PHYS 1443 – Section 003
Lecture #6

Monday, Sept. 15, 2003
Dr. Jaehoon Yu

- Motion in Two Dimensions
  - Projectile Motion
- Reference Frames
- Forces
- Newton's Laws of Motion

Announcements

- Quiz #2 this Wednesday, Sept. 17
- e-mail distribution list:29 of you have subscribed so far.
  - There will be negative extra credit from this week
    - -1 point if not done by 5pm, Friday, Sept. 12
    - -3 points if not done by 5pm, Friday, Sept. 19
    - -5 points if not done by 5pm, Friday, Sept. 26
  - A test message will be sent Wednesday for verification purpose
- Remember the 1st term exam, **<u>Monday, Sept. 29</u>**, two weeks from today
  - Covers up to chapter 6.
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: **<u>Your grade is an F.</u>**
  - Mixture of multiple choice and essay problems

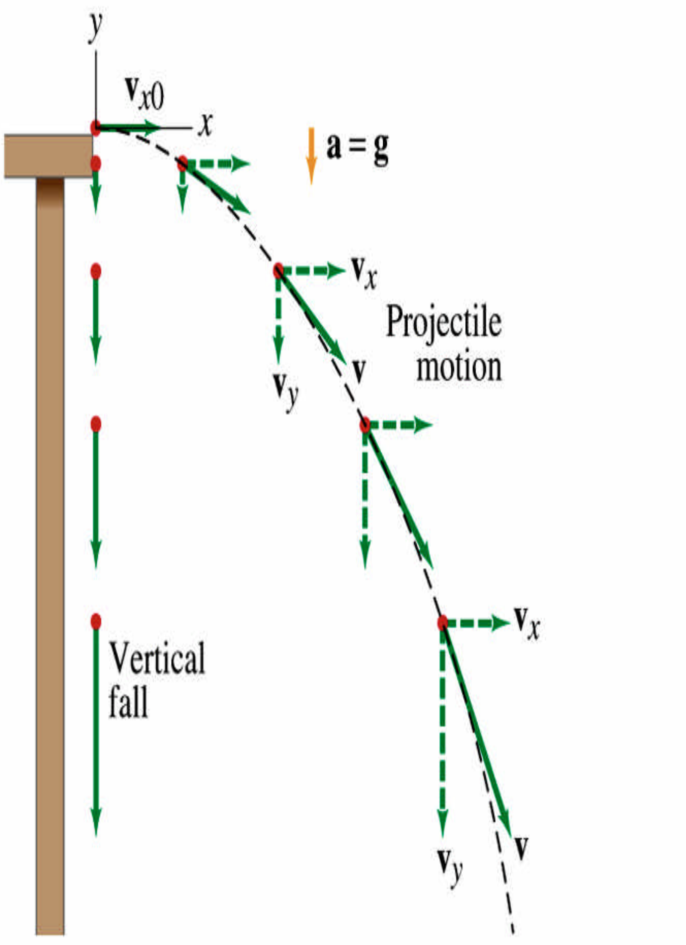
Kinetic Quantities in 1d and 2d

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

Projectile Motion

- A 2-dim motion of an object ur the gravitational acceleration v the assumptions
  - Free fall acceleration, -*g*, is con: over the range of the motion
  - Air resistance and other effects negligible
- A motion under constant acceleration!!!! ➔ Superpositi of two motions
  - Horizontal motion with constant velocity (no acceleration)
  - Vertical motion under constant acceleration (*g*)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0

$$x_f = v_{xi}t = v_i\cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i\cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

