Phys 1443 – Section 003

Lecture #3

Monday, Aug. 30, 2004

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1. One Dimensional Motion

   Average Velocity
   Acceleration
   Motion under constant acceleration
   Free Fall

2. Motion in Two Dimensions

   Vector Properties and Operations
   Motion under constant acceleration
   Projectile Motion
Announcements

• Homework: 38 of you have signed up (out of 43)
  – Roster will be locked at 5pm Wednesday
  – In order for you to obtain 100% on homework #1, you need to pickup the homework, attempt to solve it and submit it. 30 of you have done this.
  – Homework system deducts points for failed attempts.
    • So be careful when you input the answers
    • Input the answers to as many significant digits as possible
  – All homework problems are equally weighted

• e-mail distribution list:: 15 of you have subscribed so far.
  – This is the primary communication tool. So subscribe to it ASAP.
  – 5 extra credit points if done by midnight tonight and 3 by Wednesday.
  – A test message will be sent after the class today for verification purpose

• Physics Clinic (Supplementary Instructions, SH010): 12 – 6, M-F

• Labs begin today!!!
Difference between Speed and Velocity

- Let’s take a simple one dimensional translation that has many steps:

   Let’s call this line as X-axis

   Let’s have a couple of motions in a total time interval of 20 sec.

   +10m  +15m  +5m
   -5m   -10m  -15m

   Total Displacement: \( \Delta x \equiv x_f - x_i = x_i - x_f = 0(m) \)

   Average Velocity: \( v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s) \)

   Total Distance Traveled: \( D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m) \)

   Average Speed: \( v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{60}{20} = 3(m/s) \)
Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner’s position changes from $x_1=50.0 \text{ m}$ to $x_2=30.5 \text{ m}$, as shown in the figure. What was the runner’s average velocity? What was the average speed?

- **Displacement:**

  \[ \Delta x \equiv x - x = x_2 - x_1 = 30.5 - 50.0 = -19.5 \text{ (m)} \]

- **Average Velocity:**

  \[ v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50 \text{ (m/s)} \]

- **Average Speed:**

  \[ v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} \]

  \[ = \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50 \text{ (m/s)} \]
Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Mathematically: Slope of the position variation as a function of time

- Instantaneous speed is the size (magnitude) of the velocity vector:
Position vs Time Plot

It is helpful to understand motions to draw them on position vs time plots.

1. Running at a constant velocity (go from x=0 to x=x₁ in t₁, Displacement is + x₁ in t₁ time interval)
2. Velocity is 0 (go from x₁ to x₁ no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x₁ to x=0 in t₃-t₂ time interval, Displacement is - x₁ in t₃-t₂ time interval)

Does this motion physically make sense?
Instantaneous Velocity

Average Velocity

Time

Instantaneous Velocity
Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation \( x = At^2 + B \) where \( A = 2.10\, \text{m/s}^2 \) and \( B = 2.80\, \text{m} \).

(a) Determine the displacement of the engine during the interval from \( t_1 = 3.00\, \text{s} \) to \( t_2 = 5.00\, \text{s} \).

\[
x_1 = x_{t_1 = 3.00} = 2.10 \times (3.00)^2 + 2.80 = 21.7\, \text{m}
\]

\[
x_2 = x_{t_2 = 5.00} = 2.10 \times (5.00)^2 + 2.80 = 55.3\, \text{m}
\]

Displacement is, therefore:

\[
\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6\, (\text{m})
\]

(b) Determine the average velocity during this time interval.

\[
-v_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00 - 3.00} = \frac{33.6}{2.00} = 16.8\, (\text{m/s})
\]
Example 2.3 cont’d

(c) Determine the instantaneous velocity at \( t=t^2=5.00 \) s.

The derivative of the engine’s equation of motion is

\[
\frac{d}{dt} \left( C t^n \right) = n C t^{n-1}
\]

and

\[
\frac{d}{dt} (C) = 0
\]

The instantaneous velocity at \( t=5.00 \) s is

\[
v_x(t = 5.00 \text{s}) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0 \text{ (m/s)}
\]
Displacement, Velocity and Speed

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed

$$|v_x| = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{|dx|}{dt}$$
Acceleration

Change of velocity in time (what kind of quantity is this?)

- **Average acceleration:**

\[
a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}
\]

analog to

\[
v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}
\]

- **Instantaneous acceleration:**

\[
a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2}
\]

analog to

\[
v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}
\]

- In calculus terms: A slope (derivative) of velocity with respect to time or change of slopes of position as a function of time