PHYS 1443 – Section 003
Lecture #23

Monday, Dec. 1, 2003

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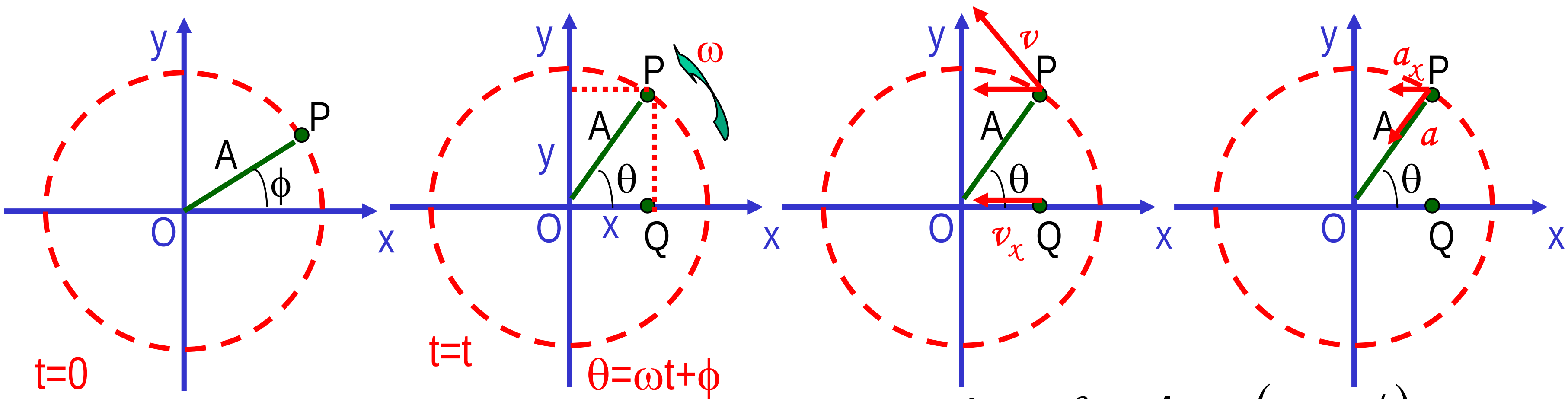
1. Simple Harmonic Motion and Uniform Circular Motion
2. Damped Oscillation
3. Waves
4. Speed of Waves
5. Sinusoidal Waves
6. Rate of Wave Energy Transfer
7. Superposition and Interference
8. Reflection and Transmission

Announcements

- Homework # 12
  - Due at 5pm, Friday, Dec. 5
- The final exam
  - On Monday, Dec. 8, 11am – 12:30pm in SH103.
  - Covers: Chap. 10 not covered in Term #2 – Ch15.
- Need to talk to me? I will be around this week.

Simple Harmonic and Uniform Circular Motions

Uniform circular motion can be understood as a superposition of two simple harmonic motions in x and y axis.



When the particle rotates at a uniform angular speed ω, x and y coordinate position become

$$x = A\cos\theta = A\cos(\omega t + \phi)$$
$$y = A\sin\theta = A\sin(\omega t + \phi)$$

Since the linear velocity in a uniform circular motion is Aω, the velocity components are

$$v_x = -v\sin\theta = -A\omega\sin(\omega t + \phi)$$
$$v_y = +v\cos\theta = A\omega\cos(\omega t + \phi)$$

Since the radial acceleration in a uniform circular motion is $v^2/A = \omega^2 A$, the components are

$$a_x = -a\cos\theta = -A\omega^2\cos(\omega t + \phi)$$
$$a_y = -a\sin\theta = -A\omega^2\sin(\omega t + \phi)$$

Example for Uniform Circular Motion

A particle rotates counterclockwise in a circle of radius 3.00m with a constant angular speed of 8.00 rad/s. At t=0, the particle has an x coordinate of 2.00m and is moving to the right. A) Determine the x coordinate as a function of time.

