PHYS 1443 – Section 003
Lecture #24

Monday, Nov. 29, 2004
Dr. Jaehoon Yu

1. Simple Harmonic Motion
2. Equation of SHM
3. Simple Block Spring System
4. Energy of SHO
5. SHO and Circular Motion
6. Pendulum
7. Damped and Forced Oscillations

Homework #12 is due midnight, Friday, Dec. 3, 2004!!

Final Exam, Monday, Dec. 6!!

Announcements

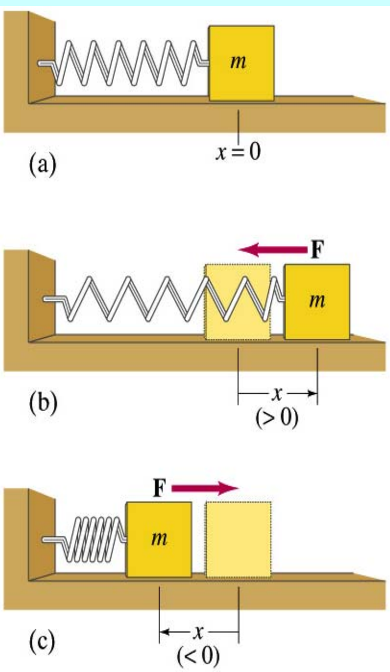
- Final Exam
  - Date: Monday, Dec. 6
  - Time: 11:00am – 12:30pm
  - Location: SH103
  - Covers: CH 10 – CH 14
- Review this Wednesday, Dec. 1!

Vibration or Oscillation

What are the things that vibrate/oscillate?

- Tuning fork
- A pendulum
- A car going over a bump
- Building and bridges
- The spider web with a prey

So what is a vibration or oscillation? A periodic motion that repeats over the same path.



A simplest case is a block attached at the end of a coil spring.

When a spring is stretched from its equilibrium position by a length x, the force acting on the mass is

$$F = -kx$$

The sign is negative, because the force resists against the change of length, directed toward the equilibrium position.

Acceleration is proportional to displacement from the equilibrium

Acceleration is opposite direction to displacement

This system is doing a simple harmonic motion (SHM).

Simple Harmonic Motion

Motion that occurs by the force that depends on displacement, and the force is always directed toward the system's equilibrium position.

What is a system that has such characteristics?

A system consists of a mass and a spring

When a spring is stretched from its equilibrium position by a length x, the force acting on the mass is

$$F = -kx$$

It's negative, because the force resists against the change of length, directed toward the equilibrium position.

From Newton's second law $F = ma = -kx$ we obtain $a = -\frac{k}{m}x$

This is a second order differential equation that can be solved but it is beyond the scope of this class.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Condition for simple harmonic motion

What do you observe from this equation?

Acceleration is proportional to displacement from the equilibrium
Acceleration is opposite direction to displacement

This system is doing a simple harmonic motion (SHM).

Equation of Simple Harmonic Motion

The solution for the 2nd order differential equation

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

$$x = A\cos\left(\boxed{\omega t + \phi}\right)$$

Amplitude

Phase

Angular Frequency

Phase constant

Generalized expression of a simple harmonic motion

Let's think about the meaning of this equation of motion

What happens when t=0 and ϕ=0?

$$x = A\cos(0+0) = A$$

What is ϕ if x is not A at t=0?

$$x = A\cos(\phi) = x'$$

$$\phi = \cos^{-1}(x')$$

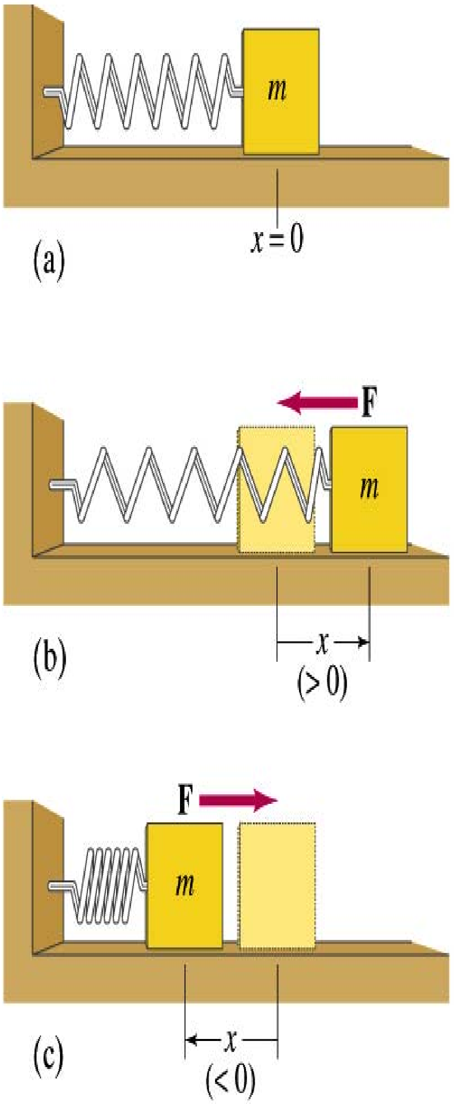
What are the maximum/minimum possible values of x?

A/-A

An oscillation is fully characterized by its:

- Amplitude
- Period or frequency
- Phase constant

Vibration or Oscillation Properties



The maximum displacement from the equilibrium is — Amplitude

One cycle of the oscillation

The complete to-and-fro motion from an initial point

Period of the motion, T

The time it takes to complete one full cycle

Unit? — s

Frequency of the motion, f

The number of complete cycles per second

Unit? — s^{-1}

Relationship between period and frequency?

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

More on Equation of Simple Harmonic Motion

What is the time for full cycle of oscillation?

Since after a full cycle the position must be the same

$$x = A\cos(\omega(t+T)+\phi) = A\cos(\omega t + 2\pi + \phi)$$

The period $T = \frac{2\pi}{\omega}$

One of the properties of an oscillatory motion

How many full cycles of oscillation does this undergo per unit time?

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Frequency

What is the unit?

1/s=Hz

Let's now think about the object's speed and acceleration.

$$x = A\cos(\omega t + \phi)$$

Speed at any given time $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$ Max speed $v_{max} = \omega A$

Acceleration at any given time $a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$

Max acceleration $a_{max} = \omega^2 A$

What do we learn about acceleration?

