PHYS 1443 – Section 003
Lecture #5

Wednesday, Sept. 8, 2004
Dr. Jaehoon Yu

1. One Dimensional Motion
   Motion under constant acceleration
   Free Fall
2. Motion in Two Dimensions
   Vector Properties and Operations
   Motion under constant acceleration
   Projectile Motion

Today’s homework is HW #4, due 1pm, next Wednesday, Sept. 15!!
Announcements

• E-mail distribution list: 25 of you have registered
  – *Important communication tool!!*
  – Next Wednesday is the last day of e-mail registration
  – -5 extra points if you don’t register by next Wednesday!
  – A test message will be sent out next Wednesday!!

• Homework: 43/47 of you are registered!

• Quiz Results
  – Class average: 8.7/15
  – Top score: 15
  – Quiz accounts for 15%. Please do not miss!
Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

\[ v_{xf}(t) = v_{xi} + a_xt \]

Velocity as a function of time

\[ x_f - x_i = \bar{v}_x t = \frac{1}{2} (v_{xf} + v_{xi}) t \]

Displacement as a function of velocities and time

\[ x_f = x_i + v_{xi}t + \frac{1}{2} a_xt^2 \]

Displacement as a function of time, velocity, and acceleration

\[ v_{xf}^2 = v_{xi}^2 + 2ax(x_f - x_i) \]

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!
How do we solve a problem using a kinematic formula for constant acceleration?

• Identify what information is given in the problem.
  – Initial and final velocity?
  – Acceleration?
  – Distance?
  – Time?

• Convert the units to SI to be consistent.

• Identify what the problem wants.

• Identify which formula is appropriate and easiest to solve for what the problem wants.
  – Frequently multiple formula can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.

• Solve the equation for the quantity wanted.
Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? As long as it takes for it to crumple.

The initial speed of the car is

\[ v_{xi} = 100 \text{km/h} = \frac{100000 \text{m}}{3600 \text{s}} = 28 \text{m/s} \]

We also know that

\[ v_{xf} = 0 \text{m/s} \quad \text{and} \quad x_f - x_i = 1 \text{m} \]

Using the kinematic formula

\[ v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \]

The acceleration is

\[ a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0 - (28 \text{m/s})^2}{2 \times 1 \text{m}} = -390 \text{m/s}^2 \]

Thus the time for air-bag to deploy is

\[ t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28 \text{m/s}}{-390 \text{m/s}^2} = 0.07 \text{s} \]
Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80\text{m/s}^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call $(-y)$; where up and down direction are indicated as the variable “$y$”
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80\text{m/s}^2$
Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at \( t=0 \) with \( +20.0 \text{m/s} \) initial velocity on the roof of a 50.0m high building.

What is the acceleration in this motion? \( g=-9.80 \text{m/s}^2 \)

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? \( V=0 \)

\[
v_f = v_i + at = +20.0 - 9.80t = 0.00 \text{m/s}
\]

Solve for \( t \)

\[
t = \frac{20.0}{9.80} = 2.04 \text{s}
\]

(b) Find the maximum height.

\[
y_f = y_i + v_i t + \frac{1}{2}at^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2
\]

\[
= 50.0 + 20.4 = 70.4 (m)
\]
Example of a Falling Object cnt’d

(c) Find the time the stone reaches back to its original height.

\[ t = 2.04 \times 2 = 4.08 \text{s} \]

(d) Find the velocity of the stone when it reaches its original height.

\[ v_{yf} = v_{yi} + a_yt = 20.0 + (-9.80) \times 4.08 = -20.0 \text{ (m/s)} \]

(e) Find the velocity and position of the stone at t=5.00s.

\[ v_{yf} = v_{yi} + a_yt = 20.0 + (-9.80) \times 5.00 = -29.0 \text{ (m/s)} \]

\[ y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2 \]

\[ = 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5 \text{ (m)} \]
Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute

\[
\begin{align*}
  x_1 &= r \cos \theta \\
  y_1 &= r \sin \theta \\
  \tan \theta &= \frac{y_1}{x_1}
\end{align*}
\]
Example

Cartesian Coordinate of a point in the xy plane are \((x,y)= (-3.50,-2.50)\text{m}\). Find the polar coordinates of this point.

\[
r = \sqrt{(x^2 + y^2)}
\]

\[
= \sqrt{((-3.50)^2 + (-2.50)^2)}
\]

\[
= \sqrt{18.5} = 4.30\text{ (m)}
\]

\[
\theta = 180 + \theta_s
\]

\[
\tan \theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}
\]

\[
\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ
\]

\[
\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ
\]
Vector and Scalar

Vector quantities have both magnitude (size) and direction

\textbf{Force, gravitational acceleration, momentum}

Normally denoted in \textbf{BOLD} letters, \textbf{\( \mathbf{F} \)}, or a letter with arrow on top \( \vec{\mathbf{F}} \)

