PHYS 1443 – Section 003
Lecture #16

Wednesday, Oct. 20, 2004
Dr. Jaehoon Yu

1. Linear Momentum
2. Linear Momentum and Forces
3. Conservation of Momentum
4. Impulse and Momentum Change

Today's homework is #9, due Wednesday, Oct. 27!!

2nd Term Exam Monday, Nov. 1!!

Announcements

- Quiz Results:
  - Class Average: 4.25/10
    - Previous quizzes?: 5.7/10 (1^{st}), 4.5/10 (2^{nd})
    - Hope this is not a trend…
  - Top score: 9
  - Quiz constitutes 15% of the total grade
  - One or two more quizzes before the end of the semester…
- Remember the 2^{nd} term exam, Monday, Nov. 1
  - 1:00 – 2:30pm, SH103
  - Covers Chapter 6 – wherever we get to (~Chapter 11?).
  - No make-up exams
    - Miss an exam without pre-approval or a good reason: Your grade is F.
  - Mixture of multiple choice and free style problems

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

$$\boxed{\sum\vec{F} = \frac{d\vec{p}}{dt}}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that does not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces and the net force in the SYSTEM is still 0.

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum p_1 and #2 has p_2 at some point of time.

Using momentum-force relationship

$$\boxed{\vec{F}_{21} = \frac{d\vec{p}_1}{dt}}$$ and $$\boxed{\vec{F}_{12} = \frac{d\vec{p}_2}{dt}}$$

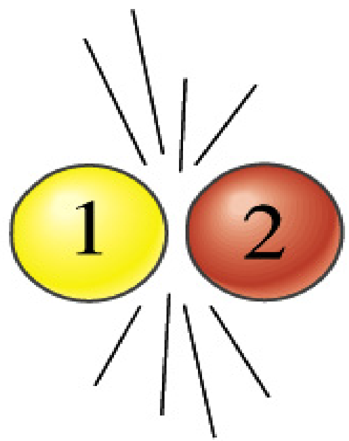
And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$

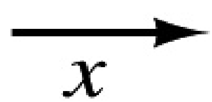
The total linear momentum of the system is conserved!!!

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$



More on Conservation of Linear Momentum in a Two Particle System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interaction

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1_i} = \vec{p}_{2f} + \vec{p}_{1f}$$

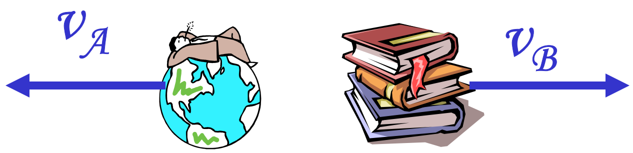
$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

Estimate an astronaut's resulting velocity after he throws his book to a direction in the space to move to a direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = -\frac{m_B \vec{v}_B}{m_A} = -\frac{1}{70}\vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\,\vec{i}\,(m/s)$$

Impulse and Linear Momentum

Net force causes change of momentum ➔ Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse I.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt = \vec{I}$$

So what do you think an impulse is?

Impulse of the force F acting on a particle over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the particle caused by that force. Impulse is the degree of which an external force changes momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\overline{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{I} \equiv \overline{\vec{F}}\Delta t$$

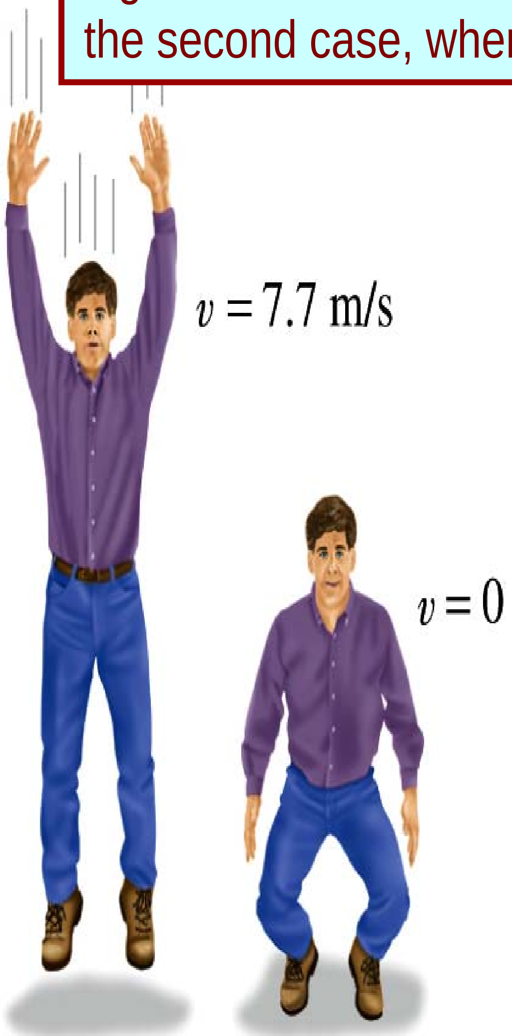
If force is constant

$$\vec{I} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example 9-6

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$I = \overline{F}\Delta t = \Delta p = p_f - p_i = 0 - mv =$$
$$= -70kg \cdot 7.7m/s = -540N \cdot s$$

Example 9 – 6 cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times10^{-3}s$$

Since the magnitude of impulse is $$I = \bar{F}\Delta t = 540N\cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6\times10^{-3}} = 2.1\times10^{5}N$$

How large is this average force? $$Weight = 70kg\cdot 9.8m/s^2 = 6.9\times10^{2}N$$

$$\bar{F} = 2.1\times10^{5}N = 304\times6.9\times10^{2}N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1\times10^{3}N = 5.9Weight$$