PHYS 1443 – Section 003
Lecture #7

Wednesday, Sept. 17, 2003
Dr. Jaehoon Yu

- Newton's Laws of Motion
  - Newton's 2nd Law of Motion
  - Newton's 3rd Law of Motion
  - The Force of Gravity
  - Freebody Diagrams
- Friction

Today's homework is homework #4, due noon, next Wednesday!!

Remember the first term exam on Monday, Sept. 29!!

Announcement

- Almost all of you have done e-mail. We only have five people not on the list.
  - I will send out a test message this afternoon ➔ Please reply if you receive the message.
  - -2 points if not done by this Friday
- Can I speak to:
  - Robyn Barber, Mark Helms & James Mann after the class?

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

PHYS 1443-003, Fall 2003
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Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs higher force.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the **1st law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.

A frame of reference that is moving at constant velocity is called an *Inertial Frame*

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

1. Independent of the object's surroundings: The same no matter where you go.
2. Independent of method of measurement: The same no matter how you measure it.

The heavier an object gets the bigger the inertia!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F_i} = m\vec{a}$$

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x \quad \sum_i F_{iy} = ma_y \quad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is $[m][a] = [M][LT^{-2}]$
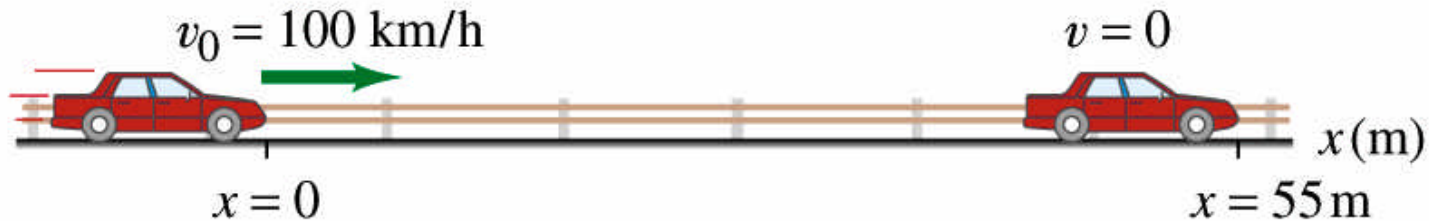
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, a Newton (N)

$$1\,N \equiv 1\,kg \cdot m/s^2 \approx \frac{1}{4}\,lbs$$

Example for Newton's 2nd Law of Motion

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?

$v_0 = 100$ km/h $v = 0$

$x = 0$ $x(m)$

                        $x = 55\,m$

What do we need to know to figure out the force?   Acceleration!!

What are given?   Initial speed: $v_{xi} = 100 \ km/h = \ 28\,m/s$ Final speed: $v_{xf} = 0\,m/s$

Displacement: $\Delta x = x_f - x_i = 55\,m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Acceleration $\Rightarrow$

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{-\left(28m/s\right)^2}{2\left(55m\right)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = \ ma_x = 1500kg \times \left(-7.1m/s^2\right) = -1.1 \times 10^4\,N$$

Given the force how long does it take to stop a car?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2ma_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2F_x}$$

•Linearly proportional to the mass of the car
•Squarely proportional to the speed of the car
•Inversely proportional to the force by the brake
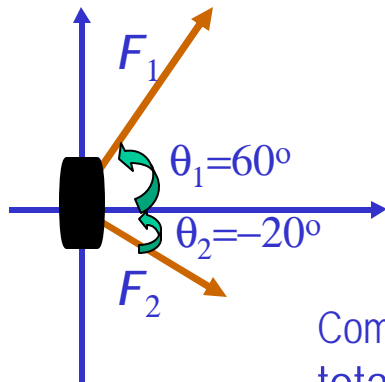
Wednesday, Sept. 17, 2003   PHYS 1443-003, Fall 2003
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Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.

Components of F_1

$$F_{1x} = \left|\vec{F_1}\right|\cos q_1 = 8.0 \times \cos(60°) = 4.0 N$$

$$F_{1y} = \left|\vec{F_1}\right|\sin q_1 = 8.0 \times \sin(60°) = 6.9 N$$

Components of F_2

$$F_{2x} = \left|\vec{F_2}\right|\cos q_2 = 5.0 \times \cos(-20°) = 4.7 N$$

$$F_{2y} = \left|\vec{F_2}\right|\sin q_2 = 5.0 \times \sin(-20°) = -1.7 N$$

Components of total force F

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7 N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2 N = ma_y$$

$\theta_1 = 60°$

$\theta_2 = -20°$

F_1

F_2

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29\, m/s^2$$

$$a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17\, m/s^2$$

$$\left|\vec{a}\right| = \sqrt{(29)^2 + (17)^2} = 34 m/s^2$$

$$q = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30°$$

Acceleration Vector a

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} = \left(29\,\hat{i} + 17\,\hat{j}\right)m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\boldsymbol{F_g}$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force, F_{12}, exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force, F_{21}, exerted on object 1 by object 2.
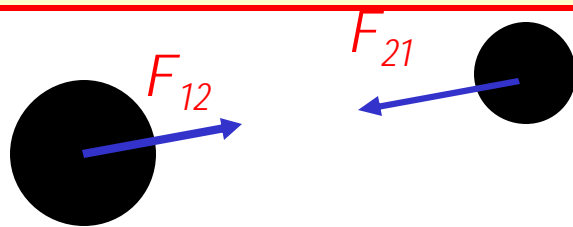
$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act on different objects.

What is the reaction force to the force of a free fall object?

The force exerted by the ground when it completed the motion.

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on **frictionless ice**. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?
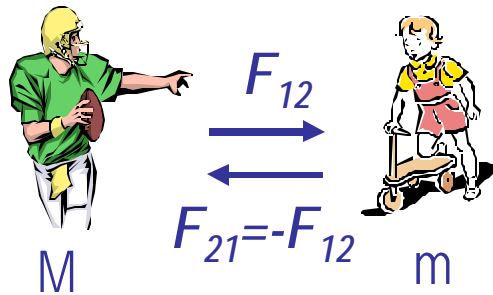
$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \qquad F_{12x} = ma_{bx} \qquad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \qquad F_{21x} = Ma_{Mx} \qquad F_{21y} = Ma_{My} = 0$$

$$\vec{F}_{12} = -\vec{F}_{21} \quad \left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F \qquad a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

F₁₂

F₂₁=-F₁₂

M   m

b) Who moves farther while their hands are in contact?

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

$$\therefore v_{bxf} \rangle v_{Mxf} \quad \text{if} \quad M \rangle m \quad \text{by the ratio of the masses}$$

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

$$x_b = v_{bxf}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{m}v_{Mxf}t + \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(v_{Mxf}t + \frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$