PHYS 1443 – Section 003
Lecture #3

Wednesday, Sept. 3, 2003

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1. One Dimensional Motion
   - Acceleration
   - Motion under constant acceleration
   - Free Fall
2. Motion in Two Dimensions
   - Vector Properties and Operations
   - Motion under constant acceleration
   - Projectile Motion

Today's homework is homework #2, due noon, next Wednesday!!

Announcements

- Homework: 33 of you have signed up (out of 36)
  - Roster will be locked at 5pm today
  - In order for you to obtain 100% on homework #1, you need to pickup the homework, attempt to solve it and submit it. ➔ Only about 27 of you have done this.
  - Homework system deducts points for failed attempts.
    - So be careful when you input the answers
    - Input the answers to as many significant digits as possible
  - All homework problems are equally weighted
- e-mail distribution list:14 of you have subscribed so far.
  - This is the primary communication tool. So subscribe to it ASAP.
  - A test message will be sent after the class today for verification purpose
- Physics Clinic (Supplementary Instructions): 11-6, M-F
- Could I speak to Steven Smith after the class?

Survey Resuts

- 25 of you have taken Physics before
  - 18 in High School
  - 10 of you had Classical Mechanics while 4 said E&M
  - 13 of you are taking this course due to degree requirements
  - About 5 of you wanted to have better understanding
- 24 of you had some level of calculus before
  - 10 in HS and 13 in college
  - You must have had it already or must be having it concurrently
- 31 of you said physics is mandatory
- 16 of you take this course because you like physics

Let's have fun together this semester!!!

Displacement, Velocity and Speed

Displacement

$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity

$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed

$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity

$$\boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed

$$\boxed{\left|v_x\right| \equiv \lim_{\Delta t \to 0} \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|}$$

Acceleration

Change of velocity in time (what kind of quantity is this?)

- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$ analogous to $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$ analogous to $$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- In calculus terms: A slope (derivative) of velocity with respect to time or change of slopes of position as a function of time

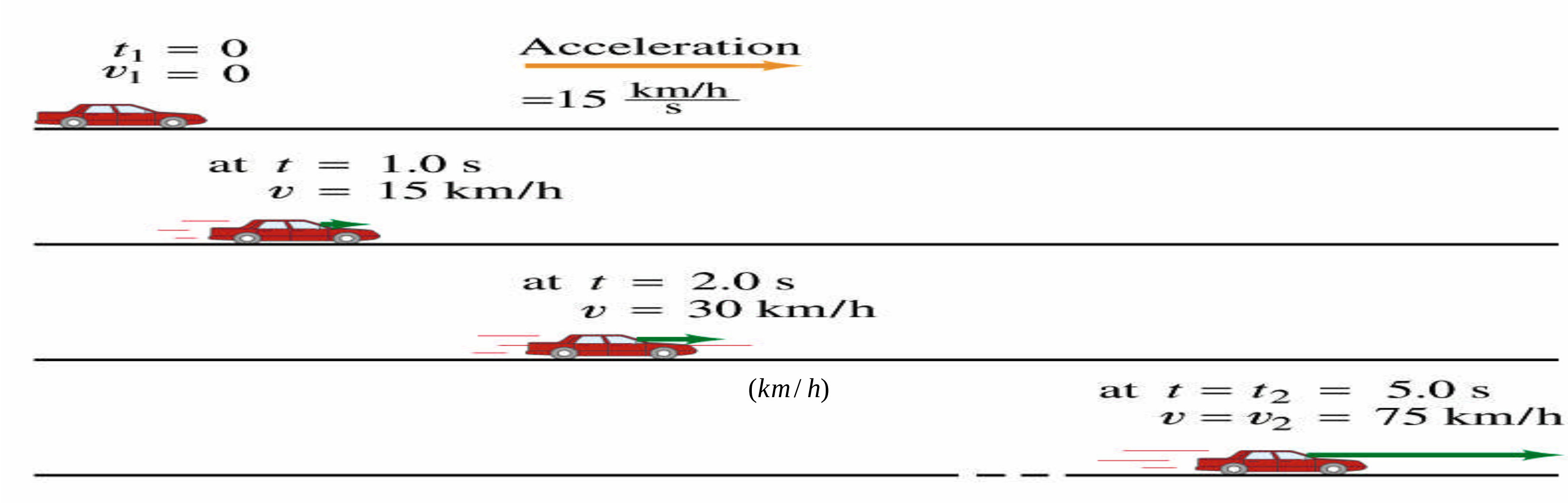
Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Could there be acceleration when an object is not moving? YES!
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$)
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

Example 2.4

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$\boxed{v_{xi} = 0\,m/s}$$

$$\boxed{v_{xf} = \frac{75000m}{3600s} = 21\,m/s}$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Example 2.7

A particle is moving on a straight line so that its position as a function of time is given by the equation $x=(2.10m/s^2)t^2+2.8m$.

(a) Compute the average acceleration during the time interval from t_1=3.00s to t_2=5.00s.

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(2.10t^2 + 2.80\right) = 4.20t$$

$$v_{xi} = v_x(t = 3.00s) = 4.20 \times 3.00 = 12.6(m/s)$$

$$v_{xf} = v_x(t = 5.00s) = 4.20 \times 5.00 = 21.0(m/s)$$

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{21.0 - 12.6}{5.00 - 3.00} = \frac{8.40}{2.00} = 4.2\left(m/s^2\right)$$

(b) Compute the particle's instantaneous acceleration as a function of time.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}(4.20t) = 4.20\left(m/s^2\right)$$

What does this mean?

