PHYS 1443 – Section 003
Lecture #5

Wednesday, Sept. 10, 2003
Dr. Jaehoon Yu

- Motion in Two Dimensions
  - Vector Components
  - 2D Motion under constant acceleration
  - Projectile Motion

Today's homework is homework #3, due noon, next Wednesday!!

Announcements

- Your lab-sessions began Monday, Sept. 8. Be sure to attend the lab classes.
- Quiz #2 next Wednesday, Sept. 17
- Homework: 100% of you have signed up
  - Very good!!!
  - If you are new to the class, please come see me after the lecture
- e-mail distribution list:26 of you have subscribed so far.
  - There will be negative extra credit from this week
    - -1 point if not done by 5pm, Friday, Sept. 12
    - -3 points if not done by 5pm, Friday, Sept. 19
    - -5 points if not done by 5pm, Friday, Sept. 26
  - A test message will be sent next Wednesday for verification purpose
- Please use office hours or send me e-mail for appointments
  - 2:30 – 3:30pm, Mondays and Wednesdays

Kinetic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t}$$

Displacement as a function of velocity and time

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

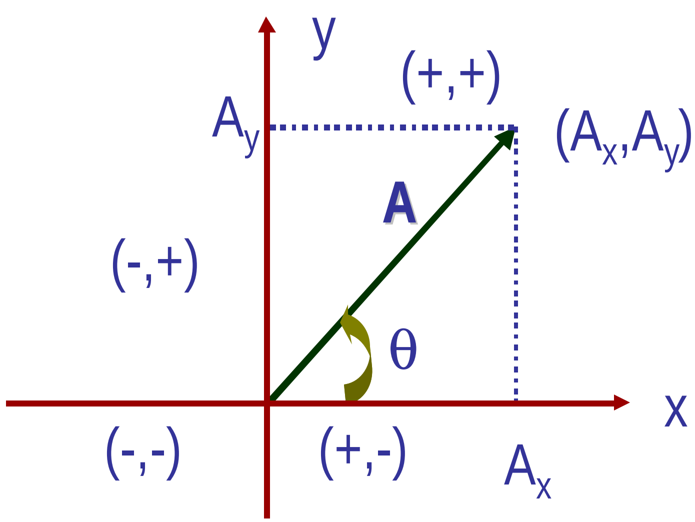
Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Vector Components

- Coordinate systems are useful in expressing vectors in their components (Sizes along coordinate axes…)
  - Expressing vectors in their components makes vector algebra easier
  - Well but then there is something missing….

  ⇒Something that tells you the direction…



$$\boxed{A_x = \left|\vec{A}\right| \cos q}$$

$$\boxed{A_y = \left|\vec{A}\right| \sin q}$$

} Components

$$\boxed{\left|\vec{A}\right| = \sqrt{A_x^{\,2} + A_y^{\,2}}}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos q\right)^2 + \left(\left|\vec{A}\right|\sin q\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2 q + \sin^2 q\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **Dimensionless**
- **Magnitudes are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{j},\ \vec{k}$$

So the vector **A** can be re-written as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos q\,\vec{i} + \left|\vec{A}\right|\sin q\,\vec{j}$$

Examples of Vector Additions

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C}=\vec{A}+\vec{B}=\left(2.0\vec{i}+2.0\vec{j}\right)+\left(2.0\vec{i}-4.0\vec{j}\right)\;=\left(2.0+2.0\right)\vec{i}+\left(2.0-4.0\right)\vec{j}=\left(4.0\vec{i}-2.0\vec{j}\right)m$$

OR

$$\left|\vec{C}\right|=\sqrt{(4.0)^2+(-2.0)^2}=\sqrt{16+4.0}=\sqrt{20}=4.5(m)$$

Magnitude

$$q=\tan^{-1}\frac{C_y}{C_x}=\tan^{-1}\frac{-2.0}{4.0}=-27^{\circ}$$

Direction

Find the resultant displacement of three consecutive displacements:
d$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_1$=(-13**i**+15**j**)cm

$$\vec{D}=\vec{d}_1+\vec{d}_2+\vec{d}_3\;=\left(15\vec{i}+30\vec{j}+12\vec{k}\right)+\left(23\vec{i}+14\vec{j}-5.0\vec{k}\right)+\left(-13\vec{i}+15\vec{j}\right)$$

$$=\left(15+23-13\right)\vec{i}+\left(30+14+15\right)\vec{j}+\left(12-5.0\right)\vec{k}\;=25\vec{i}+59\vec{j}+7.0\vec{k}(cm)$$

$$\left|\vec{D}\right|=\sqrt{(25)^2+(59)^2+(7.0)^2}=65(cm)$$

Magnitude

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?