Their sizes or magnitudes are denoted with normal letters, \( \mathbf{F} \), or absolute values: \( |\vec{\mathbf{F}}| \) or \( |\mathbf{F}| \)

Scalar quantities have magnitude only

Can be completely specified with a value and its unit

\textbf{Energy, heat, mass, time}

Normally denoted in normal letters, \( \mathbf{E} \)

Both have units!!!
Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system.

Which ones are the same vectors?

A = B = E = D

Why aren't the others?

C: The same magnitude but opposite direction: $C = -A$

F: The same direction but different magnitude
Vector Operations

• Addition:
  – Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  – Parallelogram method: Connect the tails of the two vectors and extend
  – Addition is commutative: Changing order of operation does not affect the results
    \[ A + B = B + A, \quad A + B + C + D + E = E + C + A + B + D \]

• Subtraction:
  – The same as adding a negative vector: \[ A - B = A + (-B) \]

• Multiplication by a scalar is increasing the magnitude \( A, B = 2A \)

Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!
Example for Vector Addition

A car travels 20.0 km due north followed by 35.0 km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.

\[ r = \sqrt{(A + B \cos \theta)^2 + (B \sin \theta)^2} \]
\[ = \sqrt{A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta} \]
\[ = \sqrt{A^2 + B^2 + 2AB \cos \theta} \]
\[ = \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0 \cos 60} \]
\[ = \sqrt{2325} = 48.2 \text{ (km)} \]

\[ \theta = \tan^{-1} \left( \frac{|B| \sin 60}{|A| + |B| \cos 60} \right) \]
\[ = \tan^{-1} \left( \frac{35.0 \sin 60}{20.0 + 35.0 \cos 60} \right) \]
\[ = \tan^{-1} \left( \frac{30.3}{37.5} \right) = 38.9° \text{ to W wrt N} \]
Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components

\[ A = (A_x, A_y) \]

\[ A_x = |A| \cos \theta \]
\[ A_y = |A| \sin \theta \]

So the above vector \( \vec{A} \) can be written as

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} = |\vec{A}| \cos \theta \hat{i} + |\vec{A}| \sin \theta \hat{j} \]

- Unit vectors are **dimensionless** vectors whose **magnitude are exactly 1**
- Unit vectors are usually expressed in \( \hat{i}, \hat{j}, \hat{k} \) or \( \vec{i}, \vec{j}, \vec{k} \)
- Vectors can be expressed using components and unit vectors
Examples of Vector Operations

Find the resultant vector which is the sum of \( \mathbf{A} = (2.0\mathbf{i} + 2.0\mathbf{j}) \) and \( \mathbf{B} = (2.0\mathbf{i} - 4.0\mathbf{j}) \)

\[
\mathbf{C} = \mathbf{A} + \mathbf{B} = (2.0\mathbf{i} + 2.0\mathbf{j}) + (2.0\mathbf{i} - 4.0\mathbf{j}) = (2.0 + 2.0)\mathbf{i} + (2.0 - 4.0)\mathbf{j} = 4.0\mathbf{i} - 2.0\mathbf{j}(m)
\]

\[
|\mathbf{C}| = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)
\]

\[
\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \frac{-2.0}{4.0} = -27^\circ
\]

Find the resultant displacement of three consecutive displacements: \( \mathbf{d}_1 = (15\mathbf{i} + 30\mathbf{j} + 12\mathbf{k})\text{cm} \), \( \mathbf{d}_2 = (23\mathbf{i} + 14\mathbf{j} - 5.0\mathbf{k})\text{cm} \), and \( \mathbf{d}_3 = (-13\mathbf{i} + 15\mathbf{j})\text{cm} \)

\[
\mathbf{D} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 = (15\mathbf{i} + 30\mathbf{j} + 12\mathbf{k}) + (23\mathbf{i} + 14\mathbf{j} - 5.0\mathbf{k}) + (-13\mathbf{i} + 15\mathbf{j})
\]

\[
= (15 + 23 - 13)\mathbf{i} + (30 + 14 + 15)\mathbf{j} + (12 - 5.0)\mathbf{k} = 25\mathbf{i} + 59\mathbf{j} + 7.0\mathbf{k}(cm)
\]

Magnitude \[
|\mathbf{D}| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)
\]