PHYS 1443 – Section 003
Lecture #9

Wednesday, Sept. 22, 2004

Dr. Jaehoon Yu

1. Forces of Friction
2. Uniform and Non-uniform Circular Motions
3. Resistive Forces and Terminal Velocity
4. Newton's Law of Universal Gravitation
5. Kepler's Laws

Homework #6 due at 1pm next Wednesday, Oct. 6!!

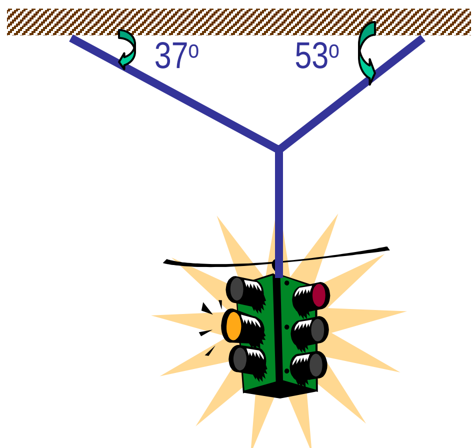
Remember the first term exam next Monday, Sept. 27!!

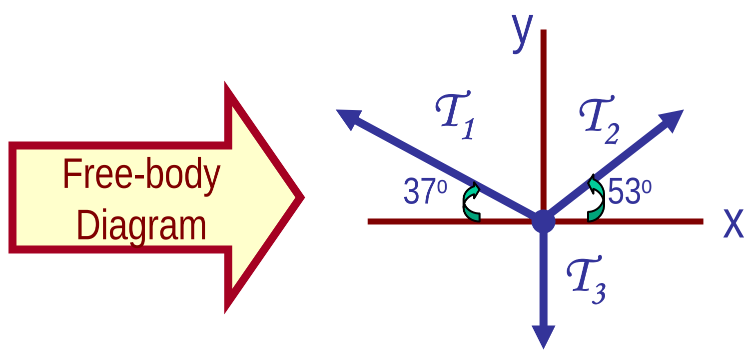
Announcements

- Quiz Results
  – Class Average: 3.6/8
  – Top score: 7
  – We have a few more quizzes through the semester
- Remember the 1st term exam, **<u>Monday, Sept. 27</u>**
  – 1:00 – 2:20pm in class
  – Covers Chapters 1 - 6.4
  – Mixture of multiple choice and free style problems
  – PLEASE DO NOT Miss the exam!!!!

Example for Using Newton's Laws

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0º and 53.0º with the horizontal. Find the tension in the three cables.





$$\vec{F}=\vec{T}_1+\vec{T}_2+\vec{T}_3=m\vec{a}=0$$ Newton's 2nd law

x-comp. of net force

$$F_x=\sum_{i=1}^{i=3}T_{ix}=0 \qquad -T_1\cos\left(37^\circ\right)+T_2\cos\left(53^\circ\right)=0 \;\therefore T_1=\frac{\cos\left(53^\circ\right)}{\cos\left(37^\circ\right)}T_2=0.754T_2$$

y-comp. of net force

$$F_y=\sum_{i=1}^{i=3}T_{iy}=0$$

$$T_1\sin\left(37^\circ\right)+T_2\sin\left(53^\circ\right)-mg=0$$

$$T_2\left[\sin\left(53^\circ\right)+0.754\times\sin\left(37^\circ\right)\right]=1.25T_2=125N$$

$$T_2=100N; \quad T_1=0.754T_2=75.4N$$

Forces of Friction

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to the velocity or the normal force.

Force of static friction, f_s: *The resistive force exerted on the object until just before the beginning of its movement*

Empirical Formula

$$\left|\vec{f}_s\right| \le \mu_s \left|\vec{n}\right|$$

What does this formula tell you?

Frictional force increases till it reaches the limit!!

Beyond the limit, the object moves, and there is NO MORE static friction but kinetic friction takes it over.