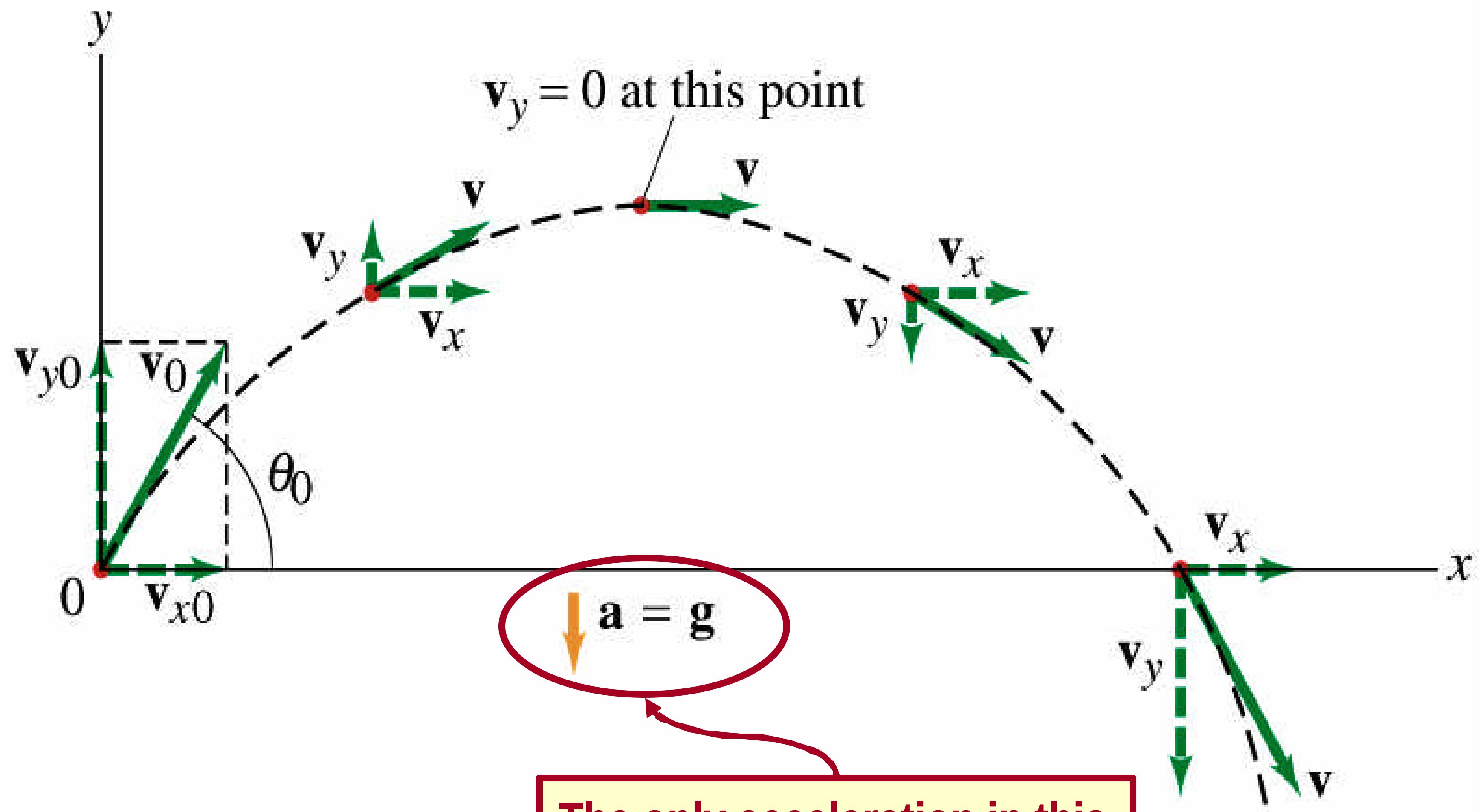
Plug t into the above

$$y_f = v_i\sin\theta_i\left(\frac{x_f}{v_i\cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f\tan\theta_i - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2}$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example of Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by *y* component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t = 0m$$

$$t(80 - gt) = 0$$

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8 \text{ sec}$$

Distance is determined by *x* component in 2-dim, because the ball is at *y=0* position when it completed it's flight.

$$x_f = v_{xi} t$$

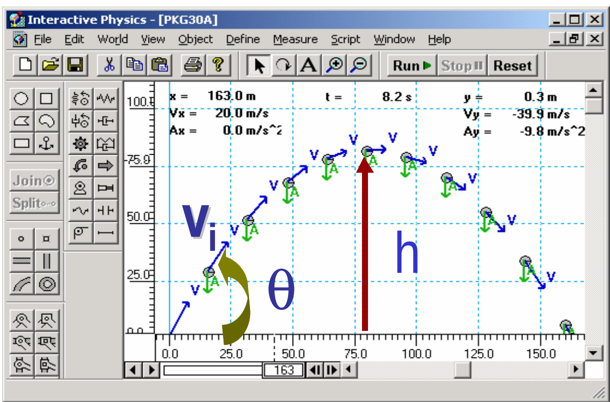
$$= 20 \times 8 = 160 \ (m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t$$

$$= v_i \sin \theta_i - g t_A = 0$$

$$\boxed{\therefore t_A = \frac{v_i \sin \theta_i}{g}}$$

Horizontal Range and Max Height

Since no acceleration in x, it still flies even if v_y=0

$$R = v_{xi}\left(2t_A\right) = 2v_i \cos q_i \left(\frac{v_i \sin q_i}{g}\right)$$

$$\boxed{R = \left(\frac{v_i^{\,2} \sin 2q_i}{g}\right)}$$

$$y_f = h = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i \sin q_i \left(\frac{v_i \sin q_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin q_i}{g}\right)^2$$

$$\boxed{y_f = \left(\frac{v_i^{\,2} \sin^2 q_i}{2g}\right)}$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$\boxed{v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s}$$

