PHYS 1443 – Section 003
Lecture #23

Wednesday, Nov. 24, 2004
Dr. Jaehoon Yu

1. Quiz Workout
2. Buoyant Force and Archimedes' Principle
3. Flow Rate and Continuity Equation
4. Bernoulli's Equation
5. Simple Harmonic Motion

Today's Homework is #12 due 5pm, Friday, Dec. 3, 2004!!

Final Exam, Monday, Dec. 6!!

Announcements

- Final Exam
  - Date: Monday, Dec. 6
  - Time: 11:00am – 12:30pm
  - Location: SH103
  - Covers: CH 10 – CH 14
- Review next Wednesday, Dec. 1!
- Quiz Result
  - Class Average: 4/7 (equivalent to 5.7 out of 10)
  - Top score: 7
  - Previous results: 4.9, 4.3, 4.3 out of 10
  - Marked improvement (a whopping 32%)!! Congratulations!!
  - Overall class average: 7.6/15

Buoyant Forces and Archimedes' Principle

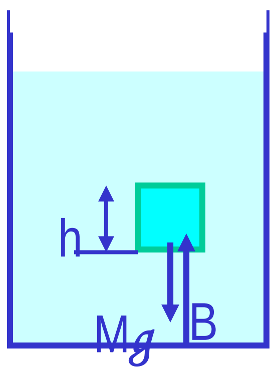
Why is it so hard to put an inflated beach ball under water while a small piece of steel sinks in the water?

The water exerts force on an object immersed in the water.
This force is called Buoyant force.

How does the Buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called, Archimedes' principle. What does this mean?



Let's consider a cube whose height is h and is filled with fluid and at in its equilibrium so that its weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

The pressure at the bottom of the cube is larger than the top by ρgh.

Therefore,

$$\Delta P = B / A = \rho gh$$

$$B = \Delta PA = \rho ghA = \rho Vg$$

$$B = \rho Vg = Mg = F_g$$

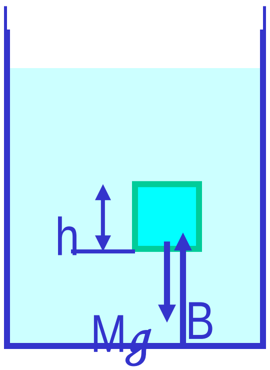
Where Mg is the weight of the fluid.

More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object

Let's consider an object of mass M, with density ρ_0, is immersed in the fluid with density ρ_f.



The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

Therefore total force of the system is $F = B - F_g = (\rho_f - \rho_0)Vg$

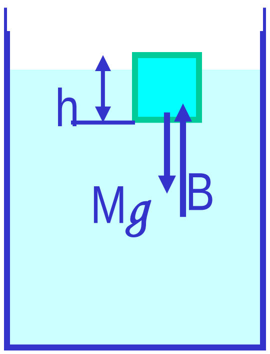
What does this tell you?

The total force applies to different directions depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger that the fluid's, the object will sink to the bottom of the fluid.

More Archimedes' Principle

Case 2: Floating object



Let's consider an object of mass M, with density ρ_0, is in static equilibrium floating on the surface of the fluid with density ρ_f, and the volume submerged in the fluid is V_f.

The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium

$$\rho_f V_f g = \rho_0 V_0 g$$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating its density is always smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

Example for Archimedes' Principle

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = \rho_w V_w g = \rho_w V_c g = 0.98N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98N}{\rho_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg/m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is not made of pure gold.

Example for Buoyant Force

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is V_i. Then the weight of the iceberg F_{gi} is

$$F_{gi} = \rho_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is V_w. The buoyant force B caused by the displaced water becomes

$$B = \rho_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$\rho_i V_i g = \rho_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\, kg/m^3}{1030\, kg/m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

