PHYS 1443 – Section 003
Lecture #14

Monday, Oct. 20, 2002
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1. Power
   - Energy loss in Automobile
2. Linear Momentum
3. Linear Momentum and Forces
4. Conservation of Linear Momentum
5. Impulse and Linear Momentum
6. Collisions

Remember the 2nd term exam (ch 6 – 12), Monday, Nov. 3!

Power

- Rate at which work is performed
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?

  $\Longrightarrow$ 8 cylinder car climbs up faster

Is the amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $\overline{P} = \frac{\Delta W}{\Delta t}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d}{dt}\left(\vec{s}\right) = \vec{F} \cdot \vec{v} = Fv \cos q$

Unit? $J/s = Watts$ $1HP = 746\,Watts$

What do power companies sell? $1kWH = 1000\,Watts \times 3600\,s = 3.6 \times 10^{6}\,J$

Energy

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% wasted in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts of the car

4% in operating other crucial parts such as oil and fuel pumps, etc

The 13% of the energy from fuel is used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg, \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$mn = mmg = 227N$$

Air Drag

$$f_a = \frac{1}{2} D r A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2 v^2 = 0.647 v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Example for Power

A compact car has a mass of 800kg, and its efficiency is rated at 18%. Find the amount of gasoline used to accelerate the car from rest to 27m/s (~60mi/h). Use the fact that the energy equivalent of 1gal of gasoline is 1.3×10^8J.

First let's compute what the kinetic energy needed to accelerate the car from rest to a speed *v*.

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\times 800\times(27)^2 = 2.9\times10^5 J$$

Since the engine is only 18% efficient we must divide the necessary kinetic energy with this efficiency in order to figure out what the total energy needed is.

$$W_E = \frac{K_f}{e} = \frac{1}{2e}mv^2 = \frac{2.9\times10^5 J}{0.18} = 16\times10^5 J$$

Then using the fact that 1gal of gasoline can put out 1.3×10^8J, we can compute the total volume of gasoline needed to accelerate the car to 60 mi/h.

$$V_{gas} = \frac{W_E}{1.3\times10^8 J/gal} = \frac{16\times10^5 J}{1.3\times10^8 J/gal} = 0.012 gal$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\vec{p} = m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$$

Linear Momentum and Forces

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right)$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force, therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

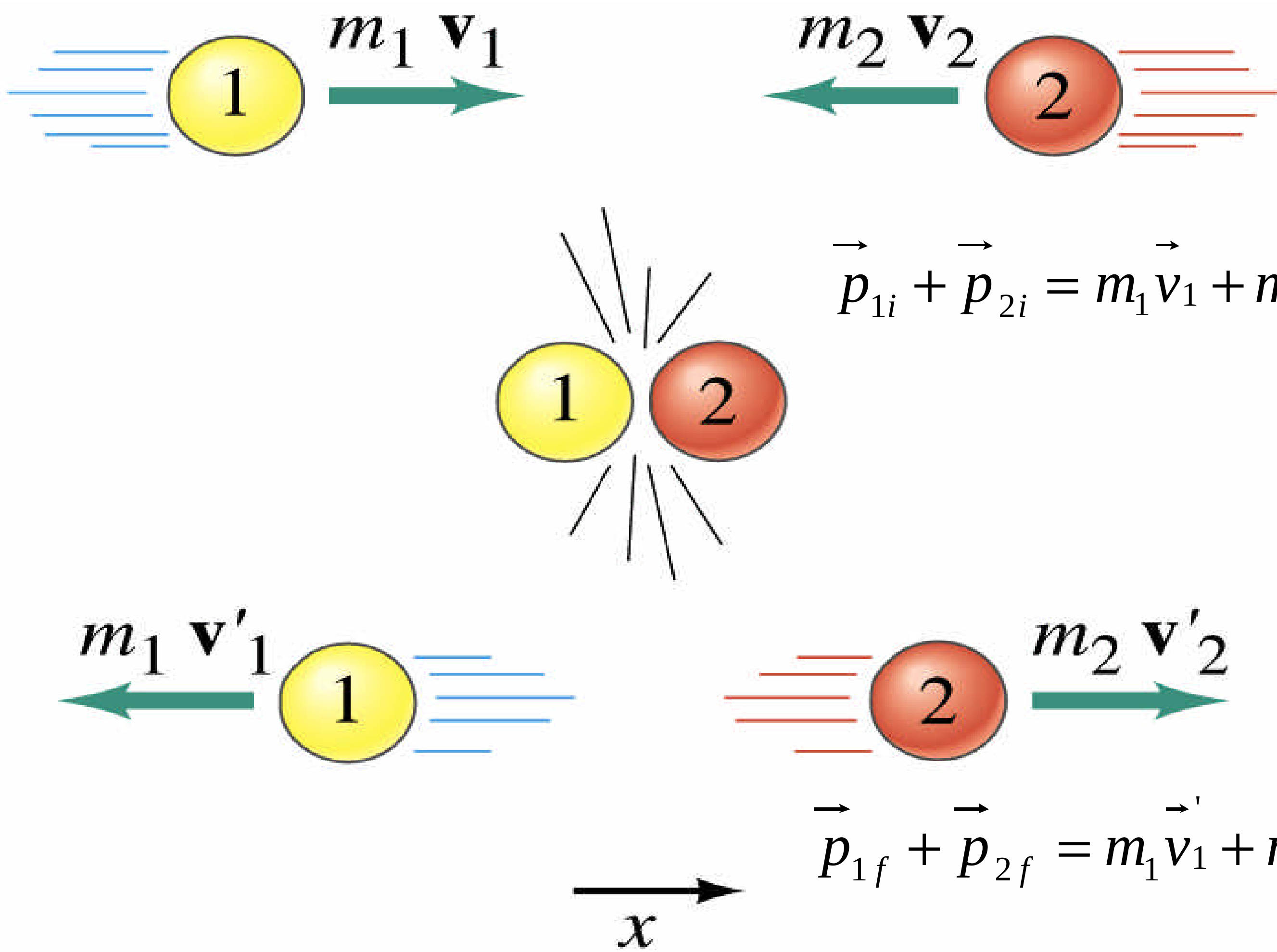
The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}_1' + m_2\vec{v}_2'$$

