PHYS 1443 – Section 003
Lecture #7

Wednesday, Sept. 15, 2004

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1. Motion in two dimension
    - Maximum ranges and heights
  - Reference Frames and relative motion
2. Newton's Laws of Motion
    - Force
    - Mass
    - Newton's Law of Inertia
    - Newton's second law of motion
    - Newton's third law of motion

Today's Homework is #5, due 1pm next Wednesday Sept. 22!!

Announcements

- Quiz #2 Next Monday, Sept. 20
  - Will cover Chapters 1 – 4.6
- e-mail distribution list: 37/47 of you have subscribed so far.
  - -3 points extra credit if not registered by midnight tonight
  - A test message will be sent Thursday for verification purpose
    - Please be sure to reply only to ME!!!
- Remember the 1st term exam, Monday, Sept. 27, two weeks from today
  - Covers up to chapter 6.
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: Your grade is F.
  - Mixture of multiple choice and free style problems

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

Becomes

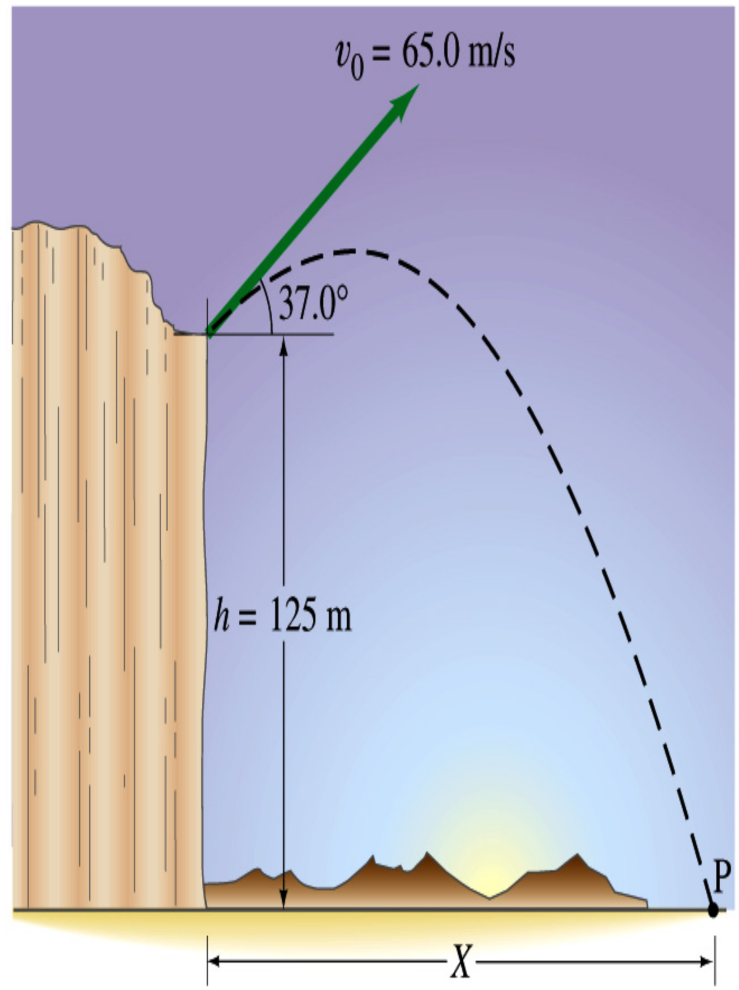
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43 s \quad \text{or} \quad t = 10.4 s$$

$$\boxed{t = 10.4 s}$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

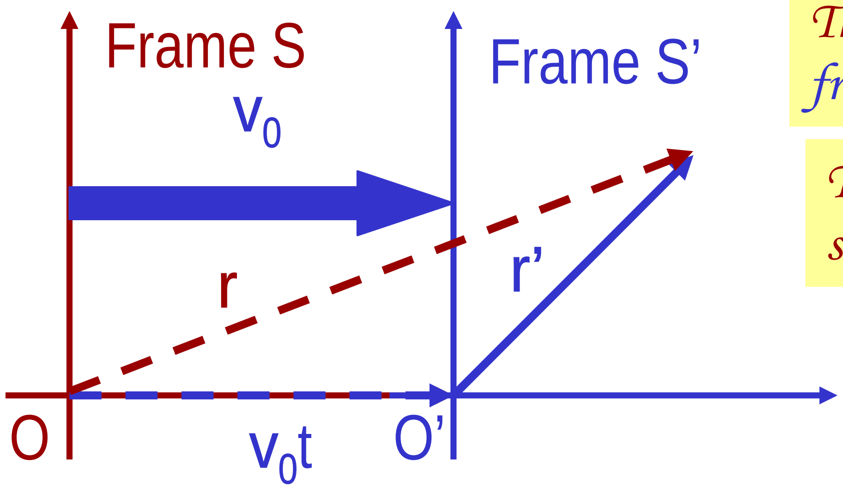
Do these yourselves at home for fun!!!

