PHYS 1443 – Section 003
Lecture #14

Wednesday, Oct. 13, 2004

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1. Conservation of Mechanical Energy
2. Work Done by Non-conservative Forces
3. Energy Diagram and Equilibrium
4. Gravitational Potential Energy
   - Escape Speed
5. Power

Homework #8, due 1pm, next Wednesday, Oct. 20!!

Quiz Monday, Oct. 18!!

Work Done by Non-conserve Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative (dissipative) force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the mechanical energy of the system is no longer conserved.

If you were to hit a free falling ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{applied} = \Delta K + \Delta U$$

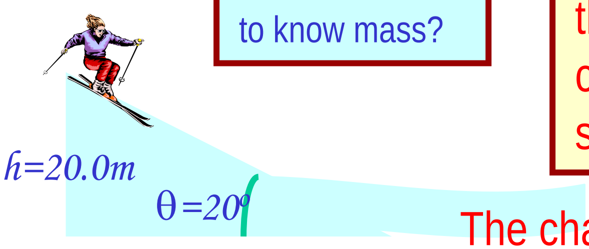
Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example of Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy to take it all away.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2 \times 0.210 \times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.

How are Conservative Forces Related to Potential Energy?

Work done by a force component on an object through a displacement Δx is

$$W = F_x \Delta x = -\Delta U$$

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

For an infinitesimal displacement Δx

$$dU = -F_x dx$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

This relationship says that any conservative force acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to position.

Does this statement make sense?

1. spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg$$

The relationship works in both the conservative force cases we have learned!!!

Energy Diagram and the Equilibrium of a System

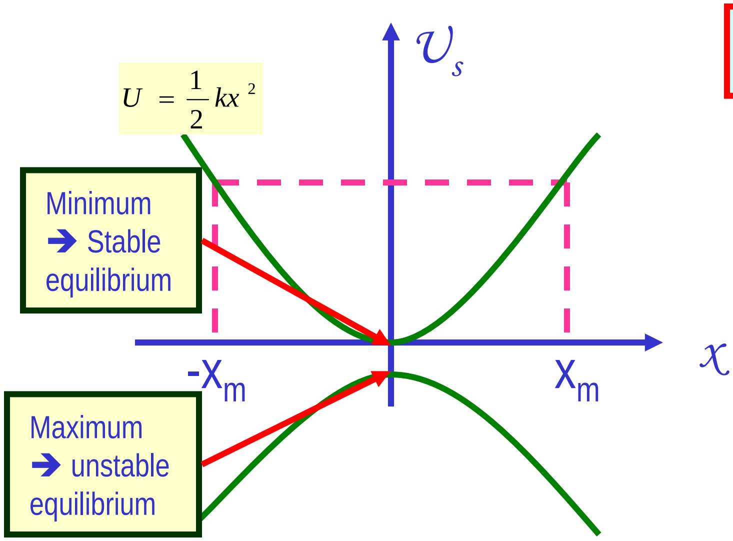
One can draw potential energy as a function of position ➔ *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is one of the stable or equilibrium of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another.
Total energy of universe is constant!!

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field g is defined as $\vec{g} \equiv \dfrac{\vec{F}_g}{m}$

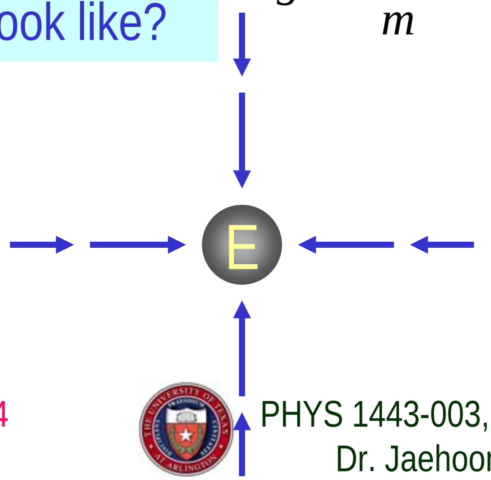
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

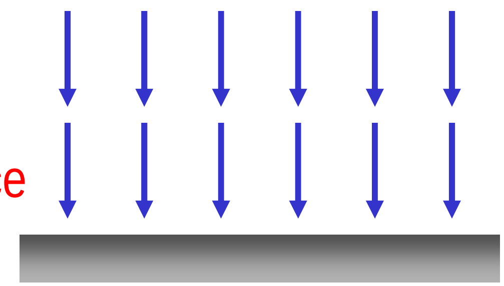
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

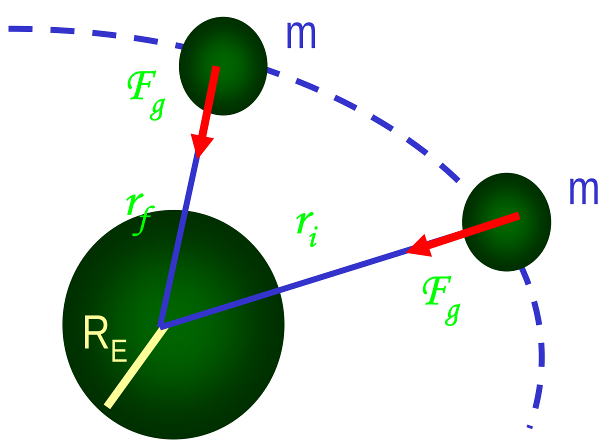
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Because gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!