PHYS 1443 – Section 003
Lecture #6

Monday, Sept. 13, 2004

Dr. Jaehoon Yu

1. Motion in two dimension
   - Motion under constant acceleration
   - Projectile motion
     - Heights and Horizontal Ranges
     - Maximum ranges and heights
   - Reference Frames and relative motion
2. Newton's Laws of Motion
   - Force
   - Law of Inertia

Quiz Next Monday, Sept. 20!!

Announcements

- Quiz #2 Next Monday, Sept. 20
  - Will cover Chapters 1 – up to what we cover this Wednesday
- e-mail distribution list: 34/47 of you have subscribed so far.
  - Remember -3 points extra credit if not registered by Wednesday
  - A test message will be sent Thursday for verification purpose
- Remember the 1st term exam, Monday, Sept. 27, two weeks from today
  - Covers up to chapter 6.
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: Your grade is F.
  - Mixture of multiple choice and free style problems
- Homework is designed to cover the most current material
  - Sometimes we don't cover them all so you might have to go beyond what is covered in the class.
  - But this is better than covering material too old.

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- Dimensionless
- Magnitudes are exactly 1
- Unit vectors are usually expressed in **i**, **j**, **k** or

$$\vec{i},\ \vec{j},\ \vec{k}$$

So the vector **A** can be re-written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta\, \vec{i} + \left|\vec{A}\right| \sin\theta\, \vec{j}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1d and 2d

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping above results in

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\left(m/s\right) \quad v_{yf}=v_{yi}+a_y t=-15+0t=-15\left(m/s\right)$$

Velocity vector

$$\vec{v}(t)=v_x\left(t\right)\vec{i}+v_y\left(t\right)\vec{j}=\left(20+4.0t\right)\vec{i}-15\vec{j}(m/s)$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_{t=5}=v_{x,t=5}\vec{i}+v_{y,t=5}\vec{j}=\left(20+4.0\times5.0\right)\vec{i}-15\vec{j}=\left(40\vec{i}-15\vec{j}\right)m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{\left(v_x\right)^2+\left(v_y\right)^2}=\sqrt{\left(40\right)^2+\left(-15\right)^2}=43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

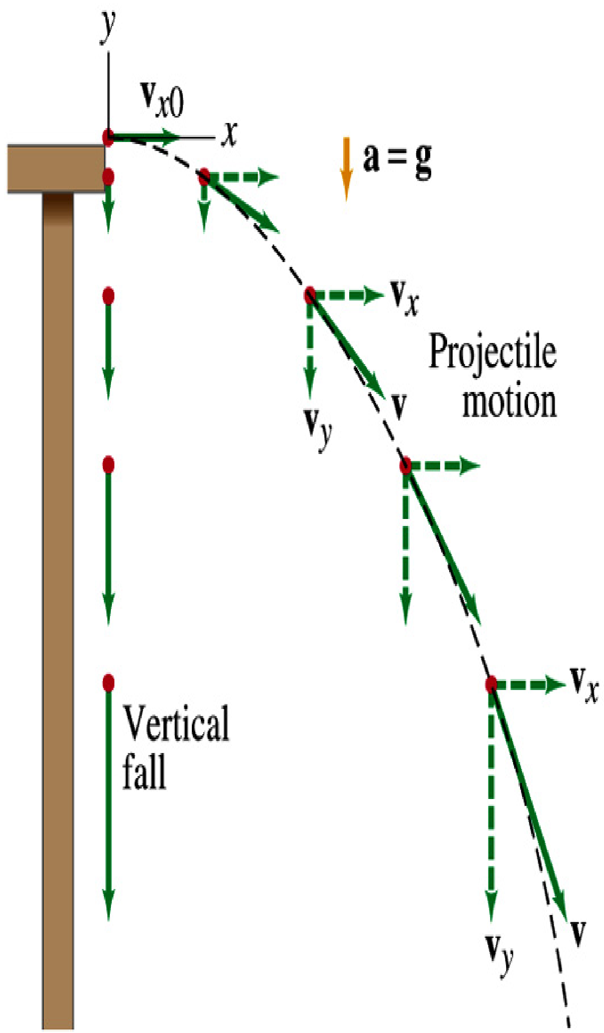
$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the assumptions
  - Free fall acceleration, -g, is constant over the range of the motion
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (no acceleration)
  - Vertical motion under constant acceleration (g)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