The acceleration of this particle is independent of time.

This particle is moving under a constant acceleration.

One Dimensional Motion

- Let's start with the simplest case: acceleration is constant (a=a_0)
- Using definitions of average acceleration and velocity, we can draw equations of motion (description of motion, velocity and position as a function of time)

$$\boxed{\overline{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xf} - v_{xi}}{t}}$$

If t_f=t and t_i=0

$$\boxed{v_{xf} = v_{xi} + a_x t}$$

For constant acceleration, simple numeric average

$$\boxed{\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t}$$

$$\boxed{\overline{v}_x = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t}}$$

If t_f=t and t_i=0

$$\boxed{x_f = x_i + \overline{v}_x t}$$

Resulting Equation of Motion becomes

$$\boxed{x_f = x_i + \overline{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

One Dimensional Motion cont'd

Average velocity $\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$ $\Rightarrow$ $x_f = x_i + \bar{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right)t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ $\Rightarrow$ $t = \frac{v_{xf} - x_{xi}}{a_x}$

Substituting t in the above equation, we obtain

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Kinetic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}(v_{xf} + v_{xi})t$$

Displacement as a function of velocity and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/s (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? $\Rightarrow$ As long as it takes for it to crumple.

The initial speed of the car is $$v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinetic formula $$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

The acceleration is $$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0 - (28m/s)^2}{2 \times 1m} = -390m/s^2$$

Thus the time for air-bag to deploy is $$t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28m/s}{-390m/s^2} = 0.07s$$

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Example for Using 1D Kinetic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at maximum height.

What is so special about the maximum height? V=0

$$\boxed{v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s}$$

$$\boxed{t = \frac{20.0}{9.80} = 2.04s}$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$
$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$\boxed{t = 2.04 \times 2 = 4.08s}$$

(d) Find the velocity of the stone when it reaches its original height.

$$\boxed{v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0 (m/s)}$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

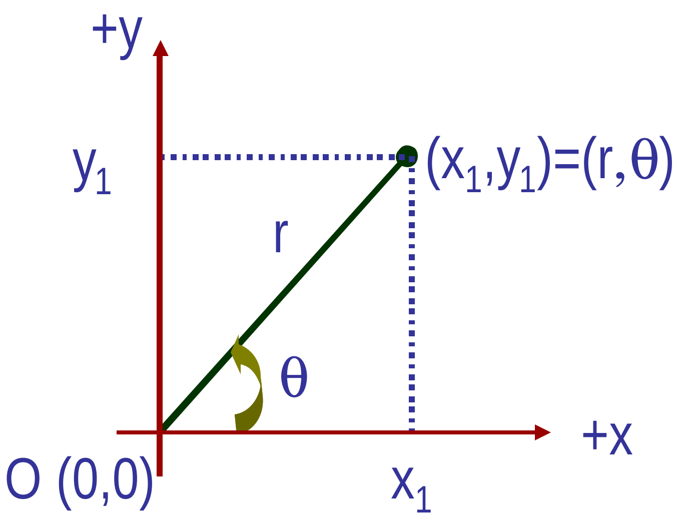
$$\boxed{v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0 (m/s)}$$

Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$
$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5 (m)$$

Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



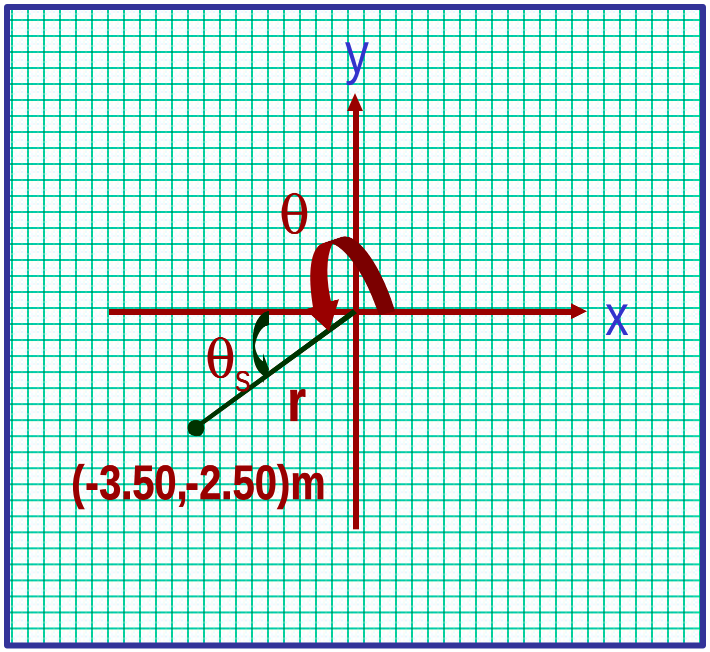
How are Cartesian and Polar coordinates related?

$$x = r\cos q$$
$$y = r\sin q$$

$$r = \sqrt{\left(x_1^2 + y_1^2\right)}$$
$$\tan q = \frac{y_1}{x_1}$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$
$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$
$$= \sqrt{18.5} = 4.30\,(m)$$

$$q = 180 + q_s$$

$$\tan q_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$q_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^{\circ}$$

$$\boxed{\therefore q = 180 + q_s = 180^{\circ} + 35.5^{\circ} = 216^{\circ}}$$