PHYS 1443 – Section 003
Lecture #15

Monday, Oct. 18, 2004

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1. Gravitational Potential Energy
    - Escape Speed
2. Power
3. Linear Momentum
4. Linear Momentum and Forces
5. Conservation of Momentum
6. Impulse

2nd Term Exam Monday, Nov. 1!!

Announcements

- Remember the 2nd term exam, **Monday, Nov. 1**, two weeks from today
  - Covers Chapter 6 – wherever we get to (~Chapter 11?).
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: Your grade is F.
  - Mixture of multiple choice and free style problems

The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

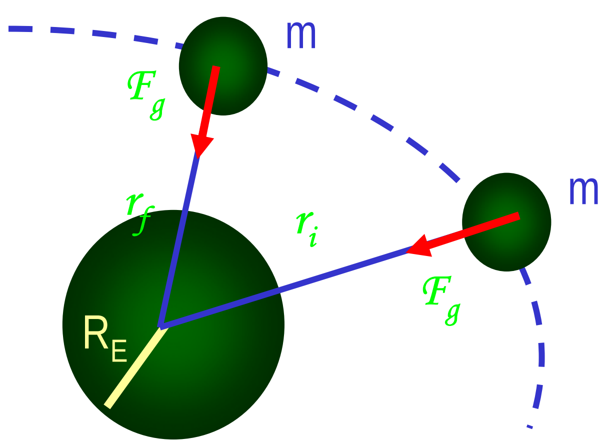
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path is radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr \xrightarrow{\text{For the whole path}} W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of work in the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is $F(r) = -\frac{GM_E m}{r^2}$

So the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain

$$U = -\frac{GM_E m}{r}$$

For any two particles?

$$U = -\frac{Gm_1 m_2}{r}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$U = \sum_{i,j} U_{i,j}$$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

The above equation becomes

$$\Delta U = -GM_E m\frac{\left(r_f - r_i\right)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$$r_i \approx R_E \quad \text{and} \quad r_f \approx R_E$$

Therefore, ΔU becomes

$$\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$$

Since on the surface of the Earth the gravitational field is

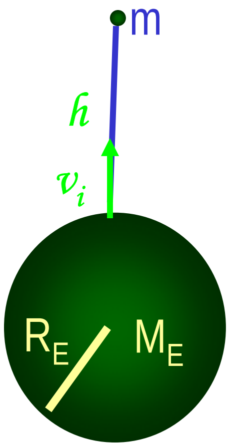
$$g = \frac{GM_E}{R_E^2}$$

The potential energy becomes

$$\Delta U = -mg\Delta y$$

Escape Speed

$v_f=0$ at $h=r_{max}$



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop $v_f=0$ at the distance r_{max}.

Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{\max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height h of the object.

$$h = r_{\max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for the object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.37\times 10^{6}}}$$

$$= 1.12\times 10^4\, m/s = 11.2\, km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object

Power

- Rate at which the work is done or the energy is transferred
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
  - ➔ 8 cylinder car climbs up faster

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinders than 4.

Average power $\boxed{\overline{P} = \frac{\Delta W}{\Delta t}}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \lim_{\Delta t \to 0} \left(\sum \vec{F}\right) \cdot \frac{\Delta \vec{s}}{\Delta t} = \left(\sum \vec{F}\right) \cdot \vec{v} = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $\boxed{J/s = Watts}$ $\boxed{1HP = 746\,Watts}$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^{6}\,J$

Energy

Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:
- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2 v^2 = 0.647 v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\boxed{\vec{p} \equiv m\vec{v}}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$