Since the radius is 3.00m, the amplitude of oscillation in x direction is 3.00m. And the angular frequency is 8.00rad/s. Therefore the equation of motion in x direction is

$$x = A\cos\theta = (3.00m)\cos(8.00t+\phi)$$

Since x=2.00, when t=0 $\quad 2.00 = (3.00m)\cos\phi; \quad \phi = \cos^{-1}\left(\frac{2.00}{3.00}\right) = 48.2^\circ$

However, since the particle was moving to the right ϕ=-48.2°,

$$x = (3.00m)\cos(8.00t - 48.2^\circ)$$

Find the x components of the particle's velocity and acceleration at any time t.

Using the displcement

$$v_x = \frac{dx}{dt} = -(3.00\cdot 8.00)\sin(8.00t-48.2) = (-24.0m/s)\sin(8.00t-48.2^\circ)$$

Likewise, from velocity

$$a_x = \frac{dv}{dt} = (-24.0\cdot 8.00)\cos(8.00t-48.2) = (-192m/s^2)\cos(8.00t-48.2^\circ)$$

Damped Oscillation

More realistic oscillation where an oscillating object loses its mechanical energy in time by a retarding force such as friction or air resistance.

Let's consider a system whose retarding force is air resistance R=-bv (b is called damping coefficient) and restoration force is -kx

$$\sum F_x = -kx - bv = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

The solution for the above 2nd order differential equation is

$$x = e^{-\frac{b}{2m}t} A\cos(\omega t + \phi)$$

Damping Term

The angular frequency ω for this motion is

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

This equation of motion tells us that when the retarding force is much smaller than restoration force, the system oscillates but the amplitude decreases, and ultimately, the oscillation stops.

We express the angular frequency as

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

Where as the natural frequency ω_0

$$\omega_0 = \sqrt{\frac{k}{m}}$$

More on Damped Oscillation

The motion is called **Underdamped** when the magnitude of the maximum retarding force $R_{max} = bv_{max} < kA$

How do you think the damping motion would change as retarding force changes?

$$-bv_{max} \rightarrow -kA$$

As the retarding force becomes larger, the amplitude reduces more rapidly, eventually stopping at its equilibrium position

Under what condition this system does not oscillate?

$$\omega = 0 \quad \omega_0 = \frac{b}{2m}$$

The system is **Critically damped**

$$b = 2m\omega_0 = 2\sqrt{mk}$$

What do you think happen?

Once released from non-equilibrium position, the object would return to its equilibrium position and stops.

If the retarding force is larger than restoration force

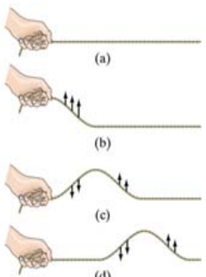
$$R_{max} = bv_{max} > kA$$

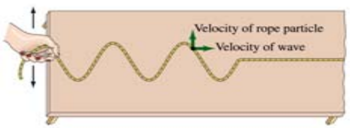
The system is **Overdamped**

Once released from non-equilibrium position, the object would return to its equilibrium position and stops, but a lot slower than before

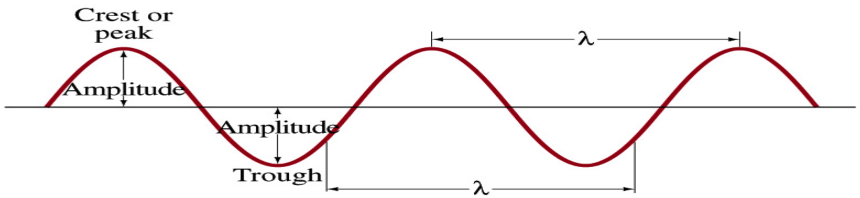
Waves

- Waves do not move medium rather carry energy from one place to another
- Two forms of waves
  - Pulse
  - Continuous or periodic wave

