

# PHYS 1443 – Section 003

## Lecture #9

*Wednesday, Sept. 24, 2003*

*Dr. Jaehoon Yu*

- Forces of Friction
- Uniform and Non-uniform Circular Motions
- Resistive Forces and Terminal Velocity
- Newton's Law of Gravitation

**Homework #5 due at midnight next Thursday, Oct. 2!!**

**Remember the first term exam on next Monday, Sept. 29!!**

Wednesday, Sept. 24, 2003



PHYS 1443-003, Fall 2003  
Dr. Jaehoon Yu

# Forces of Friction

*Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.*

*These forces are either proportional to velocity or normal force*

Force of static friction,  $f_s$ :

*The resistive force exerted on the object until just before the beginning of its movement*

Empirical  
Formula

$$|\vec{f}_s| \leq m_s |\vec{n}|$$

*What does this formula tell you?*

Frictional force increases till it reaches to the limit!!

Beyond the limit, there is no more static frictional force but kinetic frictional force takes it over.

Force of kinetic friction,  $f_k$

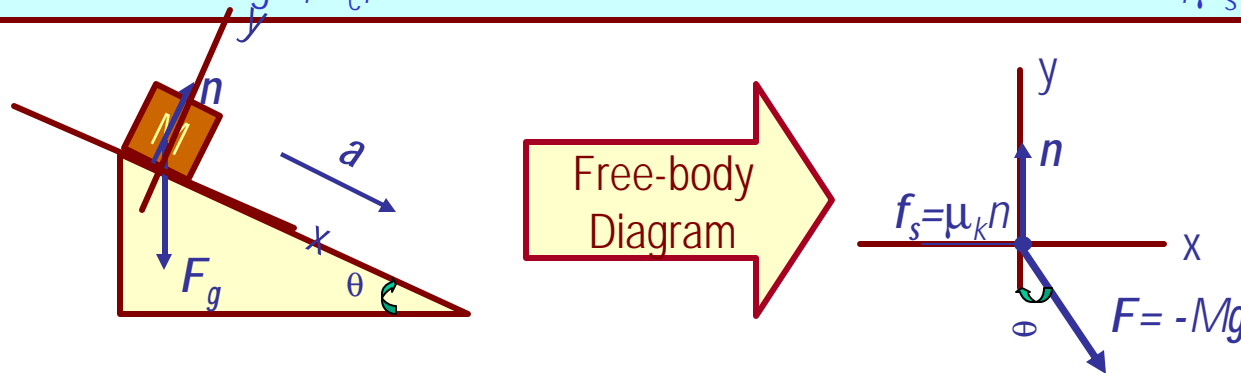
$$|\vec{f}_k| = m_k |\vec{n}|$$

*The resistive force exerted on the object during its movement*



# Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle,  $\theta_c$ , one can determine coefficient of static friction,  $\mu_s$ .



Net force

$$\vec{F} = M \vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$$

x comp.

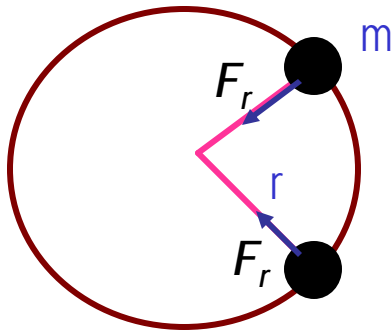
$$F_x = F_{gx} - f_s = Mg \sin q - f_s = 0 \quad f_s = m_s n = Mg \sin q_c$$

y comp.

$$F_y = Ma_y = n - F_{gy} = n - Mg \cos q_c = 0 \quad n = F_{gy} = Mg \cos q_c$$

$$m_s = \frac{Mg \sin q_c}{n} = \frac{Mg \sin q_c}{Mg \cos q_c} = \tan q_c$$

# Newton's Second Law & Uniform Circular Motion



*The centripetal acceleration is always perpendicular to velocity vector,  $v$ , for uniform circular motion.*

$$a_r = \frac{v^2}{r}$$

*Are there forces in this motion? If so, what do they do?*

*The force that causes the centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. This force is called **centripetal force**.*

$$\sum F_r = ma_r = m \frac{v^2}{r}$$

*What do you think will happen to the ball if the string that holds the ball breaks? Why?*

*Based on Newton's 1<sup>st</sup> law, since the external force no longer exist, the ball will continue its motion without change and will fly away following the tangential direction to the circle.*

# Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



*Centripetal  
acceleration:*

$$a_r = \frac{v^2}{r}$$

*When does the  
string break?*

$$\sum F_r = ma_r = m \frac{v^2}{r} > T$$

*When the centripetal force is greater than the sustainable tension.*

$$m \frac{v^2}{r} = T$$

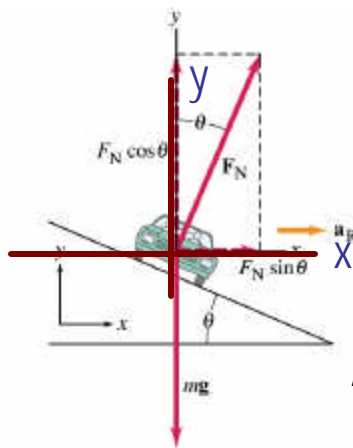
$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2 (m / s)$$

Calculate the tension of the cord  
when speed of the ball is 5.00m/s.

$$T = m \frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33 (N)$$

# Example of Banked Highway

(a) For a car traveling with speed  $v$  around a curve of radius  $r$ , determine a formula for the angle at which a road should be banked so that no friction is required to keep the car from skidding.



x comp.  $\sum F_x = n \sin \mathbf{q} - m a_r = n \sin \mathbf{q} - \frac{m v^2}{r} = 0$

$$n \sin \mathbf{q} = \frac{m v^2}{r}$$

y comp.  $\sum F_y = n \cos \mathbf{q} - m g = 0 \quad n \cos \mathbf{q} = m g$

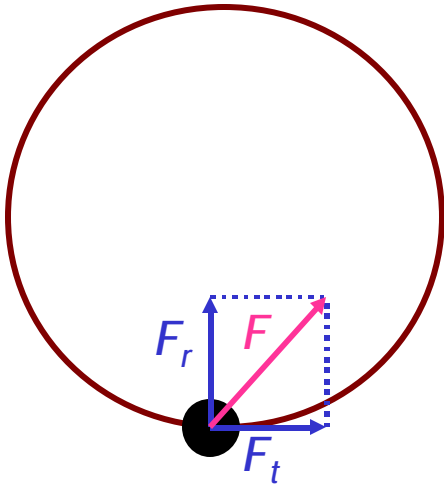
$$n = \frac{m g}{\sin \mathbf{q}} \quad n \sin \mathbf{q} = \frac{m g \sin \mathbf{q}}{\cos \mathbf{q}} = m g \tan \mathbf{q} = \frac{m v^2}{r}$$

$$\tan \mathbf{q} = \frac{v^2}{g r}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50 \text{ km} / \text{hr} = 14 \text{ m} / \text{s} \quad \tan \mathbf{q} = \frac{(14)^2}{50 \times 9.8} = 0.4 \quad \mathbf{q} = \tan^{-1}(0.4) = 22^\circ$$

# Forces in Non-uniform Circular Motion



*The object has both tangential and radial accelerations.*

*What does this statement mean?*

*The object is moving under both tangential and radial forces.*

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

*These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.*

*How does the acceleration look?*

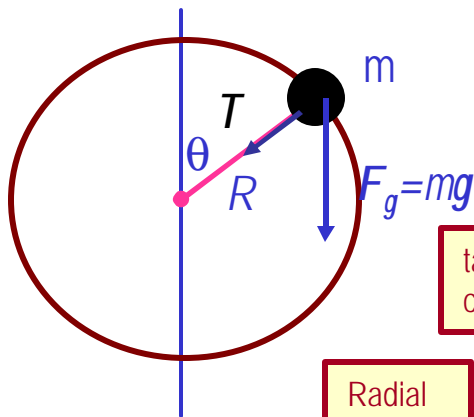
$$a = \sqrt{a_r^2 + a_t^2}$$

# Example of Non-Uniform Circular Motion

A ball of mass  $m$  is attached to the end of a cord of length  $R$ . The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is  $v$  and the cord makes an angle  $\theta$  with vertical.

*What are the forces involved in this motion?*

*The gravitational force  $F_g$  and the radial force,  $T$ , providing tension.*



tangential  
comp.

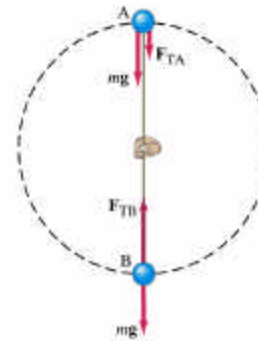
$$\sum F_t = mg \sin q = ma_t \quad a_t = g \sin q$$

Radial  
comp.

$$\sum F_r = T + mg \cos q = ma_r = m \frac{v^2}{R}$$

$$T = m \left( \frac{v^2}{R} - g \cos q \right)$$

At what angles the tension becomes maximum and minimum. What are the tensions?