Acceleration is reverse direction to displacement

Acceleration and speed are $\pi/2$ off phase:

When v is maximum, a is at its minimum

Simple Harmonic Motion continued

Phase constant determines the starting position of a simple harmonic motion.

$$x = A\cos(\omega t + \phi) \qquad \boxed{\text{At } t=0} \qquad x\big|_{t=0} = A\cos\phi$$

This constant is important when there are more than one harmonic oscillation involved in the motion and to determine the overall effect of the composite motion

Let's determine phase constant and amplitude

$$\boxed{\text{At } t=0} \qquad x_i = A\cos\phi \qquad v_i = -\omega A\sin\phi$$

By taking the ratio, one can obtain the phase constant

$$\boxed{\phi = \tan^{-1}\left(-\frac{v_i}{\omega x_i}\right)}$$

By squaring the two equation and adding them together, one can obtain the amplitude

$$x_i^2 = A^2\cos^2\phi$$

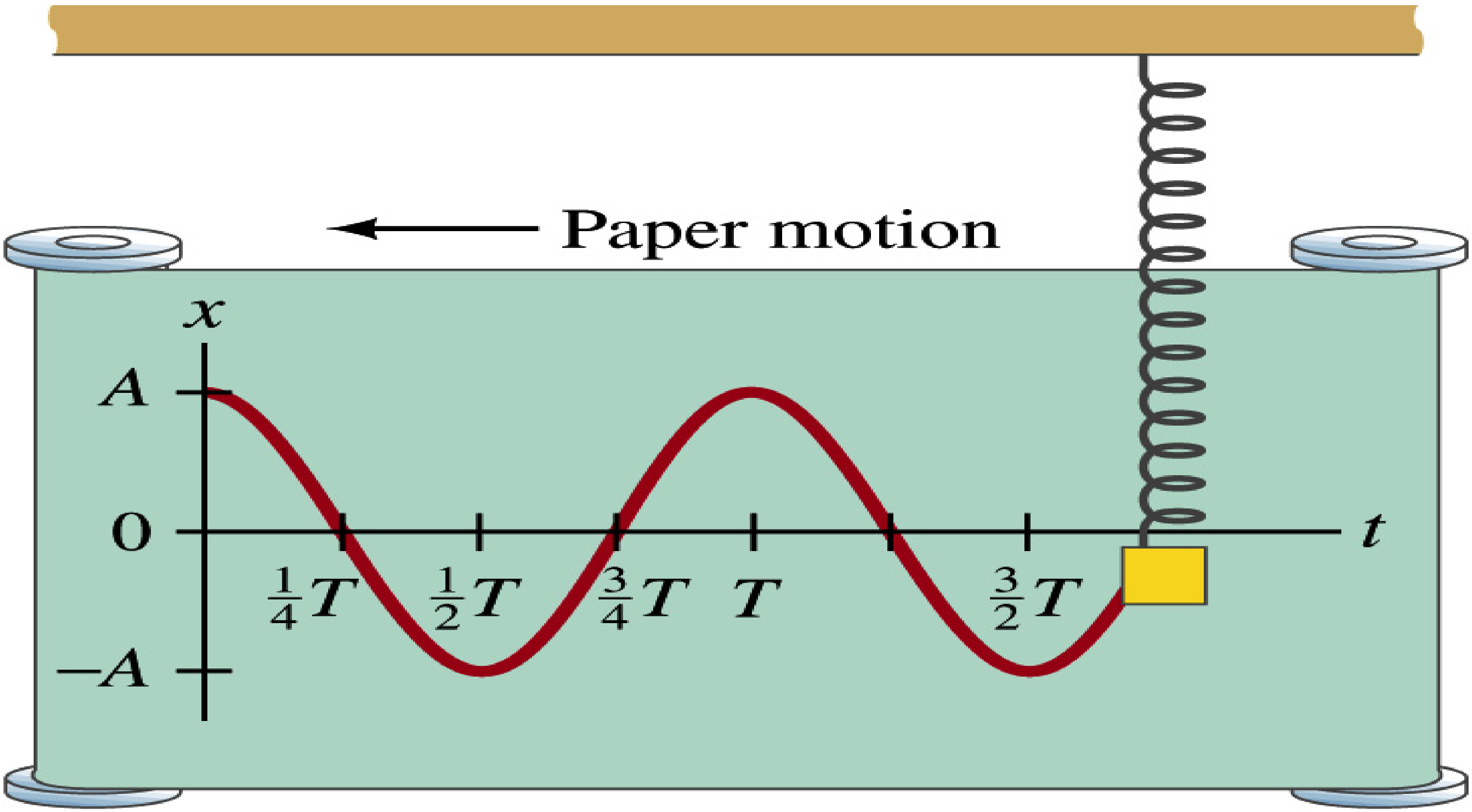
$$v_i^2 = \omega^2 A^2\sin^2\phi$$

$$A^2\left(\cos^2\phi + \sin^2\phi\right) = A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2$$

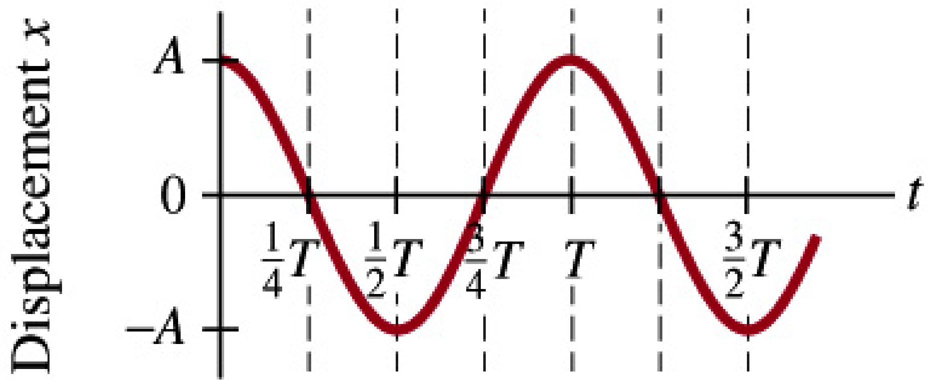
$$\boxed{A = \sqrt{x_i^2 + \left(\frac{v_i}{\omega}\right)^2}}$$