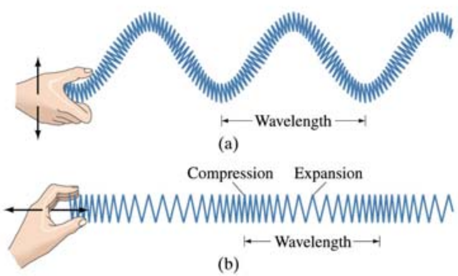


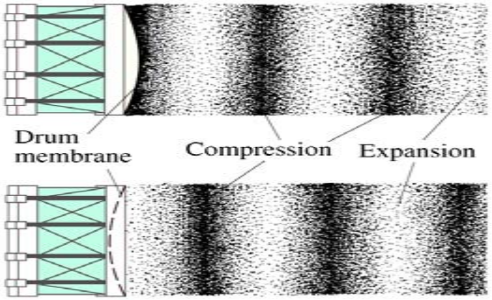


- Wave can be characterized by
  - Amplitude
  - Wave length
  - Period



- Two types of waves
  - Transverse Wave
  - Longitudinal wave
    - Sound wave





Speed of Transverse Waves on Strings

How do we determine the speed of a transverse pulse traveling on a string?

If a string under tension is pulled sideways and released, the tension is responsible for accelerating a particular segment of the string back to the equilibrium position.

So what happens when the tension increases?

The acceleration of the particular segment increases

Which means?

The speed of the wave increases.

Now what happens when the mass per unit length of the string increases?

For the given tension, acceleration decreases, so the wave speed decreases.

Which law does this hypothesis based on?

Newton’s second law of motion

Based on the hypothesis we have laid out above, we can construct a hypothetical formula for the speed of wave

$$v = \sqrt{\frac{T}{\mu}}$$

T: Tension on the string

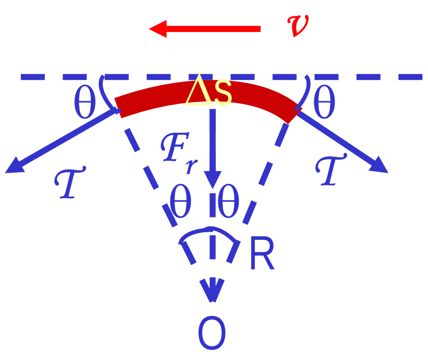
μ: Unit mass per length

Is the above expression dimensionally sound?

$T=[MLT^{-2}]$, $\mu=[ML^{-1}]$

$(T/\mu)^{1/2}=[L^2T^{-2}]^{1/2}=[LT^{-1}]$

Speed of Waves on Strings cont'd



Let's consider a pulse moving right and look at it in the frame that moves along with the the pulse.

Since in the reference frame moves with the pulse, the segment is moving to the left with the speed v, and the centripetal acceleration of the segment is

$$a_r = \frac{v^2}{R}$$

Now what do the force components look in this motion when θ is small?

$$\sum F_t = T\cos\theta - T\cos\theta = 0$$

$$\sum F_r = 2T\sin\theta \approx 2T\theta$$

What is the mass of the segment when the line density of the string is μ?

$$m = \mu\Delta s = \mu R 2\theta = 2\mu R\theta$$

Using the radial force component

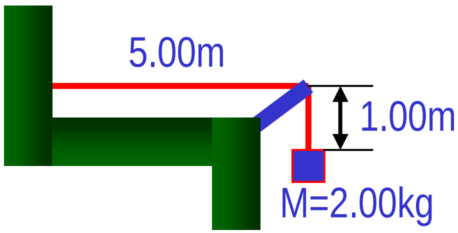
$$\sum F_r = ma = m\frac{v^2}{R} = 2\mu R\theta\frac{v^2}{R} = 2T\theta$$

Therefore the speed of the pulse is

$$v = \sqrt{\frac{T}{\mu}}$$

Example for Traveling Wave

A uniform cord has a mass of 0.300kg and a length of 6.00m. The cord passes over a pulley and supports a 2.00kg object. Find the speed of a pulse traveling along this cord.



Since the speed of wave on a string with line density μ and under the tension T is $v = \sqrt{\frac{T}{\mu}}$

The line density μ is $\mu = \frac{0.300\,kg}{6.00\,m} = 5.00 \times 10^{-2}\,kg/m$

The tension on the string is provided by the weight of the object. Therefore

$$T = Mg = 2.00 \times 9.80 = 19.6\,kg \cdot m/s^2$$

Thus the speed of the wave is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6}{5.00 \times 10^{-2}}} = 19.8\,m/s$$