PHYS 1443 – Section 003
Lecture #15

Wednesday, Oct. 22, 2002
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1. Impulse and Linear Momentum
2. Collisions
3. Two dimensional collisions
4. Center of Mass
5. Motion of a group of particles

Homework #8 is due noon, next Wednesday, Oct. 29!

Remember the 2nd term exam (ch 6 – 11), Monday, Nov. 3!

Remember the colloquium at 4:00pm today in Rm 103!!!

Power

- Rate at which work is performed
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?

  $\Longrightarrow$ 8 cylinder car climbs up faster

Is the amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $\overline{P} = \frac{\Delta W}{\Delta t}$

Instantaneous power $P \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d}{dt}\left(\vec{s}\right) = \vec{F} \cdot \vec{v} = Fv \cos q$

Unit? $J/s = Watts$ $1HP = 746\,Watts$

What do power companies sell? $1kWH = 1000\,Watts \times 3600\,s = 3.6 \times 10^{6}\,J$

Energy

Linear Momentum and Forces

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right)$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force, therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider a system with two particles that does not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle#1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces and the net force in the SYSTEM is still 0.

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum p_1 and #2 has p_2 at some point of time.

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt}$$

and

$$\vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore

$$\vec{p}_2 + \vec{p}_1 = const$$

The total linear momentum of the system is conserved!!!

Impulse and Linear Momentum

Net force causes change of momentum ➔ Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse ***I***.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt$$

$$\vec{I} \equiv \int_{t_i}^{t_f} \vec{F}dt = \Delta\vec{p}$$

So what do you think an impulse is?

*Impulse of the force **F** acting on a particle over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the particle caused by that force. Impulse is the degree of which an external force changes momentum.*

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\overline{\vec{F}} \equiv \frac{1}{\Delta t}\int_{t_i}^{t_f} \vec{F}dt$$

Impulse can be rewritten

$$\vec{I} \equiv \overline{\vec{F}}\Delta t$$

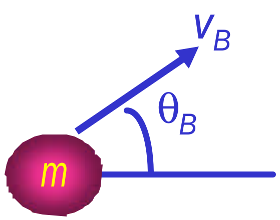
If force is constant

$$\vec{I} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example for Impulse

A golf ball of mass 50g is struck by a club. The force exerted on the ball by the club varies from 0, at the instant before contact, up to some maximum value at which the ball is deformed and then back to 0 when the ball leaves the club. Assuming the ball travels 200m, estimate the magnitude of the impulse caused by the collision.



The range R of a projectile is

$$R = \frac{v_B^2 \sin 2q_B}{g} = 200m$$

Let's assume that launch angle $\theta_i = 45^o$. Then the speed becomes:

$$v_B = \sqrt{200 \times g} = \sqrt{1960} = 44 m/s$$

Considering the time interval for the collision, t_i and t_f, initial speed and the final speed are

$$v_i = 0 \text{ (immediately before the collision)}$$

$$v_f = 44 m/s \text{ (immediately after the collision)}$$

Therefore the magnitude of the impulse on the ball due to the force of the club is

$$\left|\vec{I}\right| = \left|\Delta \vec{p}\right| = mv_{Bf} - mv_{Bi}$$

$$= 0 + 0.05 \times 44 = 2.2 kg \cdot m/s$$

Example for Impluse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i=-15.0*i* m/s and v_f=2.60*i* m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\,\vec{i}\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\,\vec{i}\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$

$$= 26400\vec{i}\ kg \cdot m/s = 2.64 \times 10^4\,\vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}$$

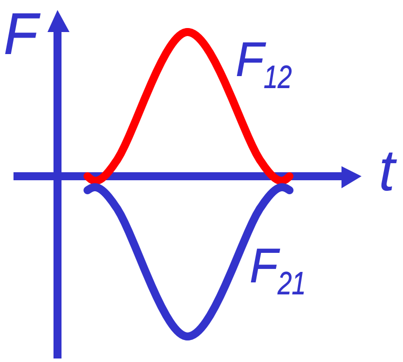
$$= 1.76 \times 10^5\,\vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5\,\vec{i}\ \mathrm{N}$$

Collisions

Generalized collisions must cover not only the physical contact but also the collisions without physical contact such as that of electromagnetic ones in a microscopic scale.

Consider a case of a collision between a proton on a helium ion.