Conservation of Linear Momentum in a Two Particle System

Consider a system with two particles that does not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle#1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces and the net force in the SYSTEM is still 0.

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum p_1 and #2 has p_2 at some point of time.

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt}$$ and $$\vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$

The total linear momentum of the system is conserved!!!

More on Conservation of Linear Momentum in a Two Particle System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interaction

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1_i} = \vec{p}_{2f} + \vec{p}_{1f}$$

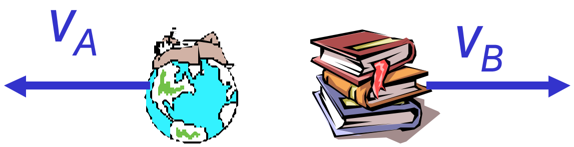
$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

Estimate an astronaut's resulting velocity after he throws his book to a direction in the space to move to a direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A\vec{v}_A + m_B\vec{v}_B$$

Assuming the astronaut's mass if 70kg, and the book's mass is 1kg and using linear momentum conservation

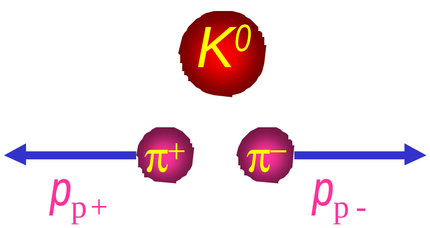
$$\vec{v}_A = -\frac{m_B\vec{v}_B}{m_A} = -\frac{1}{70}\vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i}\,(m/s)$$

Example for Linear Momentum Conservation

A type of particle, neutral kaon (K^0) decays (breaks up) into a pair of particles called pions (π^+ and π^-) that are oppositely charged but equal mass. Assuming K^0 is initially produced at rest, prove that the two pions must have mumenta that are equal in magnitude and opposite in direction.



This reaction can be written as

$$K^0 \rightarrow p^+ + p^-$$

Since this system consists of a K^0 in the initial state which results in two pions in the final state, the momentum must be conserved. So we can write

$$\vec{p}_{K^0} = \vec{p}_{p^+} + \vec{p}_{p^-}$$

Since K0 is produced at rest its momentum is 0.

$$\vec{p}_{K^0} = \vec{p}_{p^+} + \vec{p}_{p^-} = 0$$

$$\vec{p}_{p^+} = -\vec{p}_{p^-}$$

Therefore, the two pions from this kaon decay have the momanta with same magnitude but in opposite direction.