PHYS 1443 – Section 003
Lecture #12

Wednesday, Oct. 6, 2004

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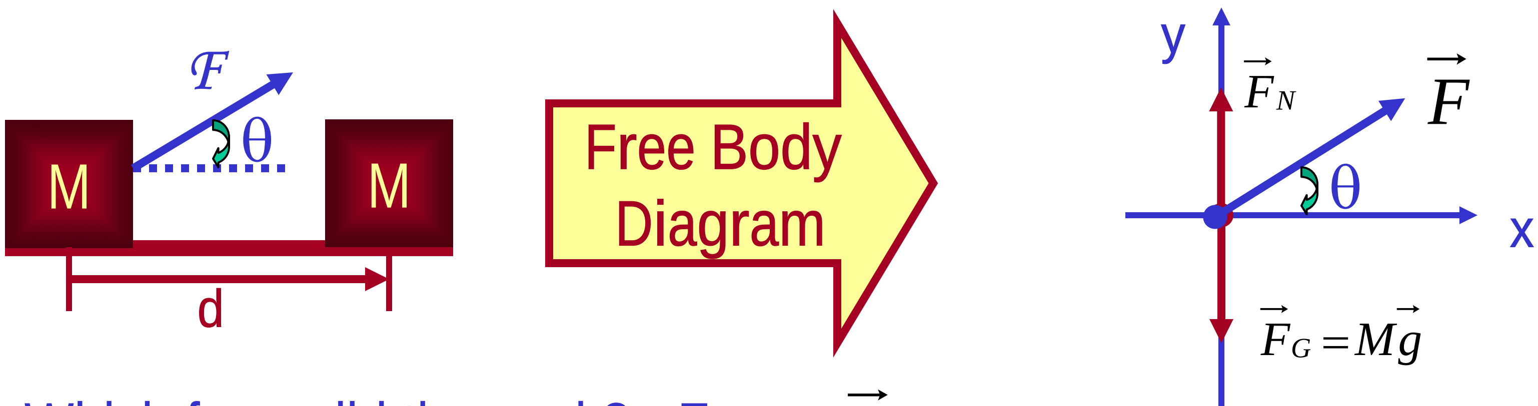
1. Work done by a constant force
2. Scalar Product of Vectors
3. Work done by a varying force
4. Work and Kinetic Energy Theorem
5. Potential Energy

Homework #7 due at 1pm next Wednesday, Oct. 13!!

Work Done by a Constant Force

Work in physics is done only when a sum of forces exerted on an object made a motion to the object.



Which force did the work? Force $\vec{F}$

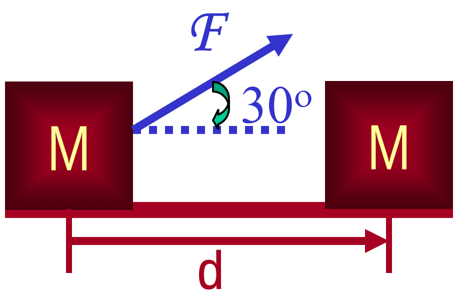
How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$ Unit? $N \bullet m = J$ (for Joule)

What does this mean? **Physical work is done only by the component of the force along the movement of the object.**

Work is an <u>energy transfer</u>!!

Example of Work w/ Constant Force

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0° with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\left(\sum \vec{F}\right)\right| \left|\vec{d}\right| \cos\theta$$

$$W = 50.0 \times 3.00 \times \cos 30^{\circ} = 130J$$

Does work depend on mass of the object being worked on?

Yes

Why don't I see the mass term in the work at all then?

It is reflected in the force. If the object has smaller mass, its would take less force to move it the same distance as the heavier object. So it would take less work. Which makes perfect sense, doesn't it?

Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A}\bullet\vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$
- Operation is commutative $\vec{A}\bullet\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta = \vec{B}\bullet\vec{A}$
- Operation follows distribution law of multiplication $\vec{A}\bullet\left(\vec{B}+\vec{C}\right) = \vec{A}\bullet\vec{B} + \vec{A}\bullet\vec{C}$
- Scalar products of Unit Vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- How does scalar product look in terms of components?

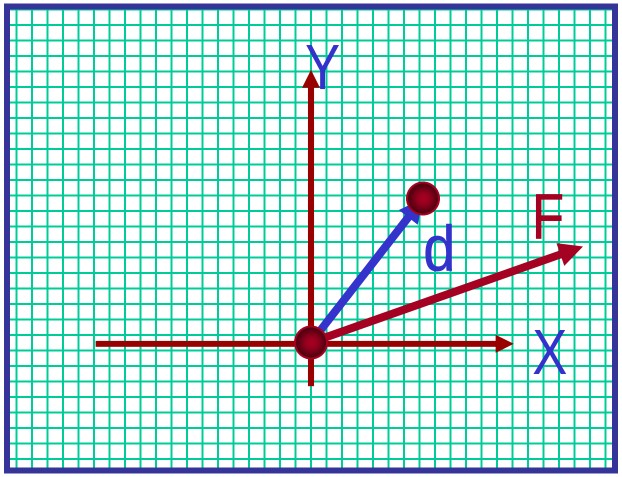
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \quad \vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

$$\vec{A}\bullet\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\bullet\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\,\hat{i}\bullet\hat{i} + A_yB_y\,\hat{j}\bullet\hat{j} + A_zB_z\,\hat{k}\bullet\hat{k}\right) + \underbrace{\text{cross terms}}_{=0}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Example of Work by Scalar Product