The collisions of these ions never involves a physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, F_{21}, changes the momentum of particle 1 by

$$\Delta\vec{p}_1 = \int_{t_i}^{t_f} \vec{F}_{21} dt$$

Likewise for particle 2 by particle 1

$$\Delta\vec{p}_2 = \int_{t_i}^{t_f} \vec{F}_{12} dt$$

Using Newton's 3rd law we obtain

$$\Delta\vec{p}_2 = \int_{t_i}^{t_f} \vec{F}_{12} dt = -\int_{t_i}^{t_f} \vec{F}_{21} dt = -\Delta\vec{p}_1$$

So the momentum change of the system in the collision is 0 and the momentum is conserved

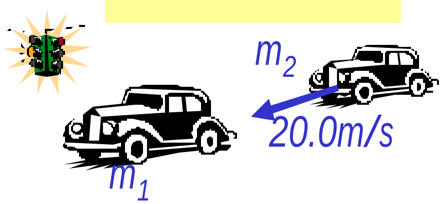
$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

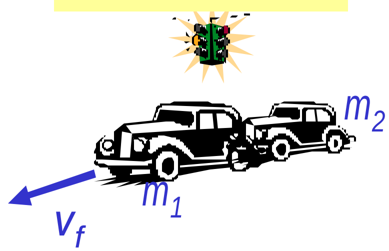
Example for Collisions

A car of mass 1800kg stopped at a traffic light is rear-ended by a 900kg car, and the two become entangled. If the lighter car was moving at 20.0m/s before the collision what is the velocity of the entangled cars after the collision?

Before collision



After collision



The momenta before and after the collision are

$$\vec{p}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = 0 + m_2\vec{v}_{2i}$$

$$\vec{p}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = (m_1 + m_2)\vec{v}_f$$

Since momentum of the system must be conserved

$$\vec{p}_i = \vec{p}_f \quad \Rightarrow \quad (m_1 + m_2)\vec{v}_f = m_2\vec{v}_{2i}$$

$$\vec{v}_f = \frac{m_2\vec{v}_{2i}}{(m_1 + m_2)} = \frac{900 \times 20.0\vec{i}}{900 + 1800} = 6.67\vec{i}\ m/s$$

What can we learn from these equations on the direction and magnitude of the velocity before and after the collision?

The cars are moving in the same direction as the lighter car's original direction to conserve momentum.

The magnitude is inversely proportional to its own mass.

Elastic and Inelastic Collisions

Momentum is conserved in any collisions as long as external forces negligible.

Collisions are classified as elastic or inelastic by the conservation of kinetic energy before and after the collisions.

Elastic Collision	*A collision in which the total kinetic energy and momentum are the same before and after the collision.*
Inelastic Collision	*A collision in which the total kinetic energy is not the same before and after the collision, but momentum is.*

Two types of inelastic collisions:Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision moving at a certain velocity together.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is constant in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In perfectly Inelastic collisions, the objects stick together after the collision, moving together. Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=(m_1+m_2)\vec{v}_f$$

$$\vec{v}_f=\frac{m_1\vec{v}_{1i}+m_2\vec{v}_{2i}}{(m_1+m_2)}$$

How about elastic collisions?

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=m_1\vec{v}_{1f}+m_2\vec{v}_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$

In elastic collisions, both the momentum and the kinetic energy are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$m_1\left(v_{1i}^2-v_{1f}^2\right)=m_2\left(v_{2i}^2-v_{2f}^2\right)$$

$$m_1\left(v_{1i}-v_{1f}\right)\left(v_{1i}+v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)\left(v_{2i}+v_{2f}\right)$$

From momentum conservation above

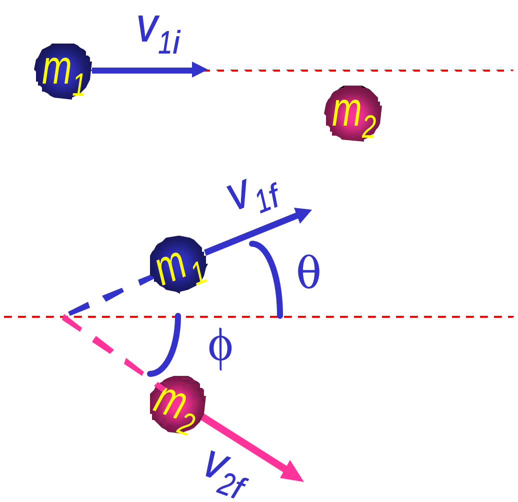
$$m_1\left(v_{1i}-v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)$$

$$v_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i}+\left(\frac{2m_2}{m_1+m_2}\right)v_{2i}$$

$$v_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)v_{1i}+\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{2i}$$

