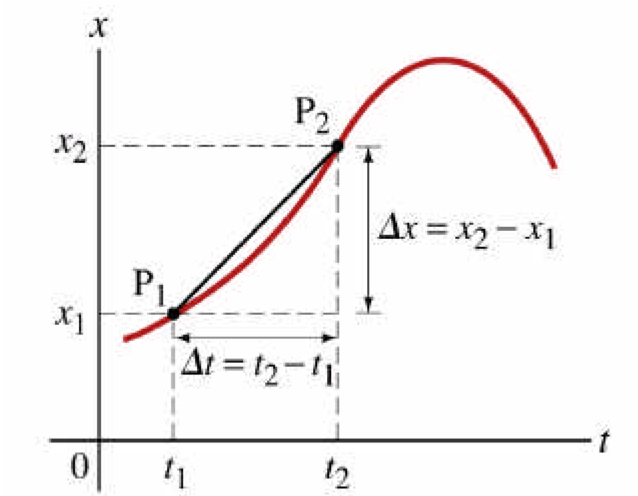
Instantaneous Velocity and Speed

Here is where calculus comes in to help understanding the concept of "instantaneous quantities"

Instantaneous velocity is defined as:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- What does this mean?
  - Displacement in an infinitesimal time interval
  - Mathematically: Slope of the position variation as a function of time
  - For a motion on a certain displacement, it might not move at the average velocity at all times.



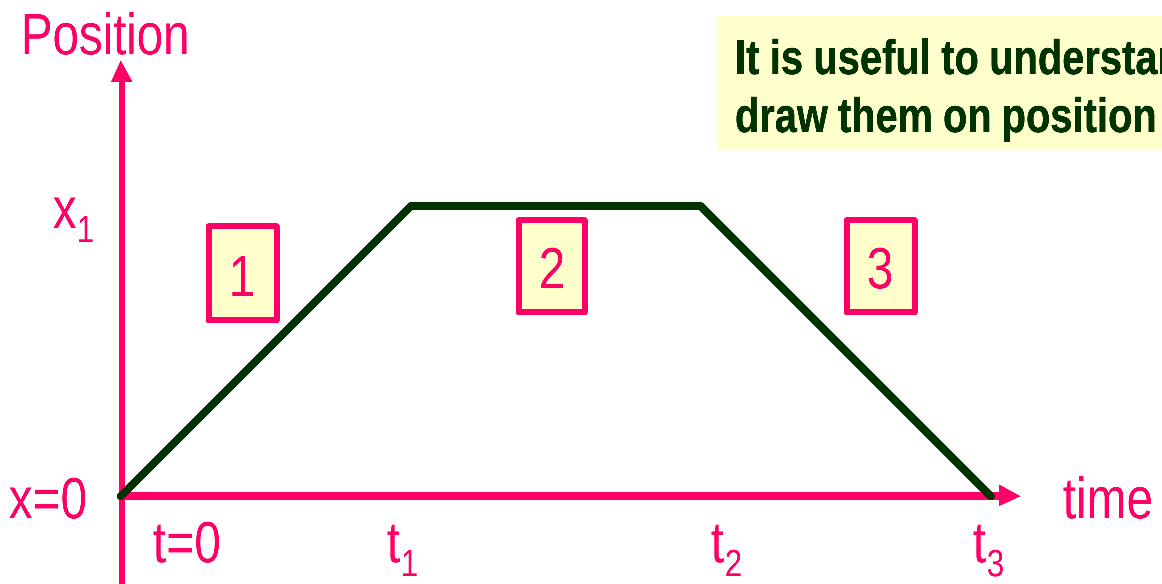
Instantaneous speed is the size (magnitude) of the instantaneous velocity:

$$\left|v_x\right| \equiv \lim_{\Delta t \to 0} \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|$$