Flow Rate and the Equation of Continuity

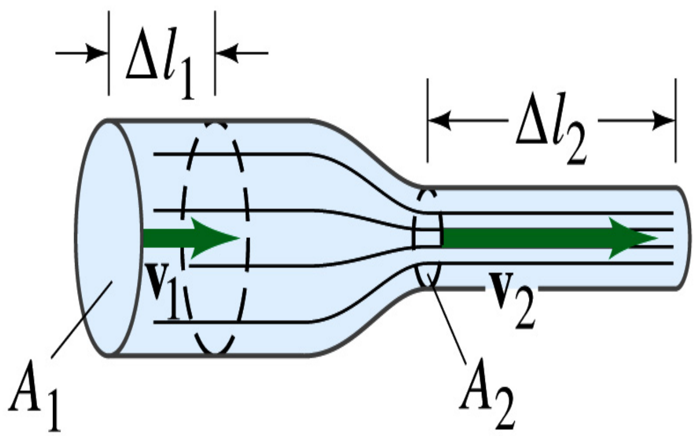
Study of fluid in motion: Fluid Dynamics

If the fluid is water: Water dynamics?? Hydro-dynamics

Two main types of flow

- **Streamline or Laminar flow**: Each particle of the fluid follows a smooth path, a streamline
- **Turbulent flow**: Erratic, small, whirlpool-like circles called eddy current or eddies which absorbs a lot of energy

Flow rate: the mass of fluid that passes a given point per unit time $\Delta m / \Delta t$



$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

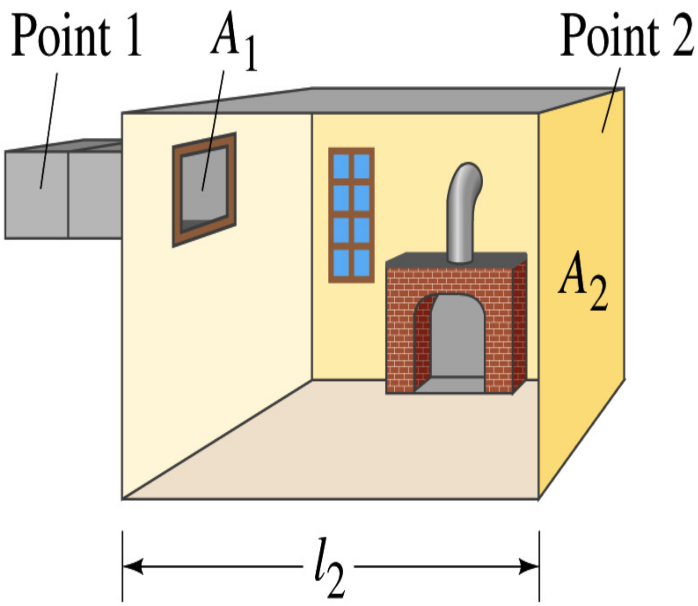
since the total flow must be conserved

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Equation of Continuity

Example for Equation of Continuity

How large must a heating duct be if air moving at 3.0m/s along it can replenish the air every 15 minutes, in a room of 300m^3 volume? Assume the air's density remains constant.



Using equation of continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Since the air density is constant