Two dimensional Collisions

In two dimension, one can use components of momentum to apply momentum conservation to solve physical problems.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x-comp. $$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-comp. $$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at fixed target accelerator experiments.) The momentum conservation tells us:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos q + m_2 v_{2f}\cos f$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin q - m_2 v_{2f}\sin f$$

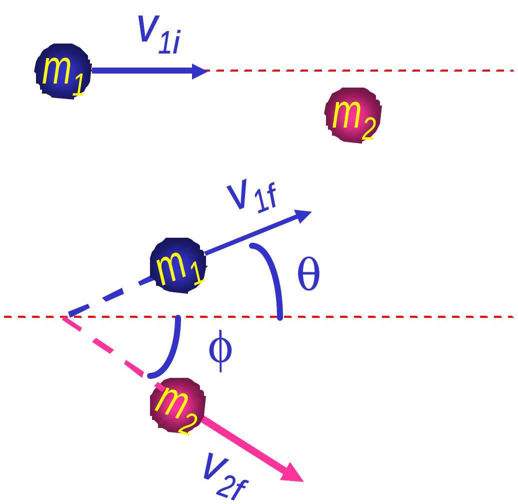
And for the elastic conservation, the kinetic energy is conserved:

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

What do you think we can learn from these relationships?

Example of Two Dimensional Collisions

Proton #1 with a speed 3.50×10^{5} m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37^{o} to the horizontal axis and proton #2 deflects at an angle ϕ to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, ϕ.



Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f}\cos\theta + m_p v_{2f}\cos\phi$

y-comp. $m_p v_{1f}\sin\theta - m_p v_{2f}\sin\phi = 0$

Canceling m_p and put in all known quantities, one obtains

$$v_{1f}\cos 37^{\circ} + v_{2f}\cos\phi = 3.50\times10^{5} \quad (1)$$

$$v_{1f}\sin 37^{\circ} = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^{5}\right)^{2} = v_{1f}^{2} + v_{2f}^{2} \quad (3)$$

Solving Eqs. 1-3 equations, one gets

$$v_{1f} = 2.80\times10^{5}\ m/s$$
$$v_{2f} = 2.11\times10^{5}\ m/s$$
$$\phi = 53.0^{\circ}$$

Do this at home☺

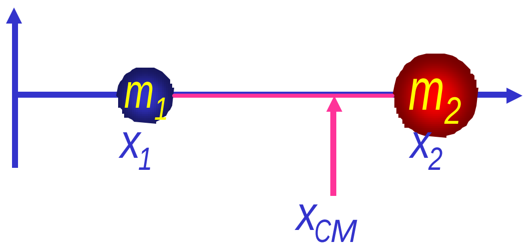
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situation objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on the point.

What does above statement tell you concerning forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Center of Mass of a Rigid Object

The formula for CM can be expanded to Rigid Object or a system of many particles

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

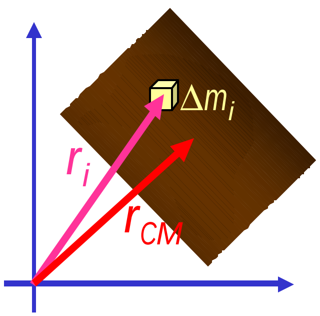
$$y_{CM} = \frac{\sum_i m_iy_i}{\sum_i m_i}$$

$$z_{CM} = \frac{\sum_i m_iz_i}{\sum_i m_i}$$

The position vector of the center of mass of a many particle system is

$$\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} + z_{CM}\vec{k} = \frac{\sum_i m_ix_i\vec{i} + \sum_i m_iy_i\vec{j} + \sum_i m_iz_i\vec{k}}{\sum_i m_i}$$

$$\vec{r}_{CM} = \frac{\sum_i m_i\vec{r}_i}{M}$$



A rigid body – an object with shape and size with mass spread throughout the body, ordinary objects – can be considered as a group of particles with mass m_i densely spread throughout the given shape of the object

$$x_{CM} \approx \frac{\sum_i \Delta m_ix_i}{M}$$

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i \Delta m_ix_i}{M} = \frac{1}{M}\int x\,dm$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$