Sinusoidal Behavior of SHM

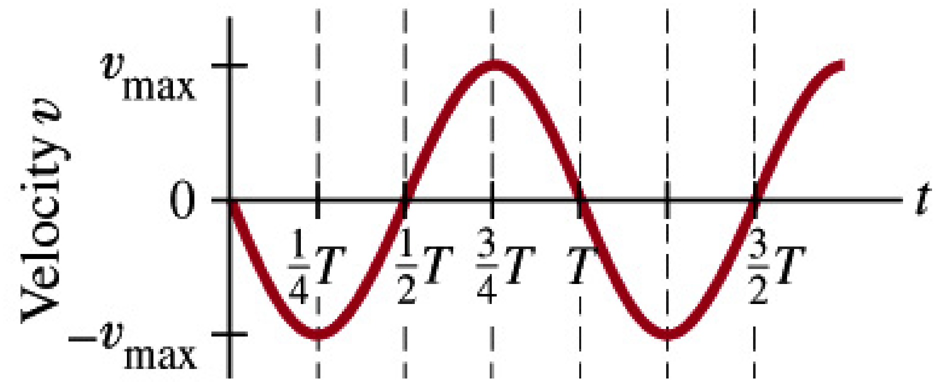
What do you think the trajectory will look if the oscillation was plotted against time?



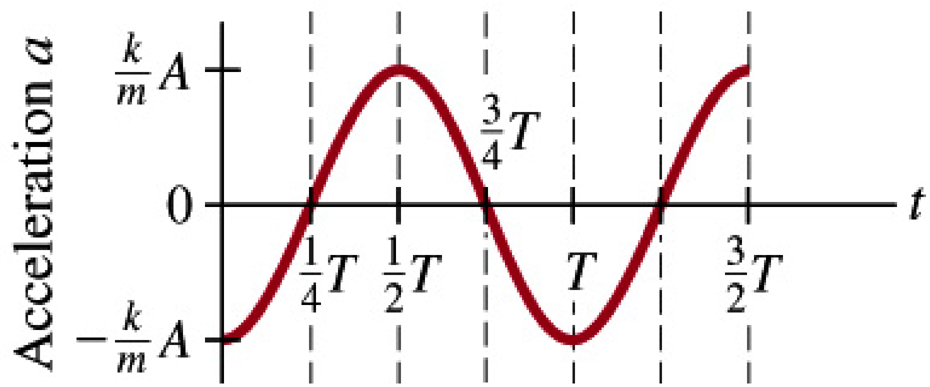
Sinusoidal Behavior of SHM



$$x = A\cos\left(2\pi\, ft\right)$$



$$v = -v_0 \sin\left(2\pi\, ft\right)$$



$$a = -a_0 \cos\left(2\pi\, ft\right)$$

Example for Simple Harmonic Motion

An object oscillates with simple harmonic motion along the x-axis. Its displacement from the origin varies with time according to the equation; $x = (4.00m)\cos\left(\pi t + \frac{\pi}{4}\right)$ where t is in seconds and the angles is in the parentheses are in radians. a) Determine the amplitude, frequency, and period of the motion.

From the equation of motion: $x = A\cos(\omega t + \phi) = (4.00m)\cos\left(\pi t + \frac{\pi}{4}\right)$

The amplitude, A, is $A = 4.00m$ The angular frequency, ω, is $\omega = \pi$

Therefore, frequency and period are $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2s$ $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}s^{-1}$

b)Calculate the velocity and acceleration of the object at any time t.

Taking the first derivative on the equation of motion, the velocity is

$$v = \frac{dx}{dt} = -(4.00 \times \pi)\sin\left(\pi t + \frac{\pi}{4}\right)m/s$$

By the same token, taking the second derivative of equation of motion, the acceleration, a, is

$$a = \frac{d^2x}{dt^2} = -(4.00 \times \pi^2)\cos\left(\pi t + \frac{\pi}{4}\right)m/s^2$$

Simple Block-Spring System

A block attached at the end of a spring on a frictionless surface experiences acceleration when the spring is displaced from an equilibrium position.

$$F_{spring} = ma$$
$$= -kx$$
$$a = -\frac{k}{m}x$$

This becomes a second order differential equation $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ If we denote $\omega^2 = \frac{k}{m}$

The resulting differential equation becomes $\frac{d^2x}{dt^2} = -\omega^2 x$

Since this satisfies condition for simple harmonic motion, we can take the solution $\boxed{x = A\cos(\omega t + \phi)}$

Does this solution satisfy the differential equation?

Let's take derivatives with respect to time $\frac{dx}{dt} = A\frac{d}{dt}(\cos(\omega t + \phi)) = -\omega A \sin(\omega t + \phi)$

Now the second order derivative becomes

$$\frac{d^2x}{dt^2} = -\omega A \frac{d}{dt}(\sin(\omega t + \phi)) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

Whenever the force acting on a particle is linearly proportional to the displacement from some equilibrium position and is in the opposite direction, the particle moves in simple harmonic motion.

More Simple Block-Spring System

How do the period and frequency of this harmonic motion look?

Since the angular frequency ω is $\omega = \sqrt{\frac{k}{m}}$

The period, T, becomes $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

So the frequency is $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

What can we learn from these?

- Frequency and period do not depend on amplitude
- Period is inversely proportional to spring constant and proportional to mass

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Special case #1

Let's consider that the spring is stretched to distance A and the block is let go from rest, giving 0 initial speed; x_i=A, v_i=0,

$$x = A\cos\omega t \quad v = \frac{dx}{dt} = -\omega A\sin\omega t \quad a = \frac{d^2x}{dt^2} = -\omega^2 A\cos\omega t \quad a_i = -\omega^2 A = -kA/m$$

This equation of motion satisfies all the conditions. So it is the solution for this motion.

Special case #2

Suppose block is given non-zero initial velocity v_i to positive x at the instant it is at the equilibrium, x_i=0

$$\phi = \tan^{-1}\left(-\frac{v_i}{\omega x_i}\right) = \tan^{-1}(-\infty) = -\frac{\pi}{2} \quad x = A\cos\left(\omega t - \frac{\pi}{2}\right) = A\sin(\omega t)$$

Is this a good solution?

Example for Spring Block System

A car with a mass of 1300kg is constructed so that its frame is supported by four springs. Each spring has a force constant of 20,000N/m. If two people riding in the car have a combined mass of 160kg, find the frequency of vibration of the car after it is driven over a pothole in the road.