# Motion in Resistive Forces

*Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.*

*Some examples?*

*Air resistance, viscous force of liquid, etc*

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.

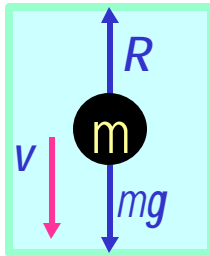
Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed



# Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write  $R=bv$



Let's consider that a ball of mass  $m$  is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R} \quad \sum F_x = 0$$

This equation also tells you that

$$\sum F_y = mg - bv = ma = m \frac{dv}{dt} \quad \frac{dv}{dt} = g - \frac{b}{m}v \quad \frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (**terminal speed**) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

How do the speed and acceleration depend on time?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

$$v = \frac{mg}{b} \left( 1 - e^{-bt/m} \right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b} \frac{b}{m} e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b} \frac{b}{m} e^{-t/\tau} = \frac{mg}{b} \frac{b}{m} \left( 1 - 1 + e^{-t/\tau} \right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant,  $\tau = m/b$ .



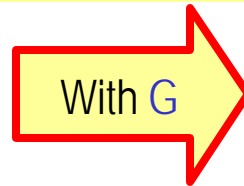
# Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. But the data people collected have not been explained until Newton has discovered the law of gravitation.

*Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.*

How would you write this principle mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$



$$F_g = G \frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

Unit?

$$N \cdot m^2 / kg^2$$

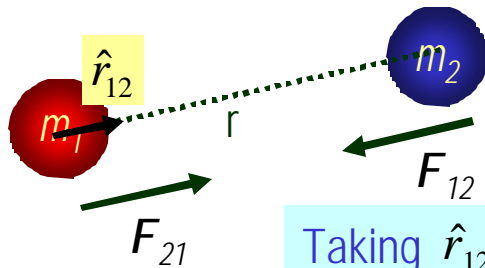
*This constant is not given by the theory but must be measured by experiment.*

This form of forces is known as an inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.



# More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.



Two objects exert gravitational force on each other following Newton's 3<sup>rd</sup> law.

Taking  $\hat{r}_{12}$  as the unit vector, we can write the force  $m_2$  experiences as

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

What do you think the negative sign mean?

It means that the force exerted on the particle 2 by particle 1 is attractive force, pulling #2 toward #1.

Gravitational force is a field force: Forces act on object without physical contact between the objects at all times, independent of medium between them.

The gravitational force exerted by a finite size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distributions was concentrated at the center.

How do you think the gravitational force on the surface of the earth look?

$$F_g = G \frac{M_E m}{R_E^2}$$



# Free Fall Acceleration & Gravitational Force

Weight of an object with mass  $m$  is  $mg$ . Using the force exerting on a particle of mass  $m$  on the surface of the Earth, one can get

$$mg = G \frac{M_E m}{R_E^2}$$

$$g = G \frac{M_E}{R_E^2}$$

What would the gravitational acceleration be if the object is at an altitude  $h$  above the surface of the Earth?

$$F_g = mg' = G \frac{M_E m}{r^2} = G \frac{M_E m}{(R_E + h)^2}$$

$$g' = G \frac{M_E}{(R_E + h)^2}$$

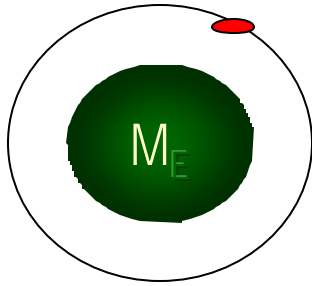
What do these tell us about the gravitational acceleration?

- The gravitational acceleration is independent of the mass of the object
- The gravitational acceleration decreases as the altitude increases
- If the distance from the surface of the Earth gets infinitely large, the weight of the object approaches 0.



# Example for Gravitational Force

The international space station is designed to operate at an altitude of 350km. When completed, it will have a weight (measured on the surface of the Earth) of  $4.22 \times 10^6 \text{ N}$ . What is its weight when in its orbit?



The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G \frac{M_E m}{R_E^2} = 4.22 \times 10^6 \text{ N}$$

Since the orbit is at 350km above the surface of the Earth, the gravitational force at that height is

$$F_O = mg' = G \frac{M_E m}{(R_E + h)^2} = \frac{R_E^2}{(R_E + h)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{(R_E + h)^2} F_{GE} = \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + 3.50 \times 10^5)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 \text{ N}$$