$$\boxed{v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s}$$

$$\boxed{y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2}$$

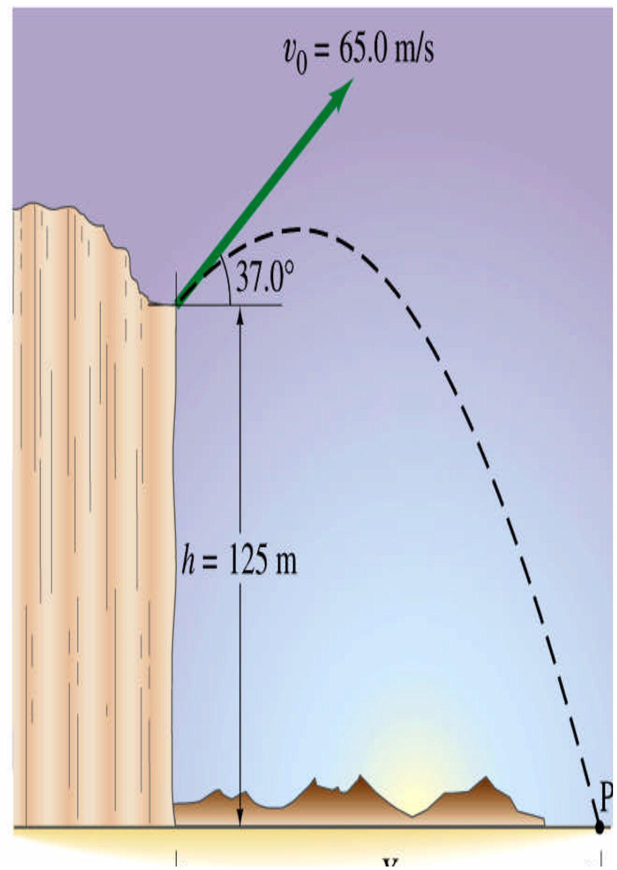
$$\boxed{gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0}$$

$$\boxed{t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}}$$

$$\boxed{t = -2.43 s \quad \text{or} \quad t = 10.4 s}$$

$$\boxed{t = 10.4 s}$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos q_i = 65.0 \times \cos 37^{\circ} = 51.9 m / s$$

$$v_{yf} = v_{yi} - gt = v_i \sin q_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m / s$$

$$\boxed{|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m / s}$$

- What are the maximum height and the maximum range of the stone?

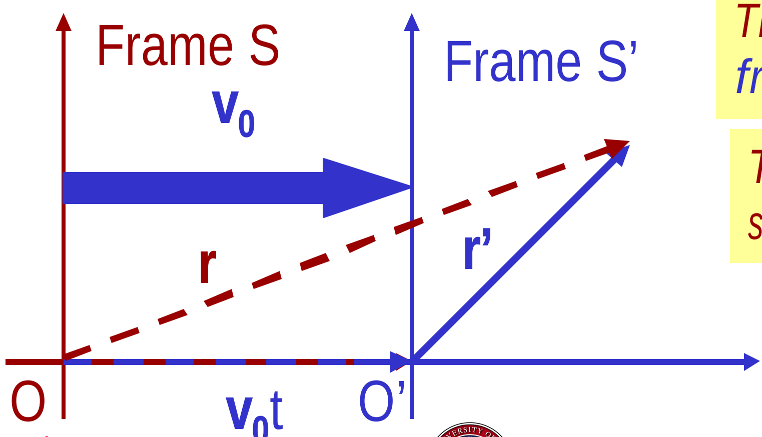
Do these yourselves at home for fun!!!

Observations in Different Reference Frames

Results of Physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you, the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



*The position vector **r'** is still **r'** in the moving frame S'.no matter how much time has passed!!*

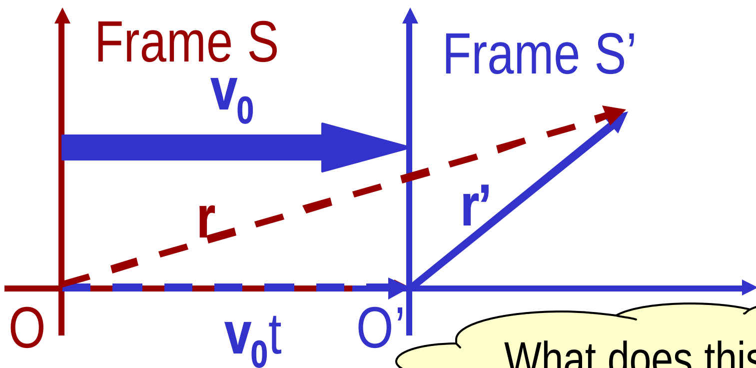
*The position vector **r** is no longer **r** in the stationary frame S when time t has passed.*

How are these position vectors related to each other?

$$\vec{r}(t)=\vec{r}'+\vec{v}_0 t$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$$\vec{a}\,' = \vec{a}, \text{ when } \vec{v}_0 \text{ is constant}$$

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion was. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCEs are what cause an object to move

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

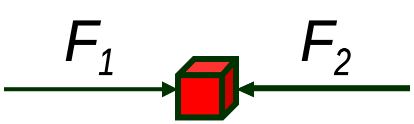
FORCEs are what cause any change in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Forces cause acceleration.

What happens there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the motion of the object.



NET FORCE,
$F = F_1 + F_2$

When net force on an objectis 0, it has constant velocity and is at its equilibrium!!

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs higher force.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the **1st law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.

A frame of reference that is moving at constant velocity is called an *Inertial Frame*