Let's assume that mass is evenly distributed to all four springs.

The total mass of the system is 1460kg.

Therefore each spring supports 365kg each.

From the frequency relationship based on Hook's law

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus the frequency for vibration of each spring is

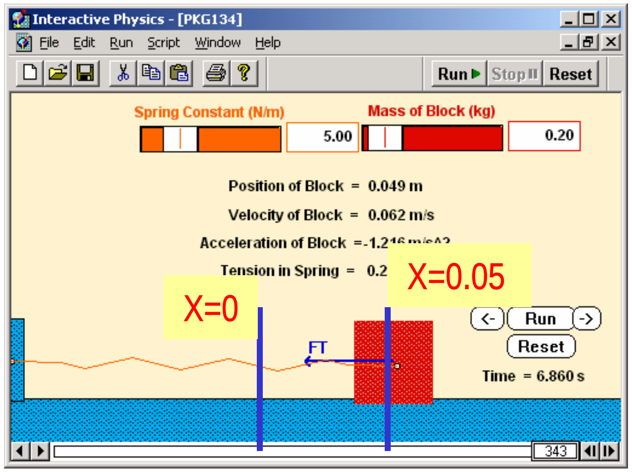
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20000}{365}} = 1.18\ s^{-1} = 1.18\ Hz$$

How long does it take for the car to complete two full vibrations?

The period is $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} = 0.849\ s$ For two cycles $2T = 1.70\ s$

Example for Spring Block System

A block with a mass of 200g is connected to a light spring for which the force constant is 5.00 N/m and is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00 cm from equilibrium and released from reset. Find the period of its motion.



From the Hook's law, we obtain

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00}{0.20}} = 5.00\ s^{-1}$$

As we know, period does not depend on the amplitude or phase constant of the oscillation, therefore the period, T, is simply

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = 1.26\ s$$

Determine the maximum speed of the block.

From the general expression of the simple harmonic motion, the speed is

$$v_{\max} = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$= \omega A = 5.00 \times 0.05 = 0.25\, m/s$$

Energy of the Simple Harmonic Oscillator

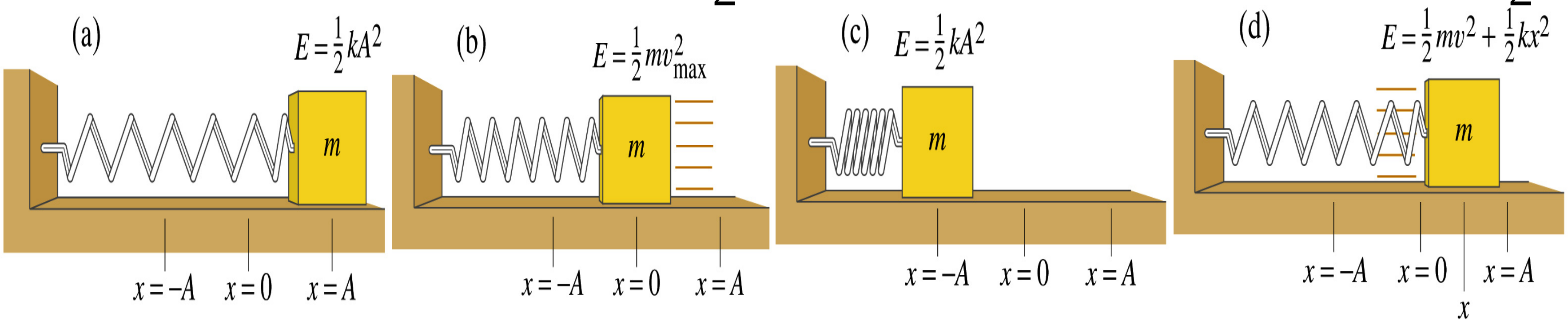
How do you think the mechanical energy of the harmonic oscillator look without friction?

Kinetic energy of a harmonic oscillator is
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

The elastic potential energy stored in the spring
$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

Therefore the total mechanical energy of the harmonic oscillator is
$$E = KE + PE = \frac{1}{2}\left[m\omega^2 A^2 \sin^2(\omega t + \phi) + kA^2 \cos^2(\omega t + \phi)\right]$$

Since $\omega = \sqrt{k/m}$
$$E = KE + PE = \frac{1}{2}\left[kA^2 \sin^2(\omega t + \phi) + kA^2 \cos^2(\omega t + \phi)\right] = \frac{1}{2}kA^2$$



Total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude.

