PHYS 1443 – Section 003
Lecture #8

Monday, Sept. 20, 2004

Dr. Jaehoon Yu

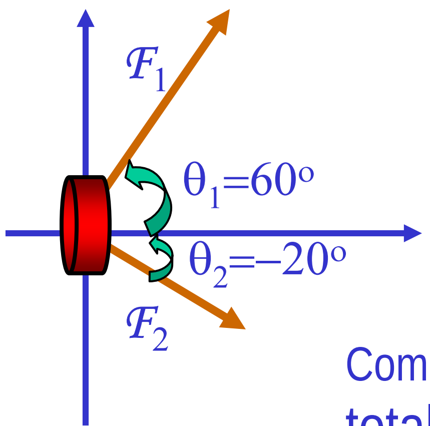
1. Newton's Laws of Motion
   - Gravitational Force and Weight
   - Newton's third law of motion
2. Application of Newton's Laws
   - Free-body diagrams
   - Application of Newton's Laws
   - Motion without friction
   - Forces of friction
   - Motion with friction

Announcements

- e-mail distribution list: 37/42 of you have subscribed so far.
  - A test message was sent Friday for verification
    - 26 of you replied! Thank you!
- Remember the 1st term exam, Monday, Sept. 27, one week from today
  - Covers up to chapter 6.
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: Your grade is F.
  - Mixture of multiple choice and free style problems

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, F1 and F2, as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

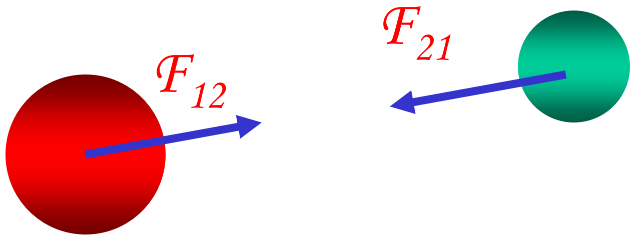
$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force F_{21} exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force F_{12} exerted on object 2 by object 1.



$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act <u>on different objects</u>.

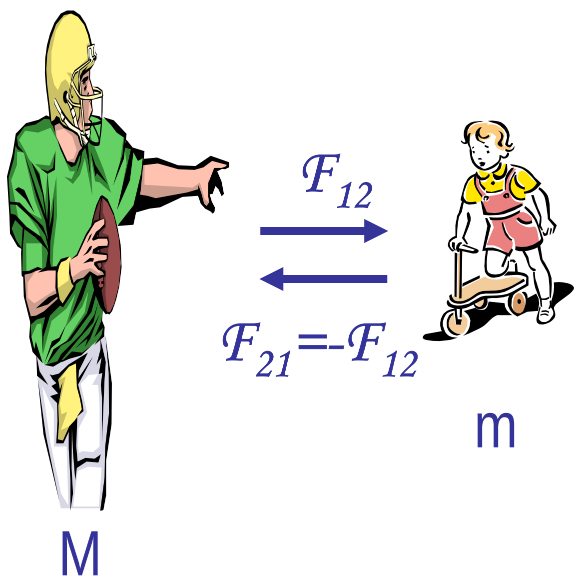
What is the reaction force to the force of a free fall object?

The gravitational force exerted by the object to the Earth!

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \quad\Rightarrow\quad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \quad\Rightarrow\quad F_{12x} = ma_{bx}, \quad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \quad\Rightarrow\quad F_{21x} = Ma_{Mx}, \quad F_{21y} = Ma_{My} = 0$$

Since $\vec{F}_{12} = -\vec{F}_{21}$ and $\left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F$

Establish the equation $\Rightarrow$ $ma_{bx} = F = Ma_{Mx}$ Divide by m $\Rightarrow$ $a_{bx} = \dfrac{F}{m} = \dfrac{M}{m} a_{Mx}$

Example of Newton's 3rd Law

Man's velocity

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

Boy's velocity

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

So boy's velocity if higher than man's, if M>m, by the ratio of the masses.

b) Who moves farther while their hands are in contact?

Boy's displacement

$$x_b = v_{bxi}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(\frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Man's displacement

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external force is applied.

Normal Force, n:

Reaction force that reacts to gravitational force due to the surface structure of an object. Its direction is perpendicular to the surface.

Tension, T:

The reactionary force by a stringy object against an external force exerted on it.

Free-body diagram

A graphical tool which is a ***diagram of external forces on an object*** *and is extremely useful analyzing forces and motion!! Drawn only on an object.*

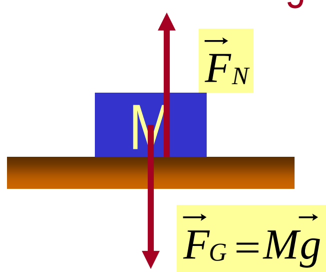
Free Body Diagrams and Solving Problems

- Free-body diagram: A diagram of vector forces acting on an object

⇒ A great tool to solve a problem using forces or using dynamics

1. Select a point on an object in the problem
2. Identify all the forces acting only on the selected object
3. Define a reference frame with positive and negative axes specified
4. Draw arrows to represent the force vectors on the selected point
5. Write down net force vector equation
6. Write down the forces in components to solve the problems

⇒ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion

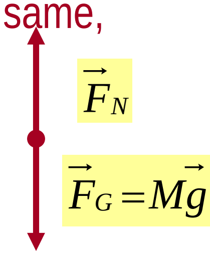


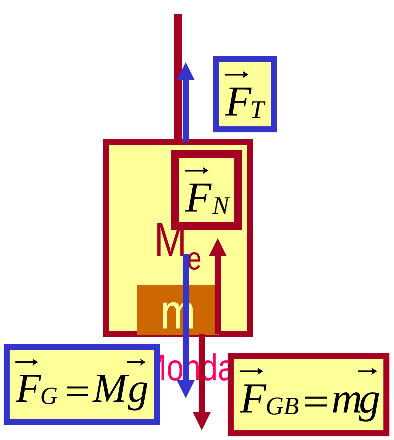
Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?