Observations in Different Reference Frames

Results of Physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you, the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



The position vector r' *is still* r' *in the moving frame S'.no matter how much time has passed!!*

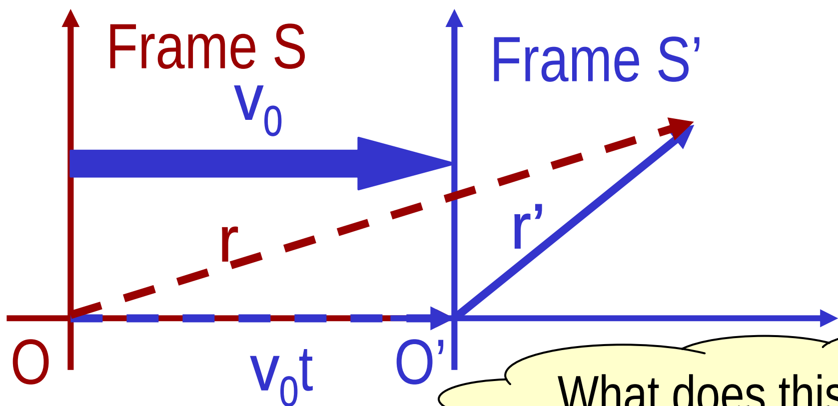
The position vector r *is no longer* r *in the stationary frame S when time t has passed.*

How are these position vectors related to each other?

$$\vec{r}(t) = \vec{r}' + \vec{v}_0 t$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$\vec{a}\,' = \vec{a}$, when $\vec{v}_0$ is constant

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCE is what cause an object to move.

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

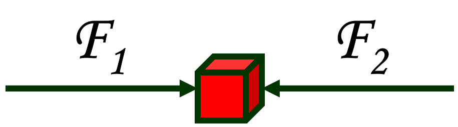
FORCEs are what cause any change in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Force causes acceleration.

What happens there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.



NET FORCE, $F = F_1 + F_2$

When net force on an object is 0, it has constant velocity and is at its equilibrium!!

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs higher force.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the 1st **law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.

A frame of reference that is moving at constant velocity is called an *Inertial Frame*

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of method of measurement: The same no matter how you measure it.

The heavier an object the bigger the inertia gets!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

Since it's a vector expression, each component should also satisfy:

$$\boxed{\sum_i F_{ix} = ma_x} \quad \boxed{\sum_i F_{iy} = ma_y} \quad \boxed{\sum_i F_{iz} = ma_z}$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is $[m][a] = [M][LT^{-2}]$

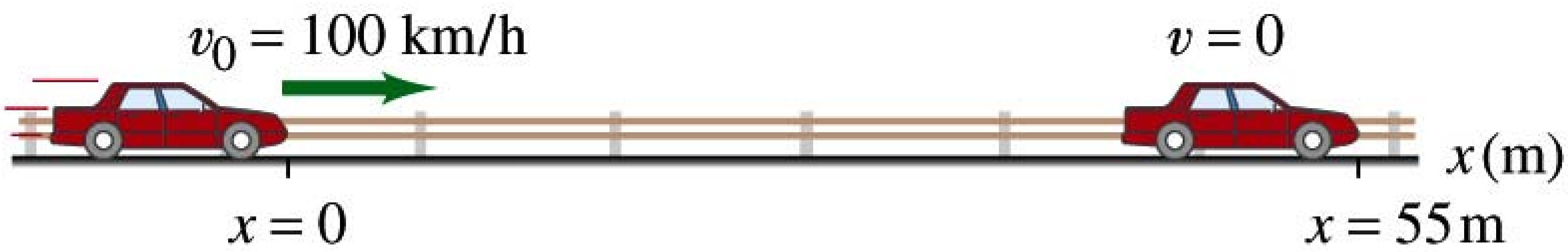
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, a Newton (N)

$$\boxed{1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs}$$

Example 4.2

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force?

Acceleration!!

What are given? Initial speed: $v_{xi} = 100\,km/h = 28\,m/s$ Final speed: $v_{xf} = 0\,m/s$

Displacement: $\Delta x = x_f - x_i = 55\,m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Acceleration

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{-\left(28m/s\right)^2}{2\left(55m\right)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times \left(-7.1m/s^2\right) = -1.1 \times 10^4 N$$

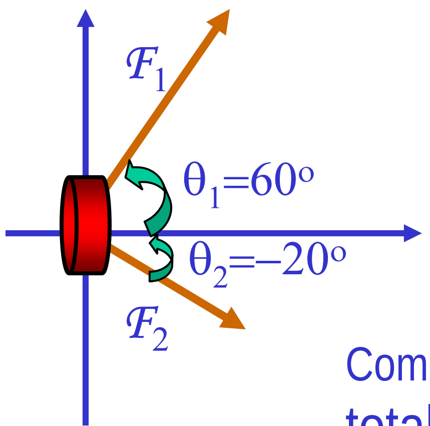
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2ma_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, F1 and F2, as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = \left(29\hat{i} + 17\hat{j}\right) m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using spring scale.