Energy of the Simple Harmonic Oscillator cont'd

Maximum KE is when PE=0

$$KE_{\max} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} k A^2$$

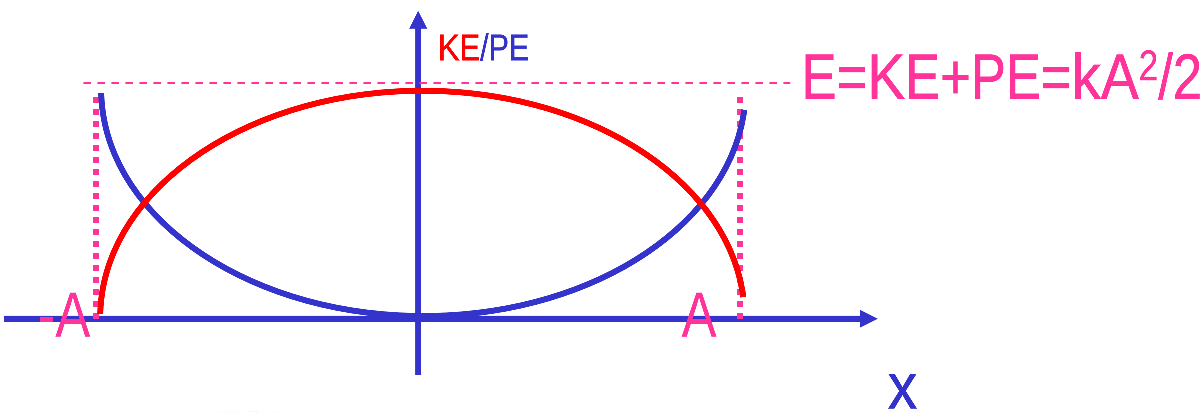
Maximum speed

$$v_{\max} = \sqrt{\frac{k}{m}} A$$

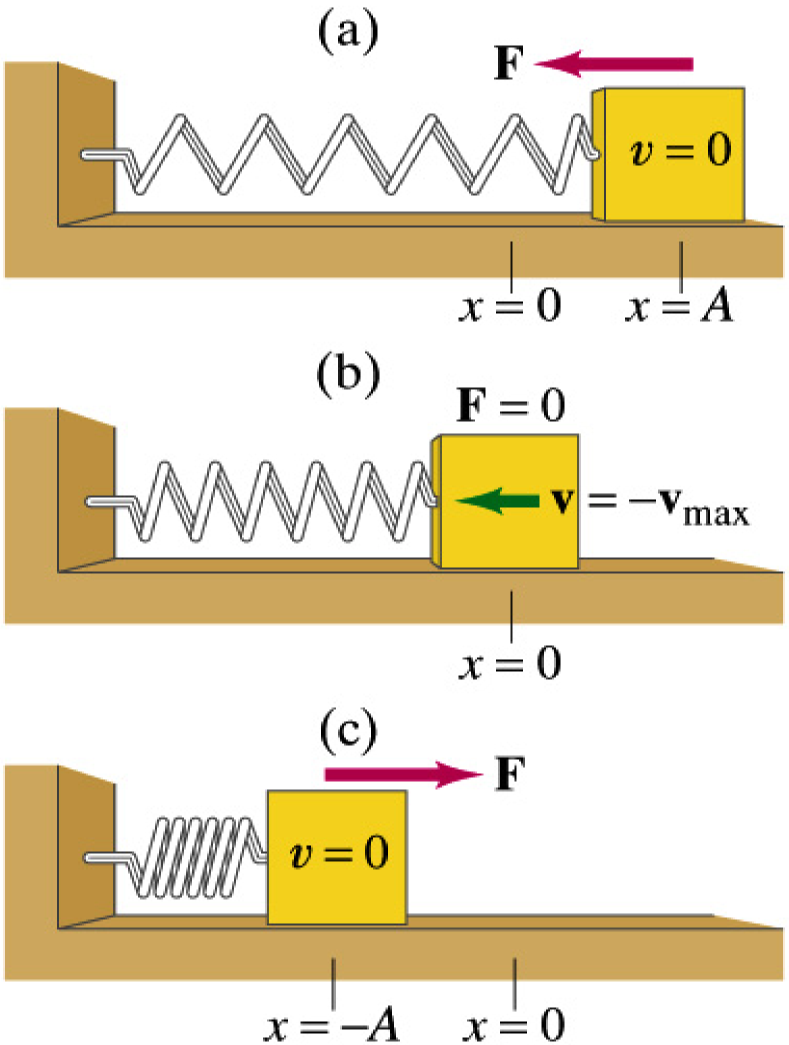
The speed at any given point of the oscillation

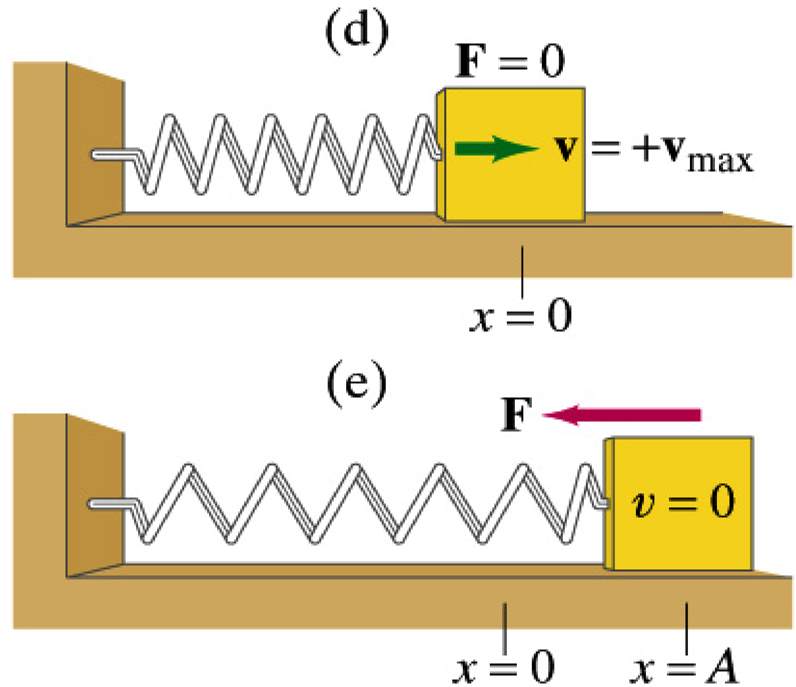
$$E = KE + PE = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = v_{\max}\sqrt{1 - \left(\frac{x}{A}\right)^2}$$



Oscillation Properties





Amplitude? A

- When is the force greatest?
- When is the speed greatest?
- When is the acceleration greatest?
- When is the potential energy greatest?
- When is the kinetic energy greatest?

Example for Energy of Simple Harmonic Oscillator

A 0.500kg cube connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless track. a) Calculate the total energy of the system and the maximum speed of the cube if the amplitude of the motion is 3.00 cm.

From the problem statement, A and k are

$$k = 20.0N/m$$
$$A = 3.00cm = 0.03m$$

The total energy of the cube is

$$E = KE + PE = \frac{1}{2}kA^2 = \frac{1}{2}(20.0)\times(0.03)^2 = 9.00\times10^{-3}J$$

Maximum speed occurs when kinetic energy is the same as the total energy

$$KE_{\max} = \frac{1}{2}mv_{\max}^2 = E = \frac{1}{2}kA^2$$
$$v_{\max} = A\sqrt{\frac{k}{m}} = 0.03\sqrt{\frac{20.0}{0.500}} = 0.190m/s$$

b) What is the velocity of the cube when the displacement is 2.00 cm.

velocity at any given displacement is

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = \sqrt{20.0\cdot\left(0.03^2 - 0.02^2\right)/0.500} = 0.141m/s$$

c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm.