Force of kinetic friction, f_k: *The resistive force exerted on the object during its movement*

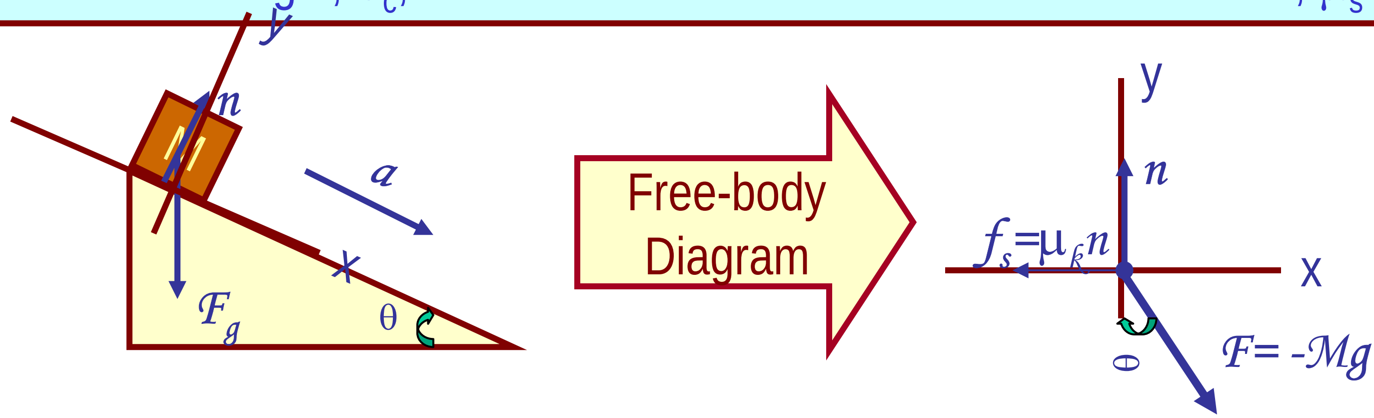
$$\left|\vec{f}_k\right| = \mu_k \left|\vec{n}\right|$$

Which direction does kinetic friction apply?

Opposite to the motion!

Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.



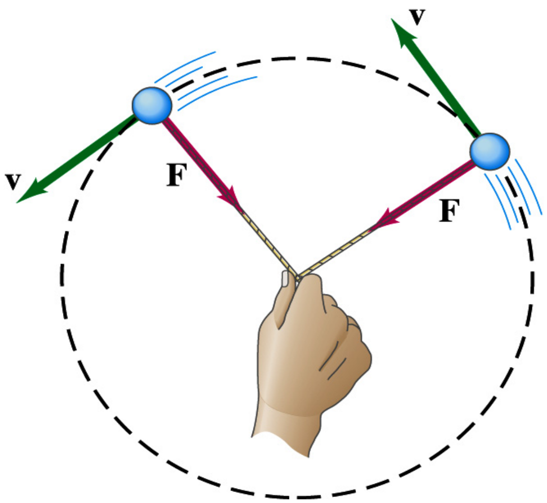
Net force $\vec{F} = M\vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$

x comp. $F_x = F_{gx} - f_s = Mg\sin\theta - f_s = 0 \qquad f_s = \mu_s n = Mg\sin\theta_c$

y comp. $F_y = Ma_y = n - F_{gy} = n - Mg\cos\theta_c = 0 \qquad n = F_{gy} = Mg\cos\theta_c$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$

Newton's Second Law & Uniform Circular Motion



The centripetal acceleration is always perpendicular to velocity vector, v, for uniform circular motion.

$$a_r = \frac{v^2}{r}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. This force is called **centripetal force.**

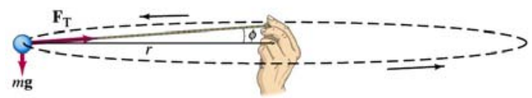
$$\sum F_r = ma_r = m\frac{v^2}{r}$$

What do you think will happen to the ball if the string that holds the ball breaks? Why?

Based on Newton's 1st law, since the external force no longer exist, the ball will continue its motion without change and will fly away following the tangential direction to the circle.

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the centripetal force is greater than the sustainable tension.

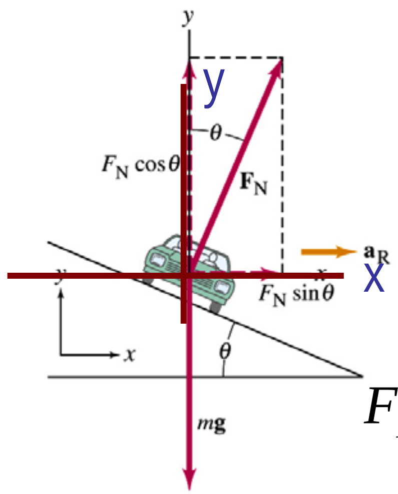
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Example of Banked Highway