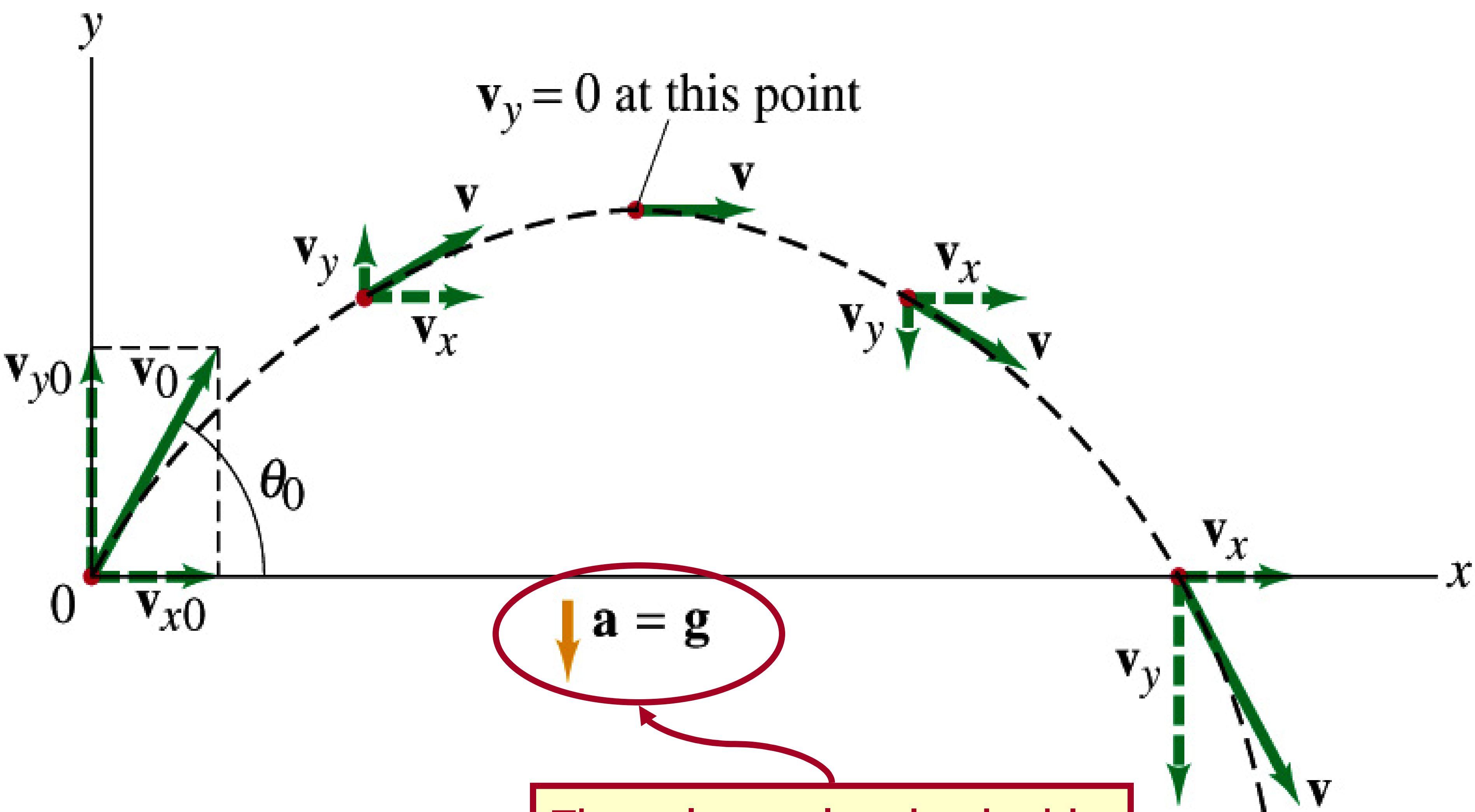
Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore\ t \approx 8\sec$$

Distance is determined by x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

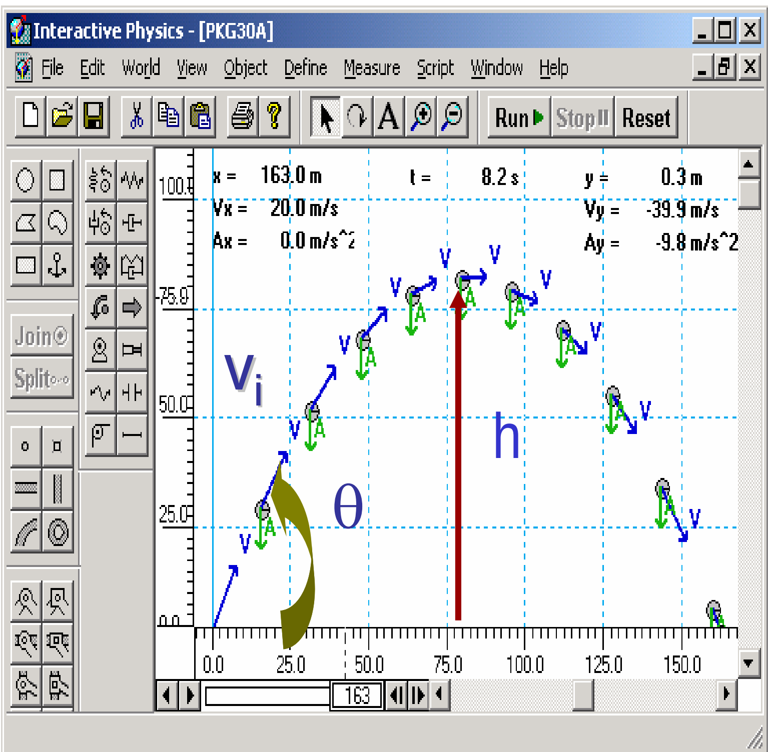
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t$$

$$= v_i \sin\theta_i - g t_A = 0$$

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

Horizontal Range and Max Height

Since no acceleration in x direction, it still flies even if v_y=0.

$$R = v_{xi}t = v_{xi}\left(2t_A\right) = 2v_i\cos\theta_i\left(\frac{v_i\sin\theta_i}{g}\right)$$

Range

$$R = \left(\frac{v_i^2\sin 2\theta_i}{g}\right)$$

$$y_f = h = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i\sin\theta_i\left(\frac{v_i\sin\theta_i}{g}\right) - \frac{1}{2}g\left(\frac{v_i\sin\theta_i}{g}\right)^2$$

Height

$$y_f = h = \left(\frac{v_i^2\sin^2\theta_i}{2g}\right)$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!