*Magnitude of Vectors are Expressed in absolute values

Position vs Time Plot

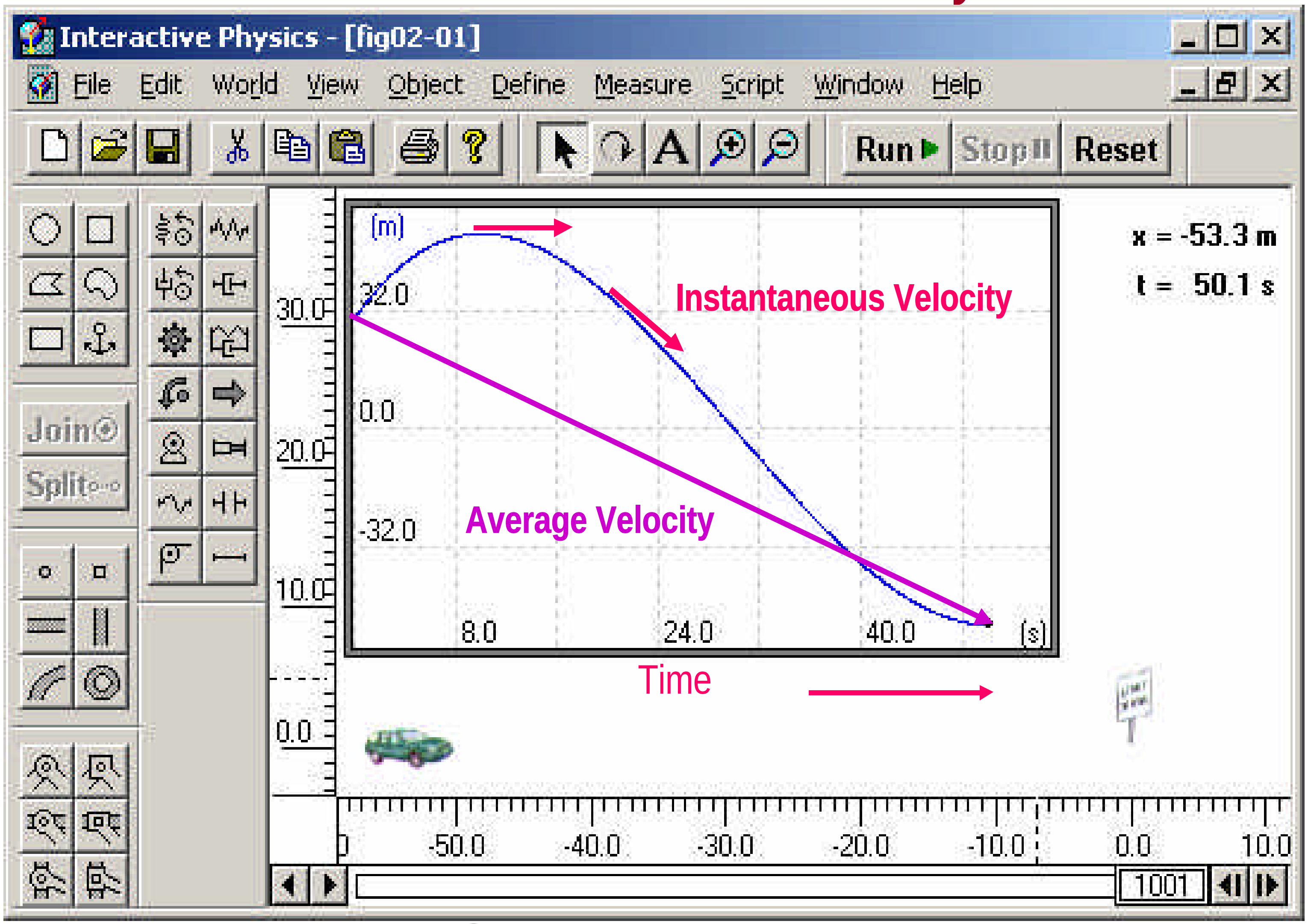
It is useful to understand motions to draw them on position vs time plots.



