PHYS 1443 – Section 003
Lecture #18

Monday, Nov. 8, 2004

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1. Torque
2. Torque and Vector Product
3. Moment of Inertia
4. Torque and Angular Acceleration
5. Parallel Axis Theorem
6. Rotational Kinetic Energy

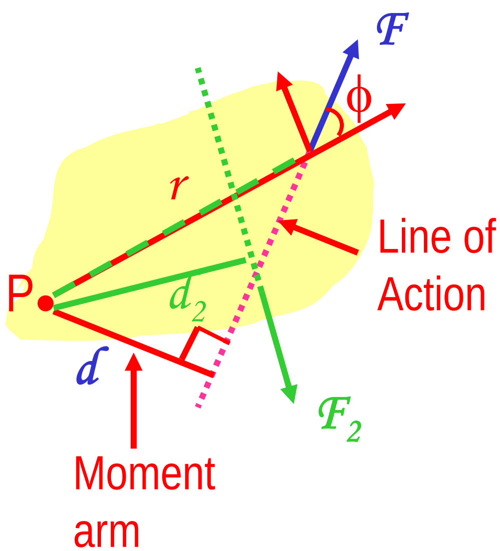
Today's homework is HW #10 on the class web page!!

Announcements

- Homework site will be down for at least two weeks
  - Homework problems will be posted on the class web page one HW at a time per week till HW site turns back on.
  - These problems will not be identical to your final HW problems but all the procedures will be identical. So I suggest you to work out the formula for your problem so that you can plug in numbers for your actual problems.
  - I have your HW grade up to HW9.
- Class Average went up to 47.3 after the fix
- Will spend about an hour today to discuss your mid-semester progress discussions

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force F being exerted at a distance r.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called Moment arm.

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

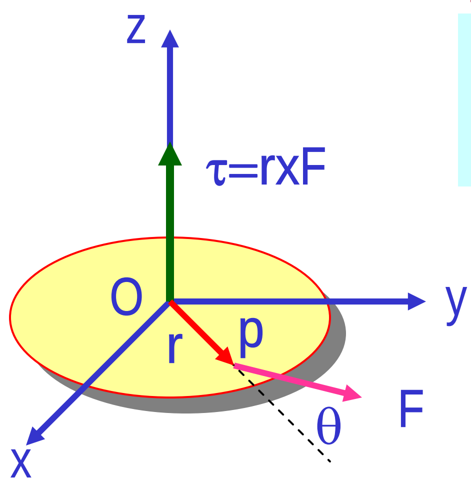
$$\tau \equiv rF\sin\phi = Fd$$

When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is positive if rotation is in counter-clockwise and negative if clockwise.

$$\sum \tau = \tau_1 + \tau_2$$

$$= F_1 d_1 - F_2 d_2$$

Torque and Vector Product



Let's consider a disk fixed onto the origin O and the force $\mathcal{F}$ exerts on the point p. What happens?

The disk will start rotating counter clockwise about the Z axis

The magnitude of torque given to the disk by the force $\mathcal{F}$ is

$$\tau = Fr\sin\phi$$

But torque is a vector quantity, what is the direction?
How is torque expressed mathematically?

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

What is the direction?

The direction of the torque follows the right-hand rule!!

The above operation is called
Vector product or Cross product

$$\vec{C} \equiv \vec{A} \times \vec{B}$$

$$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta$$

What is the result of a vector product?

Another vector

What is another vector operation we've learned?

Scalar product

$$C \equiv \vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$$

Result? A scalar

Properties of Vector Product

Vector Product is Non-commutative

What does this mean?

If the order of operation changes the result changes

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

Following the right-hand rule, the direction changes

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

Vector Product of two parallel vectors is 0.

$$\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta = \left|\vec{A}\right|\left|\vec{B}\right|\sin 0 = 0$$

Thus,

$$\vec{A}\times\vec{A} = 0$$

If two vectors are perpendicular to each other

$$\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta = \left|\vec{A}\right|\left|\vec{B}\right|\sin 90^\circ = \left|\vec{A}\right|\left|\vec{B}\right| = AB$$

Vector product follows distribution law

$$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$$

The derivative of a Vector product with respect to a scalar variable is

$$\frac{d\left(\vec{A}\times\vec{B}\right)}{dt} = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$$

More Properties of Vector Product

The relationship between unit vectors, $\vec{i}, \vec{j}$ and $\vec{k}$

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k}$$

$$\vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}$$

$$\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

Vector product of two vectors can be expressed in the following determinant form

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \left(A_y B_z - A_z B_y\right)\vec{i} - \left(A_x B_z - A_z B_x\right)\vec{j} + \left(A_x B_y - A_y B_x\right)\vec{k}$$