A particle moving in the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force F=(5.0**i**+2.0**j**) N acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

$$W = \vec{F} \cdot \vec{d} = \left(2.0\hat{i} + 3.0\hat{j}\right) \cdot \left(5.0\hat{i} + 2.0\hat{j}\right) = 2.0 \times 5.0\,\hat{i} \cdot \hat{i} + 3.0 \times 2.0\,\hat{j} \cdot \hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between **d** and F?

$$\boxed{W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta}$$

Work Done by Varying Force

- If the force depends on position of the object through the motion
  - one must consider work in small segments of the position where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

  - Then add all work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

  - If more than one force is acting, the net work is done by the net force

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

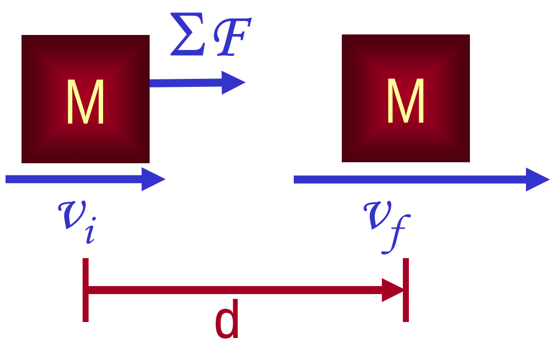
One of the forces depends on position is force by a spring $F_s = -kx$

The work done by the spring force is

$$W = \int_{-x_{\max}}^{0} F_s dx = \int_{-x_{\max}}^{0} (-kx) dx = \frac{1}{2} kx^2$$

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on the object during the motion are so complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force $\Sigma\mathcal{F}$ was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = (ma)\,d\cos 0 = (ma)\,d$$

Displacement $$d = \frac{1}{2}\left(v_f + v_i\right)t$$

Acceleration $$a = \frac{v_f - v_i}{t}$$

Work $$W = (ma)d = \left[m\left(\frac{v_f - v_i}{t}\right)\right]\frac{1}{2}\left(v_f + v_i\right)t = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Kinetic Energy $$\boxed{KE \equiv \frac{1}{2}mv^2}$$

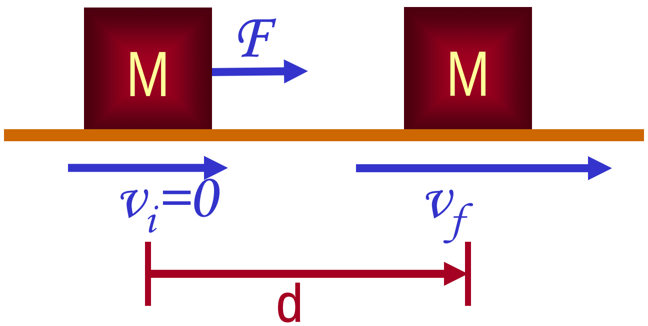
Work $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

The work done by the net force caused change of object's kinetic energy.

Work-Kinetic Energy Theorem

Example of Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36 (J)$$

From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

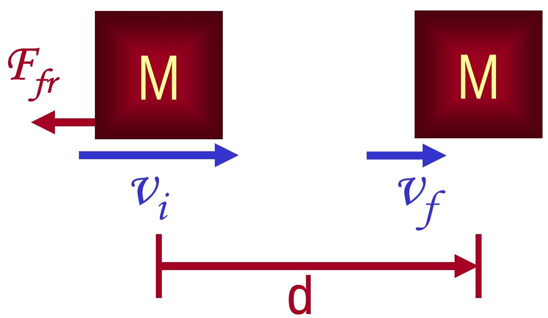
Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

Solving the equation for v_f, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Why doesn't static friction matter?

Because it isn't there while the object is moving.



Friction force F_{fr} works on the object to slow down

The work on the object by the friction F_{fr} is

$$W_{fr} = F_{fr} d\cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

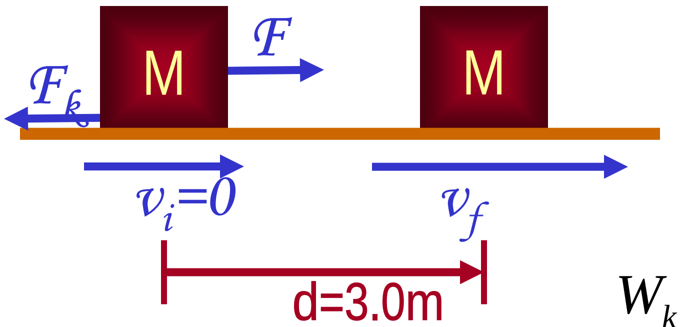
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$