Gravitational force

A force supporting the object exerted by the floor



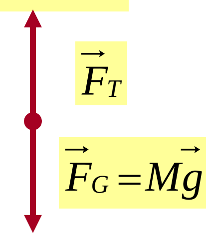


Which one would you like to select to draw FBD?

What do you think are the forces acting on this elevator?

Gravitational force

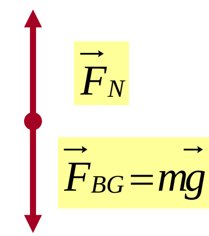
The force pulling the elevator (Tension)



What about the box in the elevator?

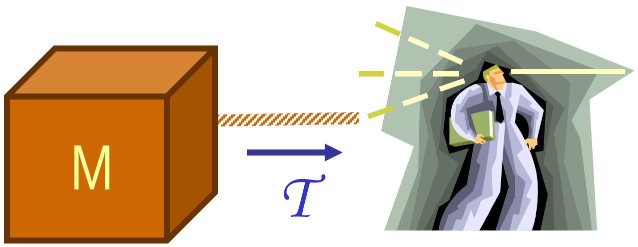
Gravitational force

Normal force



Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



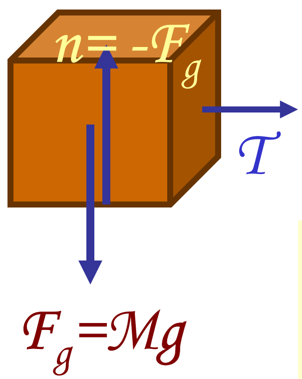
What are the forces being exerted on the box?

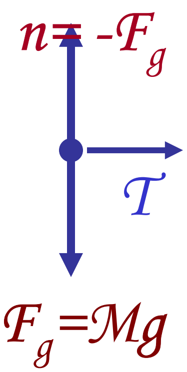
Gravitational force: F_g

Normal force: n

Tension force: T

Free-body diagram





Total force: $F=F_g+n+T=T$

$$\sum F_x = T = Ma_x \qquad \boxed{a_x = \frac{T}{M}}$$

$$\sum F_y = -F_g + n = Ma_y = 0 \qquad \boxed{a_y = 0}$$

If T is a constant force, a_x, is constant

$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

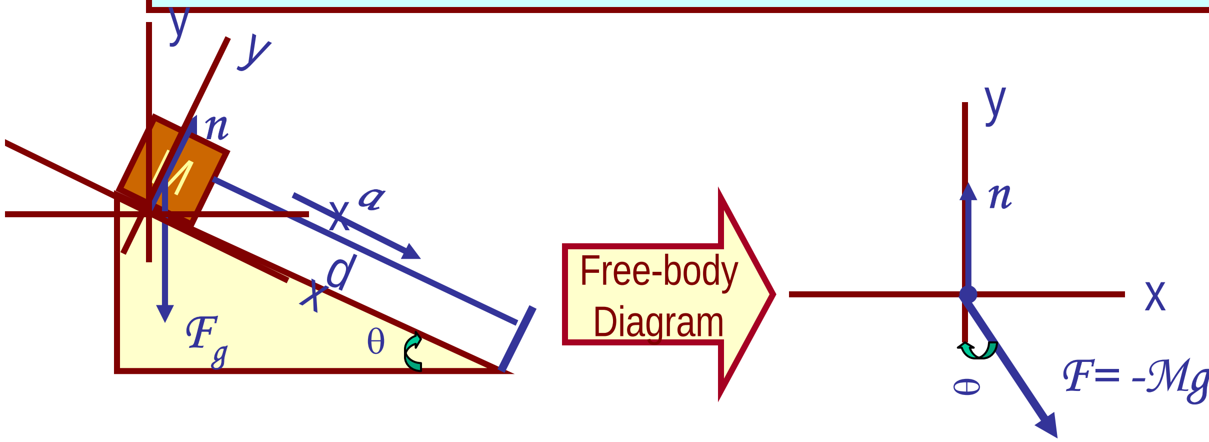
$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

What happened to the motion in y-direction?

Example w/o Friction

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.



$$\vec{F} = \vec{F}_g + \vec{n} = m\vec{a}$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$\boxed{a_x = g\sin\theta}$$

$$F_y = Ma_y = n - F_{gy} = n - mg\cos\theta = 0$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta\, t^2$$

$$\boxed{\therefore t = \sqrt{\frac{2d}{g\sin\theta}}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$

$$\boxed{\therefore v_{xf} = \sqrt{2dg\sin\theta}}$$