Kinetic energy, KE

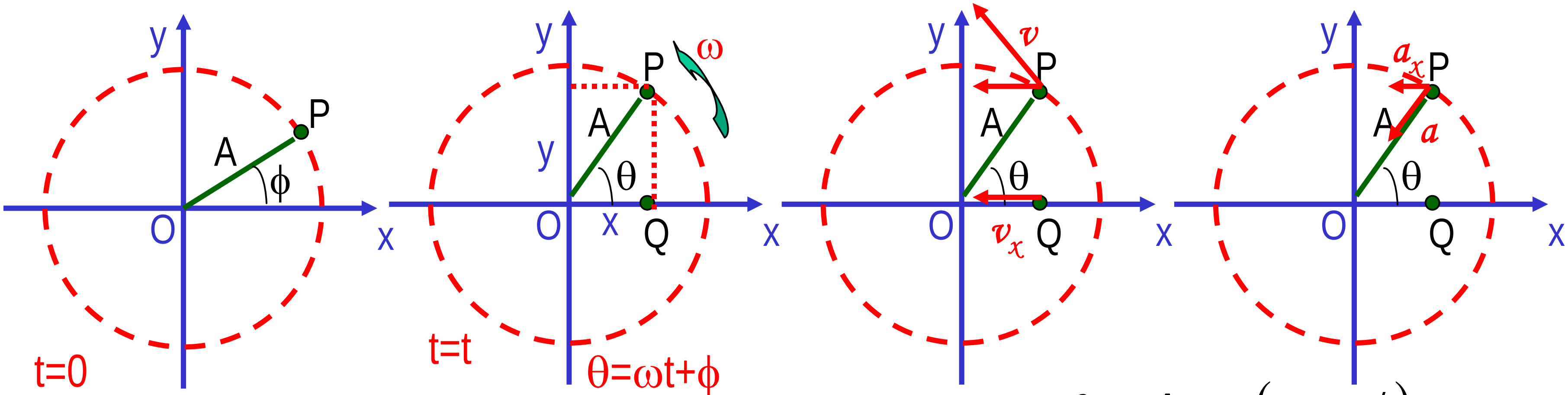
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}0.500\times(0.141)^2 = 4.97\times10^{-3}J$$

Potential energy, PE

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}20.0\times(0.02)^2 = 4.00\times10^{-3}J$$

Simple Harmonic and Uniform Circular Motions

Uniform circular motion can be understood as a superposition of two simple harmonic motions in x and y axis.



When the particle rotates at a uniform angular speed ω, x and y coordinate position become

$$x = A\cos\theta = A\cos(\omega t+\phi)$$
$$y = A\sin\theta = A\sin(\omega t+\phi)$$

Since the linear velocity in a uniform circular motion is Aω, the velocity components are

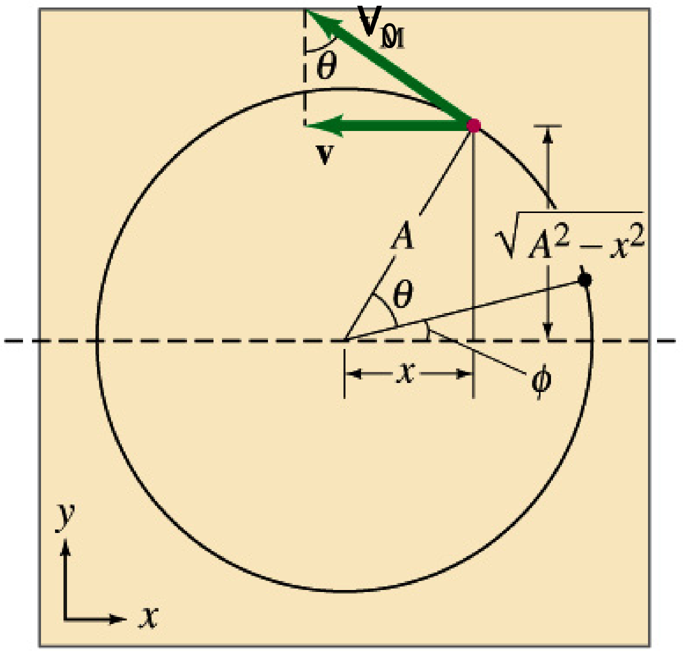
$$v_x = -v\sin\theta = -A\omega\sin(\omega t+\phi)$$
$$v_y = +v\cos\theta = A\omega\cos(\omega t+\phi)$$

Since the radial acceleration in a uniform circular motion is $v^2/A=\omega^2 A$, the components are

$$a_x = -a\cos\theta = -A\omega^2\cos(\omega t+\phi)$$
$$a_y = -a\sin\theta = -A\omega^2\sin(\omega t+\phi)$$

The Period and Sinusoidal Nature of SHM

Consider an object moving on a circle with a constant angular speed ω



$$\sin\theta = \frac{v}{v_0} = \frac{\sqrt{A^2-x^2}}{A} = \sqrt{1-\left(\frac{x}{A}\right)^2} \Rightarrow v = v_0\sqrt{1-\left(\frac{x}{A}\right)^2}$$

Since it takes T to complete one full circular motion

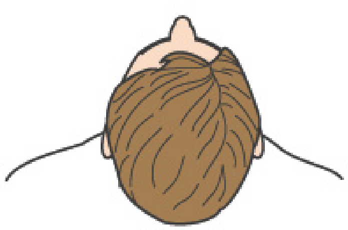
$$v_0 = \frac{2\pi A}{T} = 2\pi A f \Rightarrow T = \frac{2\pi A}{v_0}$$

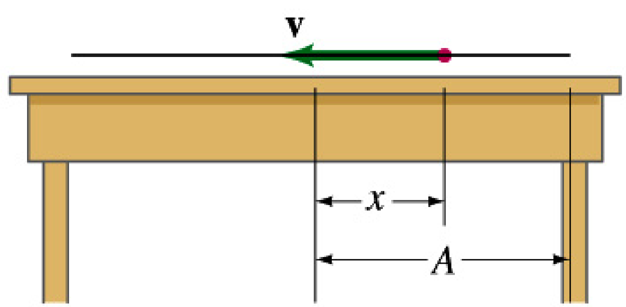
From an energy relationship in a spring SHM

$$\frac{1}{2}mv_0^2 = \frac{1}{2}kA^2 \Rightarrow v_0 = \sqrt{\frac{k}{m}}A$$

Thus, T is

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$





Natural Frequency

If you look at it from the side, it looks as though it is doing a SHM

Example for Uniform Circular Motion

A particle rotates counterclockwise in a circle of radius 3.00m with a constant angular speed of 8.00 rad/s. At t=0, the particle has an x coordinate of 2.00m and is moving to the right. A) Determine the x coordinate as a function of time.

