PHYS 1443 – Section 003
Lecture #12

Monday, Oct. 13, 2002
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1. Conservation of Mechanical Energy
2. Work done by non-conservative forces
3. How are conservative forces and potential energy related?

Don't miss the Quiz this Wednesday, Oct. 15!!

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

Potential energy function

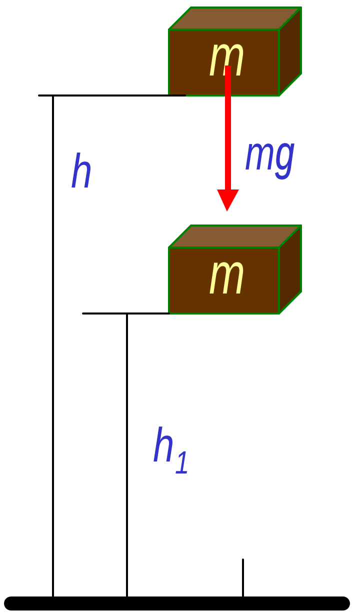
What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv K + U$$



Let's consider a brick of mass *m* at a height *h* from the ground

What is its potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

The brick gains speed

By how much?

$$v = gt$$

So what?

The brick's kinetic energy increased

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

And?

The lost potential energy converted to kinetic energy

What does this mean?

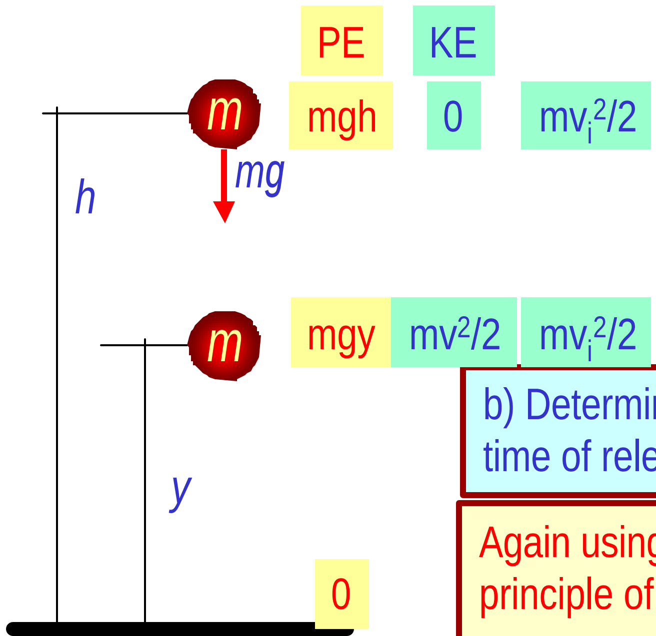
The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example

A ball of mass *m* is dropped from a height *h* above the ground. Neglecting air resistance determine the speed of the ball when it is at a height *y* above the ground.



Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \qquad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at *y* if it had initial speed v_i at the time of release at the original height *h*.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

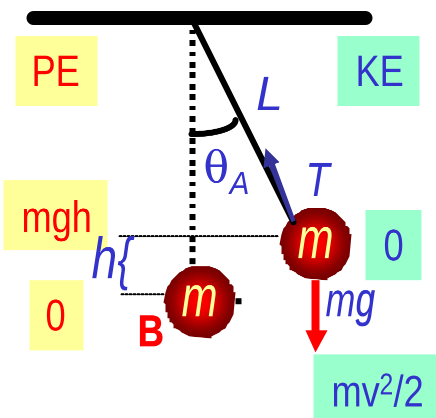
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example

A ball of mass *m* is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an angle θ_A with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B.



Compute the potential energy at the maximum height, *h*. Remember where the 0 is.

$$h = L - L\cos\theta_A = L(1-\cos\theta_A)$$

$$U_i = mgh = mgL(1-\cos\theta_A)$$

Using the principle of mechanical energy conservation

$$\boxed{K_i + U_i = K_f + U_f}$$

$$0 + mgh = mgL(1-\cos\theta_A) = \frac{1}{2}mv^2$$

$$v^2 = 2gL(1-\cos\theta_A) \quad \therefore v = \sqrt{2gL(1-\cos\theta_A)}$$

b) Determine tension T at the point B.

Using Newton's 2nd law of motion and recalling the centripetal acceleration of a circular motion

$$\sum F_r = T - mg = ma_r = m\frac{v^2}{r} = m\frac{v^2}{L}$$

$$T = mg + m\frac{v^2}{L} = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL(1-\cos\theta_A)}{L}\right)$$

$$= m\frac{gL + 2gL(1-\cos\theta_A)}{L}$$

$$\boxed{\therefore T = mg(3 - 2\cos\theta_A)}$$

Cross check the result in a simple situation. What happens when the initial angle θ_A is 0? $T = mg$

Work Done by Non-conserve Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the mechanical energy of the system is no longer conserved.

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \; W_g = -\Delta U$$

$$W_{you} = W_{app} = \Delta K + \Delta U$$

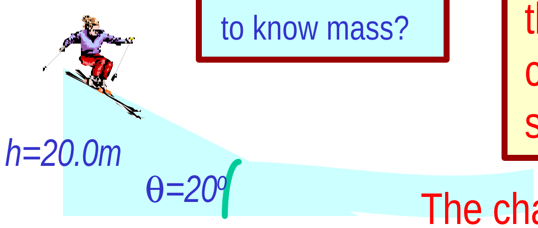
Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example of Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0*m* and the inclination angle is 20º. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 9.8\times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = m_k n = m_k mg$$

$$d = \frac{K_i}{m_k mg} = \frac{\frac{1}{2}mv^2}{m_k mg} = \frac{v^2}{2m_k g} = \frac{(19.8)^2}{2\times 0.210\times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.