(a) For a car traveling with speed *v* around a curve of radius *r*, determine a formula for the angle at which a road should be banked so that no friction is required to keep the car from skidding.



x comp. $$\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \frac{mv^2}{r} = 0$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $$\sum F_y = F_N \cos\theta - mg = 0 \qquad F_N \cos\theta = mg$$

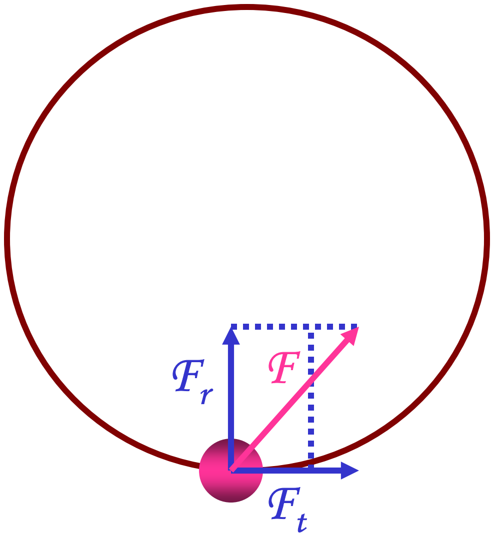
$$F_N = \frac{mg}{\sin\theta} \quad F_N \sin\theta = \frac{mg \sin\theta}{\cos\theta} = mg \tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50km/hr = 14m/s \qquad \tan\theta = \frac{(14)^2}{50 \times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22^{\circ}$$

Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

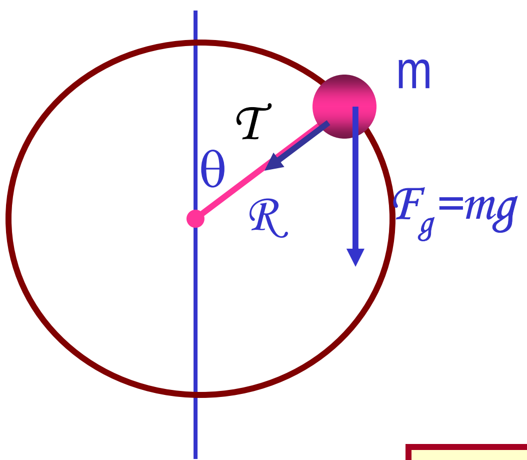
These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.

What is the magnitude of the net acceleration?

$$a = \sqrt{a_r^2 + a_t^2}$$

Example for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is v and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

- The gravitational force F_g
- The radial force, T, providing tension.

tangential comp.

$$\sum F_t = mg\sin\theta = ma_t$$

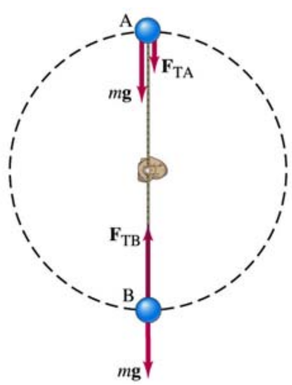
$$a_t = g\sin\theta$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} - g\cos\theta\right)$$

At what angles the tension becomes maximum and minimum. What are the tensions?



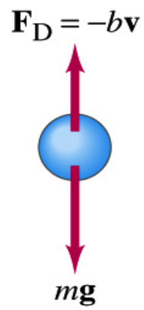
Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.

Some examples? | Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

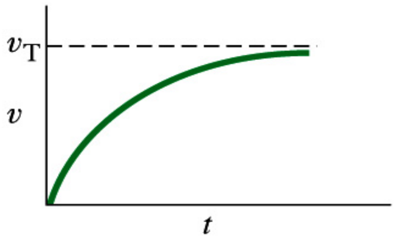
These forces are proportional to such factors as speed. They almost always increase with increasing speed.



(a)

Two different cases of proportionality:

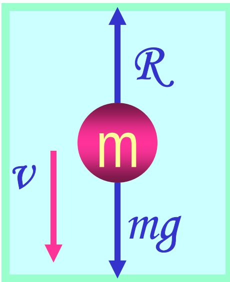
1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed



(b)

Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv.



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R} = m\vec{a} \qquad \sum F_x = 0$$

In other words

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt} \qquad \frac{dv}{dt} = g - \frac{b}{m}v \qquad \frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (**terminal speed**) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, $\tau = m/b$.