Kinetic Quantities in 1d and 2d

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\boxed{\vec{r}_i = x_i \vec{i} + y_i \vec{j}} \qquad \boxed{\vec{r}_f = x_f \vec{i} + y_f \vec{j}}$$

- Velocity vectors in x-y plane:

$$\boxed{\vec{v}_i = v_{xi} \vec{i} + v_{yi} \vec{j}} \qquad \boxed{\vec{v}_f = v_{xf} \vec{i} + v_{yf} \vec{j}}$$

Velocity vectors in terms of acceleration vector

Velocity vector components

$$\boxed{v_{xf} = v_{xi} + a_x t} \qquad \boxed{v_{yf} = v_{yi} + a_y t}$$

Putting them together in a vector form

$$\boxed{\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j}} \qquad \boxed{\vec{v}_f = \vec{v}_i + \vec{a}t}$$

2-dim Motion Under Constant Acceleration

- How are the position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping above results in

$$\vec{r}_f = \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2}$$

Example in 2-D Kinetic EoM

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\,(m/s)$$

$$v_{yf} = v_{yi} + a_y t = -15\,(m/s)$$

$$\vec{v}(t) = \left\{(20+4.0t)\vec{i} - 15\vec{j}\right\} m/s$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v} = \left\{(20+4.0\times 5.0)\vec{i} - 15\vec{j}\right\} m/s = \left(40\vec{i} - 15\vec{j}\right) m/s$$

$$speed = \left|\vec{v}\right| = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(40)^2 + (-15)^2} = 43m/s$$

Magnitude

Example in 2-D Kinetic EoM cont'd

Direction $q = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^o$

Determine the *x* and *y* components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

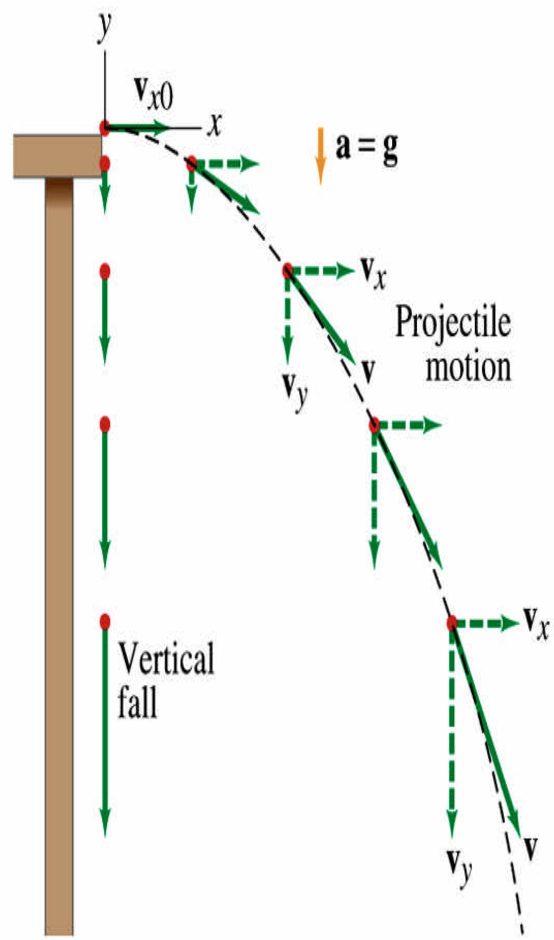
$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = \left(150\vec{i} - 75\vec{j}\right)m$$

Projectile Motion

- A 2-dim motion of an object ur the gravitational acceleration v the assumptions
  - Free fall acceleration, -*g*, is con: over the range of the motion
  - Air resistance and other effects negligible
- A motion under constant acceleration!!!! ➔ Superpositi of two motions
  - Horizontal motion with constant velocity (<u>no acceleration</u>)
  - Vertical motion under constant acceleration (*g*)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos q_i$

y-component $v_{yi} = v_i \sin q_i$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0

$$x_f = v_{xi}t = v_i \cos q_i t$$

$$t = \frac{x_f}{v_i \cos q_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i \sin q_i t - \frac{1}{2}gt^2$$

Plug t into the above

$$y_f = v_i \sin q_i \left(\frac{x_f}{v_i \cos q_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos q_i}\right)^2$$

$$y_f = x_f \tan q_i - \left(\frac{g}{2v_i^2 \cos^2 q_i}\right)x_f^2$$

What kind of parabola is this?