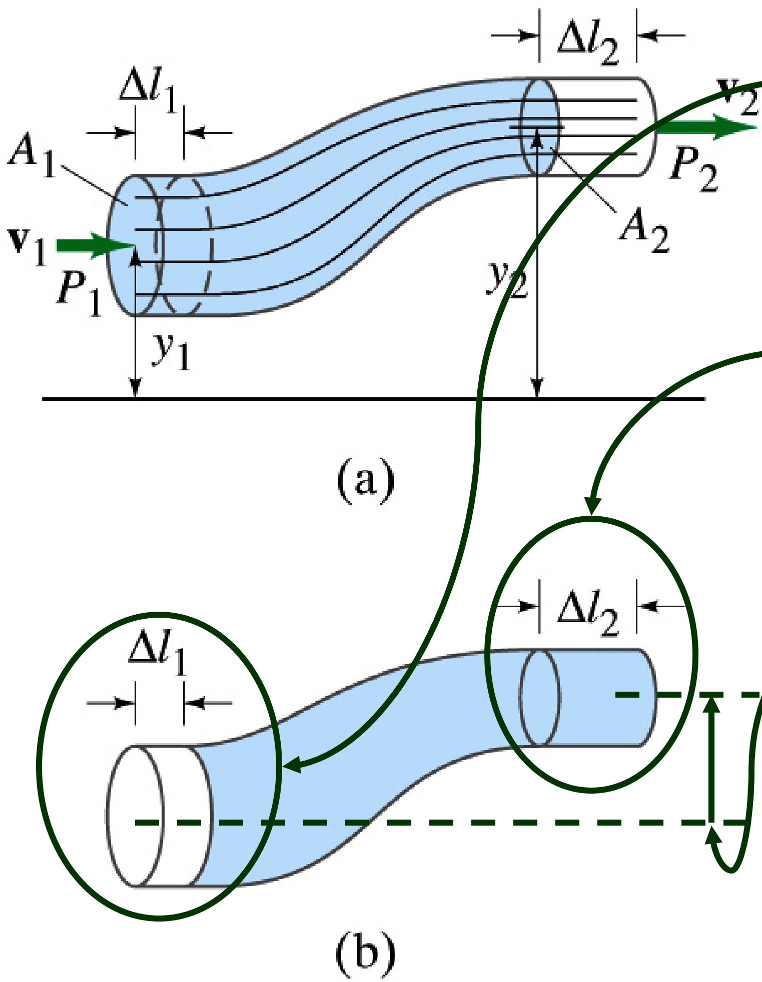
$$A_1 v_1 = A_2 v_2$$

Now let's imagine the room as the large section of the duct

$$A_1 = \frac{A_2 v_2}{v_1} = \frac{A_2 l_2 / t}{v_1} = \frac{V_2}{v_1 \cdot t} = \frac{300}{3.0 \times 900} = 0.11 m^2$$

Bernoulli's Equation

Bernoulli's Principle: Where the velocity of fluid is high, the pressure is low, and where the velocity is low, the pressure is high.



Amount of work done by the force, F_1, that exerts pressure, P_1, at point 1

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$$

Amount of work done on the other section of the fluid is

$$W_2 = -P_2 A_2 \Delta l_2$$

Work done by the gravitational force to move the fluid mass, m, from y_1 to y_2 is

$$W_3 = -mg\left(y_2 - y_1\right)$$

Bernoulli's Equation cont'd

The net work done on the fluid is

$$W = W_1 + W_2 + W_3 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

From the work-energy principle

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

Since mass, m, is contained in the volume that flowed in the motion

$$A_1 \Delta l_1 = A_2 \Delta l_2 \quad \text{and} \quad m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$$

Thus,

$$\frac{1}{2}\rho A_2 \Delta l_2 v_2^2 - \frac{1}{2}\rho A_1 \Delta l_1 v_1^2$$

$$= P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A_2 \Delta l_2 g y_2 + \rho A_1 \Delta l_1 g y_1$$

Bernoulli's Equation cont'd

Since

$$\frac{1}{2}\rho \cancel{A_2}\Delta l_2 v_2^2 - \frac{1}{2}\rho \cancel{A_1}\Delta l_1 v_1^2 = P_1 \cancel{A_1}\Delta l_1 - P_2 \cancel{A_2}\Delta l_2 - \rho \cancel{A_2}\Delta l_2 g y_2 + \rho \cancel{A_1}\Delta l_1 g y_1$$

We obtain

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Re-organize

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's Equation

Thus, for any two points in the flow

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \ const.$$

Result of Energy conservation!

For static fluid

$$P_2 = P_1 + \rho g \left(y_1 - y_2 \right) = P_1 + \rho g h$$

Pascal's Law

For the same heights

$$P_2 = P_1 + \frac{1}{2}\rho \left(v_1^2 - v_2^2 \right)$$

The pressure at the faster section of the fluid is smaller than slower section.

Example for Bernoulli's Equation

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5m/s through a 4.0cm diameter pipe in the basement under a pressure of 3.0atm, what will be the flow speed and pressure in a 2.6cm diameter pipe on the second 5.0m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left(\frac{0.020}{0.013}\right)^2 = 1.2 m/s$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

$$= 3.0\times10^5 + \frac{1}{2}1\times10^3\left(0.5^2 - 1.2^2\right) + 1\times10^3\times9.8\times(-5)$$

$$= 2.5\times10^5 N/m^2$$