PHYS 1443 – Section 003
Lecture #22

Monday, Nov. 22, 2004

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1. Density and Specific Gravity
2. Fluid and Pressure
3. Absolute and Relative Pressure
4. Pascal's Law
5. Buoyant Force and Archimedes' Principle
6. Flow Rate and Continuity Equation
7. Bernoulli's Equation

Announcements

- Evaluation today!!
- Class this Wednesday

Density and Specific Gravity

Density, ρ (rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 ^oC (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{substance}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface

Fluid and Pressure

What are the three states of matter?

Solid, Liquid, and Gas

How do you distinguish them?

By the time it takes for a particular substance to change its shape in reaction to external forces.

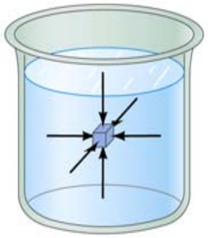
What is a fluid?

A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

We will first learn about mechanics of fluid at rest, *fluid statics*.

In what way do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the forces perpendicular to the surfaces of the object.



This force by the fluid on an object usually is expressed in the form of the force on a unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and dimension of pressure?

Unit: N/m^2

Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0^{o}C and 1 atm) is $1000 kg/m^3$. So the total mass of the water in the mattress is

$$m = \rho_W V_M = 1000 \times 2.00 \times 2.00 \times 0.300 = 1.20 \times 10^3 kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20 \times 10^3 \times 9.8 = 1.18 \times 10^4 N$$

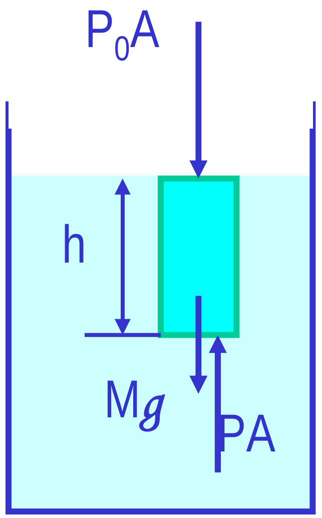
b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is 4.00 m^2, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18 \times 10^4}{4.00} = 2.95 \times 10^3$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine a liquid contained in a cylinder with height h and cross sectional area A immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho A h$

Since the system is in its equilibrium $PA - P_0A - Mg = PA - P_0A - \rho A h g = 0$

Therefore, we obtain $P = P_0 + \rho g h$

The pressure at the depth h below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by $\rho g h$.

Atmospheric pressure P_0 is

$$1.00\,atm = 1.013\times 10^{5}\,Pa$$

What else can you learn from this?

Pascal's Principle and Hydraulics

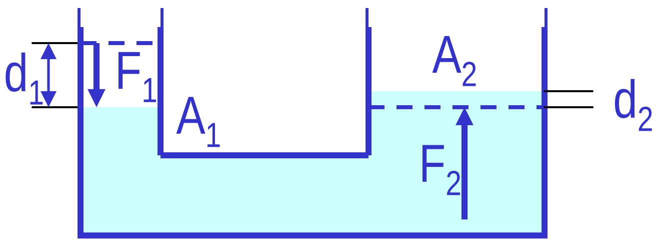
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho g h$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied on to the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force gets multiplied by the ratio of the areas A_2/A_1 and is transmitted to the force F_2 on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2} F_2 \quad = \frac{\pi (0.05)^2}{\pi (0.15)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3 \, N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5 \, Pa$$

Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

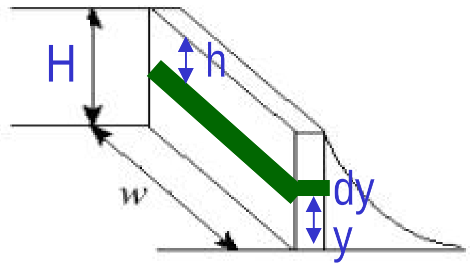
$$P - P_0 = \rho_W gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 Pa$$

Estimating the surface area of the eardrum at $1.0cm^2=1.0x10^{-4}$ m^2, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9N$$

Example for Pascal's Principle

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.



Since the water pressure varies as a function of depth, we will have to do some calculus to figure out the total force.

The pressure at the depth h is

$$P = \rho g h = \rho g\left(H - y\right)$$

The infinitesimal force dF exerting on a small strip of dam dy is

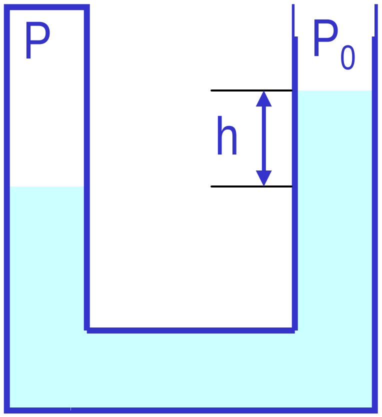
$$dF = PdA = \rho g\left(H - y\right)wdy$$

Therefore the total force exerted by the water on the dam is

$$F = \int_{y=0}^{y=H} \rho g\left(H - y\right)wdy = \rho g w\left[Hy - \frac{1}{2}y^2\right]_{y=0}^{y=H} = \frac{1}{2}\rho g w H^2$$

Absolute and Relative Pressure

How can one measure the pressure?



One can measure the pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure P_0.

The measured pressure of the system is $$P = P_0 + \rho g h$$

This is called the **absolute pressure**, because it is the actual value of the system's pressure.

In many cases we measure pressure difference with respect to atmospheric pressure due to changes in P_0 depending on the environment. This is called **gauge or relative pressure**.

$$P_G = P - P_0 = \rho g h$$

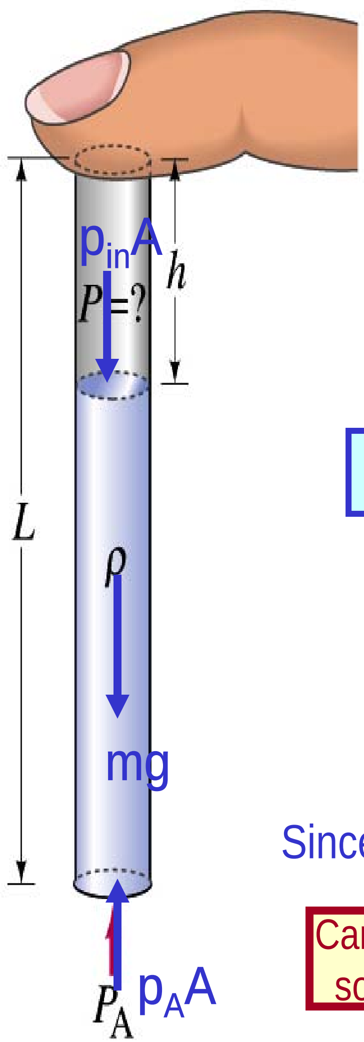
The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm is

$$P_0 = \rho g h = (13.595 \times 10^3 kg/m^3)(9.80665 m/s^2)(0.7600 m)$$
$$= 1.013 \times 10^5 Pa = 1 atm$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.

Finger Holds Water in Straw



You insert a straw of length L into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is h. Does the air in the space between your finger and the top of the liquid have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure P_A outside the straw?

Less

What are the forces in this problem?

Gravitational force on the mass of the liquid $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by outside air $F_{out} = -p_A A$

Since it is at equilibrium $F_{out} + F_g + F_{in} = 0$ ⇒ $-p_A A + \rho g(L-h)A + p_{in}A = 0$

Cancel A and solve for p_{in} ⇒ $p_{in} = p_A - \rho g(L-h)$

So p_{in} is less than P_A by ρgh.