Since the radius is 3.00m, the amplitude of oscillation in x direction is 3.00m. And the angular frequency is 8.00rad/s. Therefore the equation of motion in x direction is

$$x = A\cos\theta = (3.00m)\cos(8.00t+\phi)$$

Since x=2.00, when t=0 $\quad 2.00 = (3.00m)\cos\phi; \quad \phi = \cos^{-1}\left(\frac{2.00}{3.00}\right) = 48.2^\circ$

However, since the particle was moving to the right ϕ=-48.2°,

$$x = (3.00m)\cos\left(8.00t - 48.2^\circ\right)$$

Find the x components of the particle's velocity and acceleration at any time t.

Using the displcement

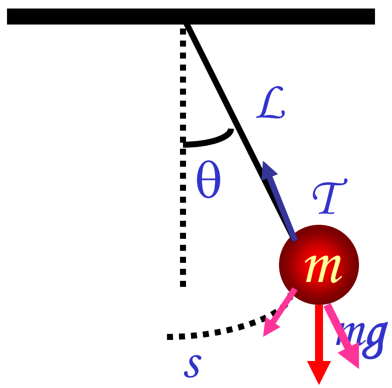
$$v_x = \frac{dx}{dt} = -(3.00\cdot 8.00)\sin(8.00t - 48.2) = (-24.0m/s)\sin\left(8.00t - 48.2^\circ\right)$$

Likewise, from velocity

$$a_x = \frac{dv}{dt} = (-24.0\cdot 8.00)\cos(8.00t - 48.2) = \left(-192m/s^2\right)\cos\left(8.00t - 48.2^\circ\right)$$

The Pendulum

A simple pendulum also performs periodic motion.



The net force exerted on the bob is

$$\sum F_r = T - mg\cos\theta_A = 0$$

$$\sum F_t = -mg\sin\theta_A = ma = m\frac{d^2 s}{dt^2}$$

Since the arc length, s, is $s = L\theta$

$$\frac{d^2 s}{dt^2} = L\frac{d^2\theta}{dt^2} = -g\sin\theta$$ results $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Again became a second degree differential equation, satisfying conditions for simple harmonic motion

If θ is very small, sinθ~θ $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta = -\omega^2\theta$ giving angular frequency $\omega = \sqrt{\frac{g}{L}}$

The period for this motion is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

The period only depends on the length of the string and the gravitational acceleration

Example for Simple Pendulum

Grandfather clock. (a) Estimate the length of the pendulum in a grandfather clock that ticks once per second.

Since the period of a simple pendulum motion is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The length of the pendulum in terms of T is

$$L = \frac{T^2 g}{4\pi^2}$$

Thus the length of the pendulum when T=1s is

$$L = \frac{T^2 g}{4\pi^2} = \frac{1\times 9.8}{4\pi^2} = 0.25m$$

(b) What would be the period of the clock with a 1m long pendulum?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.0}{9.8}} = 2.0s$$

Example for Pendulum

Christian Huygens (1629-1695), the greatest clock maker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1s. How much shorter would out length unit be had this suggestion been followed?

Since the period of a simple pendulum motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

The length of the pendulum in terms of T is

$$L = \frac{T^2 g}{4\pi^2}$$

Thus the length of the pendulum when T=1s is

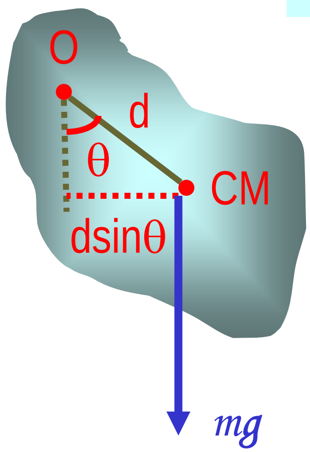
$$L = \frac{T^2 g}{4\pi^2} = \frac{1 \times 9.8}{4\pi^2} = 0.248\,m$$

Therefore the difference in length with respect to the current definition of 1m is

$$\Delta L = 1 - L = 1 - 0.248 = 0.752\,m$$

Physical Pendulum

Physical pendulum is an object that oscillates about a fixed axis which does not go through the object's center of mass.



Consider a rigid body pivoted at a point O that is a distance d from the CM.

The magnitude of the net torque provided by the gravity is

$$\sum \tau = -mgd \sin\theta$$

Then $$\sum \tau = I\alpha = I\frac{d^2\theta}{dt^2} = -mgd \sin\theta$$

Therefore, one can rewrite $$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\sin\theta \approx -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

Thus, the angular frequency ω is $$\omega = \sqrt{\frac{mgd}{I}}$$

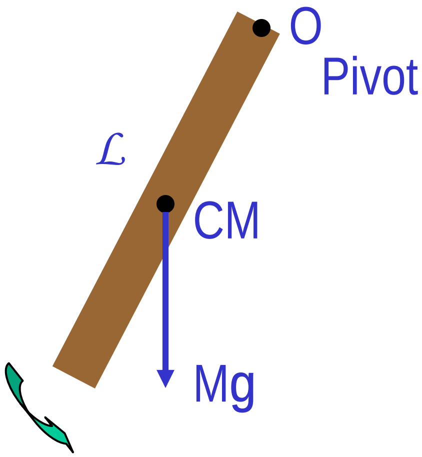
And the period for this motion is $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

By measuring the period of physical pendulum, one can measure moment of inertia.

Does this work for simple pendulum?

Example for Physical Pendulum

A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane. Find the period of oscillation if the amplitude of the motion is small.



Moment of inertia of a uniform rod, rotating about the axis at one end is $I = \frac{1}{3}ML^2$

The distance d from the pivot to the CM is L/2, therefore the period of this physical pendulum is

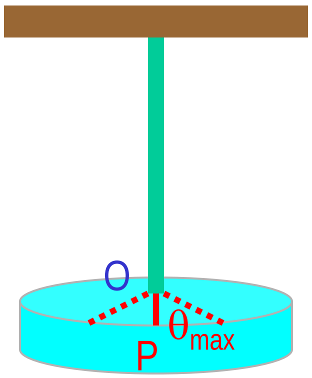
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{2ML^2}{3MgL}} = 2\pi\sqrt{\frac{2L}{3g}}$$

Calculate the period of a meter stick that is pivot about one end and is oscillating in a vertical plane.