1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

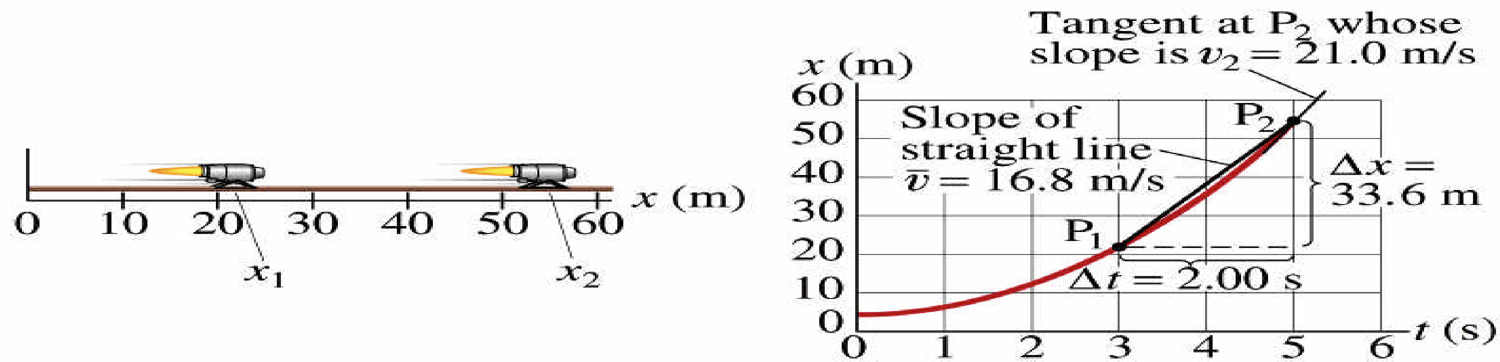
Does this motion physically make sense?

Instantaneous Velocity



Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation $x=At^2+B$ where A=2.10m/s^2 and B=2.80m.



(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2+2.80 = 21.7m$$

$$x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2+2.80 = 55.3m$$

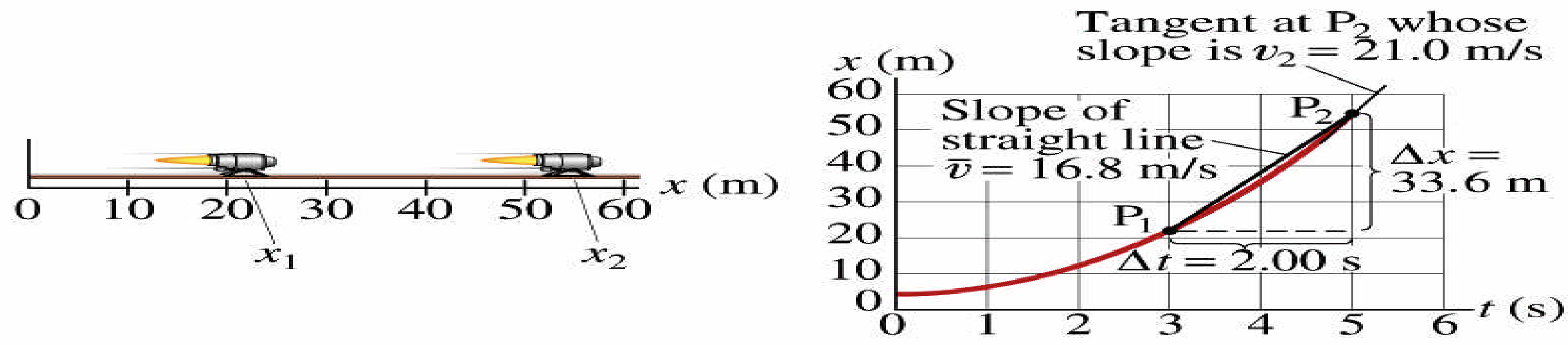
Displacement is, therefore:

$$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00-3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example 2.3 cont'd



(c) Determine the instantaneous velocity at $t=t^2=5.00s$.

Calculus formula for derivative $\boxed{\frac{d}{dt}\left(Ct^n\right)=nCt^{n-1}}$ and $\boxed{\frac{d}{dt}\left(C\right)=0}$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$\boxed{v_x\left(t = 5.00s\right) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0\left(m/s\right)}$$