Since L=1m, the period is $T = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2}{3 \cdot 9.8}} = 1.64s$

So the frequency is $f = \frac{1}{T} = 0.61s^{-1}$

Torsion Pendulum



When a rigid body is suspended by a wire to a fixed support at the top and the body is twisted through some small angle θ, the twisted wire can exert a restoring torque on the body that is proportional to the angular displacement.

The torque acting on the body due to the wire is

$$\tau = -\kappa\theta$$

κ is the torsion constant of the wire

Applying the Newton's second law of rotational motion

$$\sum \tau = I\alpha = I\frac{d^2\theta}{dt^2} = -\kappa\theta$$

Then, again the equation becomes

$$\frac{d^2\theta}{dt^2} = -\left(\frac{\kappa}{I}\right)\theta = -\omega^2\theta$$

Thus, the angular frequency ω is

$$\omega = \sqrt{\frac{\kappa}{I}}$$

And the period for this motion is

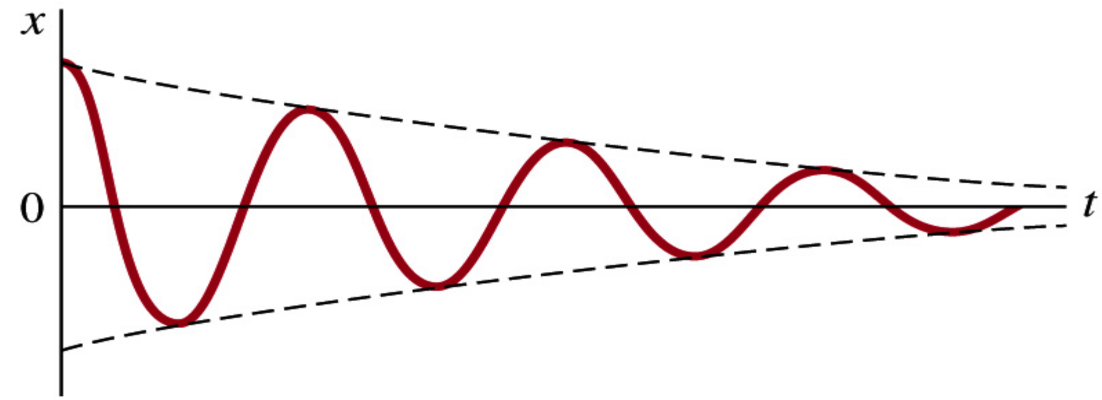
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$

This result works as long as the elastic limit of the wire is not exceeded

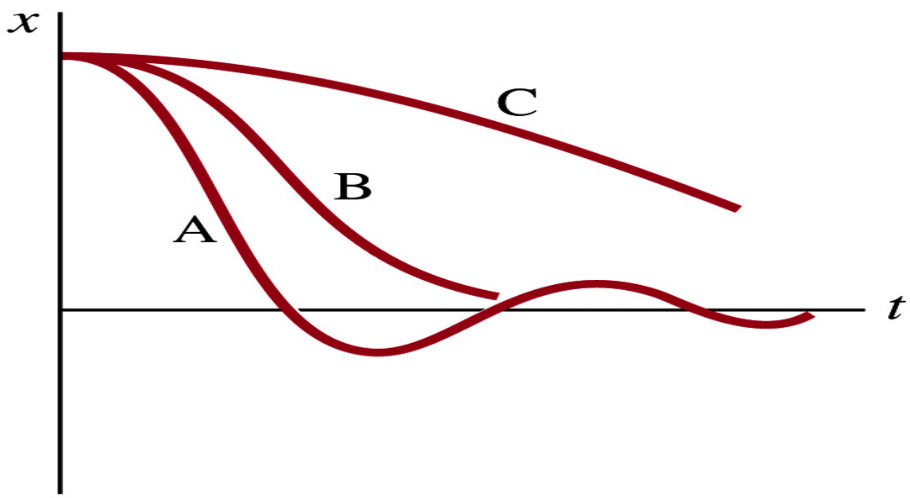
Damped Oscillation

More realistic oscillation where an oscillating object loses its mechanical energy in time by a retarding force such as friction or air resistance.

How do you think the motion would look?



Amplitude gets smaller as time goes on since its energy is spent.



Types of damping

A: Underdamped

B: Critically damped

C: Overdamped

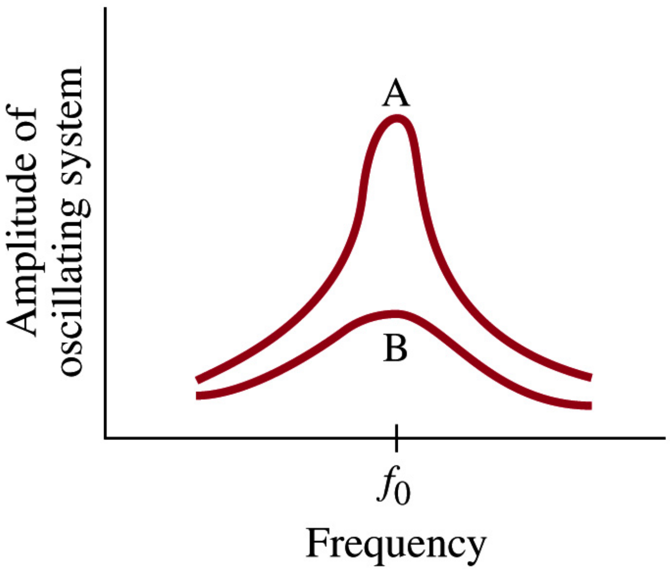
Forced Oscillation; Resonance

When a vibrating system is set into motion, it oscillates with its natural frequency f_0.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

However a system may have an external force applied to it that has its own particular frequency (f), causing forced vibration.

For a forced vibration, the amplitude of vibration is found to be dependent on the difference between f and f_0. and is maximum when $f=f_0$.



A: light damping

B: Heavy damping

The amplitude can be large when $f=f_0$, as long as damping is small.

This is called resonance. The natural frequency f_0 is also called resonant frequency.