# PHYS 1444 – Section 003
# Lecture #3

*Monday, Sept. 7, 2005*
*Dr. Jaehoon Yu*

- Motion of a Charged Particle in an Electric Field
- Electric Dipoles
- Electric Flux
- Gauss' Law

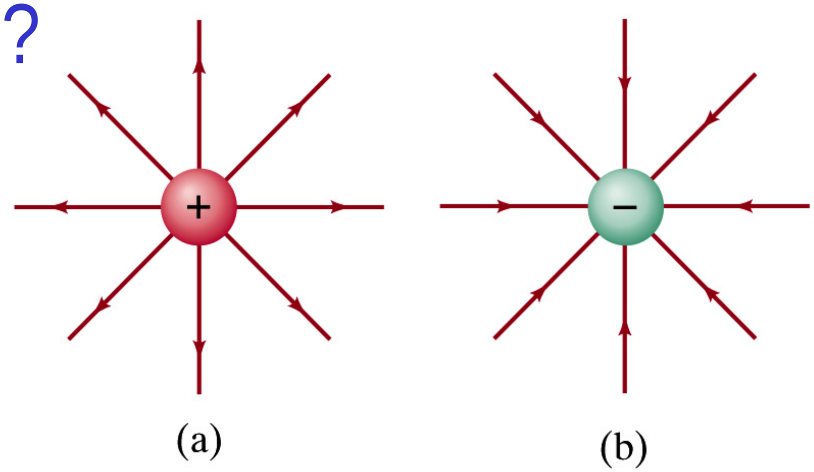
# Announcements

- Your three extra credit points for e-mail subscription is till midnight tonight. Please take a full advantage of the opportunity.
- 23/28 of you have submitted homework #2
  - Good job!!
  - Some of you lost EID
  - Are there anyone who need EID information?
- Reading assignments
  - Sec. 21 – 7
  - Sec. 22 – 3

# Motion of a Charged Particle in an Electric Field

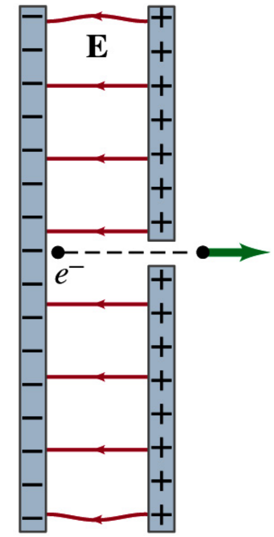
- If an object with an electric charge q is at a point in space where electric field is E, the force exerting on the object is $\vec{F} = q\vec{E}$

- What do you think will happen?
  - Let's think about the cases like these on the right.
  - The object will move along the field line…Which way?
  - The charge gets accelerated.



# Example 21 – 14

- **Electron accelerated by electric field.** An electron (mass m = 9.1x10$^{-31}$kg) is accelerated in the uniform field E (E=2.0x10$^{4}$N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.




The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $a = \frac{F}{m} = \frac{qE}{m}$

Between the plates the field E is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6\times10^{-19}C\right)\left(2.0\times10^{4}N/C\right)}{\left(9.1\times10^{-31}kg\right)} = 3.5\times10^{15}\,m/s^{2}$$

# Example 21 – 14

Since the travel distance is $1.5 \times 10^{-2}$m, using one of the kinetic eq. of motion,

$$v^2 = v_0^2 + 2ax \quad \therefore v = \sqrt{2ax} = \sqrt{2 \cdot 3.5 \times 10^{15} \cdot 1.5 \times 10^{-2}} = 1.0 \times 10^7 \, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right) = 3.2 \times 10^{-15} N$$

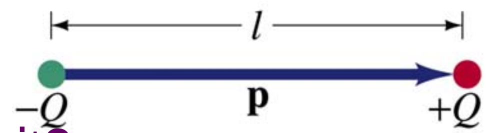
The magnitude of the gravitational force on the electron is

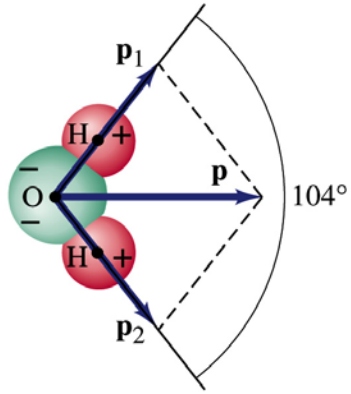
$$F_G = mg = 9.8 \, m/s^2 \times \left(9.1 \times 10^{-31} kg\right) = 8.9 \times 10^{-30} N$$

Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

# Electric Dipoles

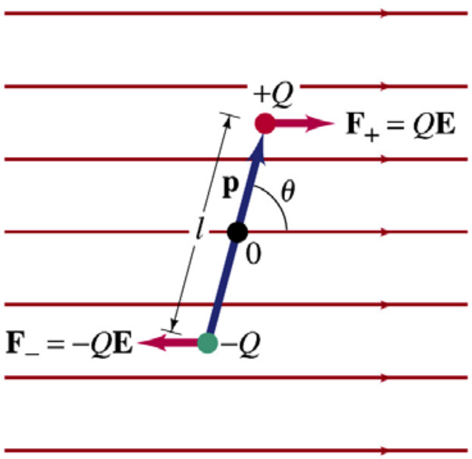
- An electric dipole is the combination of two equal charges of opposite sign, +Q and –Q, separated by a distance $\ell$, which behaves as one entity.
- The quantity $Q\ell$ is called the dipole moment and is represented by the symbol $p$.
  - The dipole moment is a vector quantity, $p$
  - The magnitude of the dipole moment is $Q\ell$. Unit?
  - Its direction is from the negative to the positive charge.
  - Many of diatomic molecules like CO have a dipole moment. ➔ These are referred as polar molecules.
    - Symmetric diatomic molecules, such as $O_2$, do not have dipole moment.
  - The water molecule also has a dipole moment which is the vector sum of two dipole moments between Oxygen and each of Hydrogen atoms.



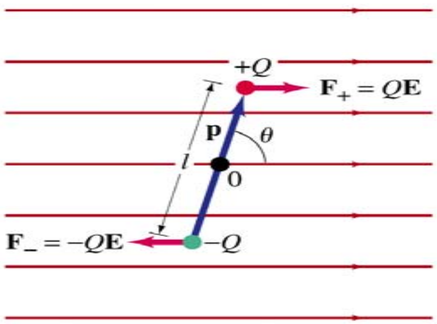
# Dipoles in an External Field

- Let's consider a dipole placed in a uniform electric field E.




- What do you think will happen?
  - Forces will be exerted on the charges.
    - The positive charge will get pushed toward right while the negative charge will get pulled toward left.
  - What is the net force acting on the dipole?
    - Zero
  - So will the dipole not move?
    - Yes, it will.
  - Why?
    - There is torque applied on the dipole.

# Dipoles in an External Field, cnt'd




- How much is the torque on the dipole?
  - Do you remember the formula for torque?
    - $\vec{\tau} = \vec{r} \times \vec{F}$
  - The magnitude of the torque exerting on each of the charges is
    - $\tau_{+Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(\frac{l}{2}\right)(QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
    - $\tau_{-Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(-\frac{l}{2}\right)(-QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
  - Thus, the total torque is
    - $\tau_{Total} = \tau_{+Q} + \tau_{-Q} = \frac{l}{2}QE\sin\theta + \frac{l}{2}QE\sin\theta = lQE\sin\theta = pE\sin\theta$
  - So the torque on a dipole in vector notation is $\vec{\tau} = \vec{p} \times \vec{E}$
- The effect of the torque is to try to turn the dipole so that the dipole moment is parallel to E. Which direction?

# Potential Energy of a Dipole in an External Field

- What is the work done on the dipole by the electric field to change the angle from $\theta_1$ to $\theta_2$?

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_1}^{\theta_2} -\tau d\theta$$

Why negative?

Because $\tau$ and $\theta$ are opposite directions to each other.

- The torque is $\tau = pE\sin\theta$.
- Thus the work done on the dipole by the field is

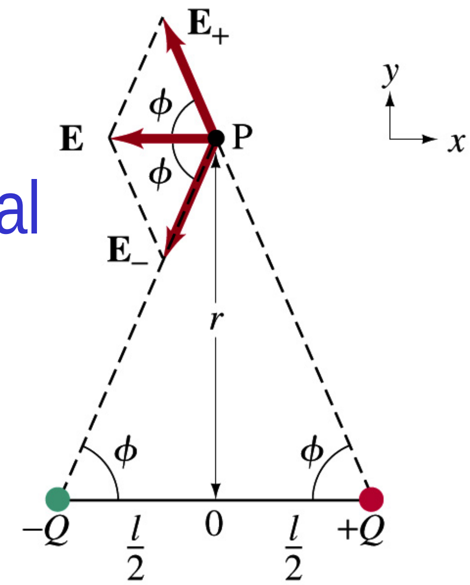
$$W = \int_{\theta_1}^{\theta_2} -pE\sin\theta d\theta = pE\left[\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left(\cos\theta_2 - \cos\theta_1\right)$$

- What happens to the dipole's potential energy, U, when a positive work is done on it by the field?
  - It decreases.
- If we choose U=0 when $\theta_1$=90 degrees, then the potential energy at $\theta_2$=$\theta$ becomes $U = -W = -pE\cos\theta = -\vec{p} \cdot \vec{E}$

# Electric Field by a Dipole

- Let's consider the case in the picture.
- There are fields by both the charges. So the total electric field by the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{r^2+(l/2)^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2+(l/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2+l^2/4}$$




- Now we must work out the x and y components of the total field.
  - Sum of the two y components is
    - Zero since they are the same but in opposite direction
  - So the magnitude of the total field is the same as the sum of the two x-components is

$$E = 2E_+\cos\phi = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r^2+l^2/4}\frac{l}{2\sqrt{r^2+l^2/4}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2+l^2/4\right)^{3/2}}$$

# Example 21 – 16

- **Dipole in a field.** The dipole moment of a water molecule is $6.1\times10^{-30}$C-m. A water molecule is placed in a uniform electric field with magnitude $2.0\times10^{5}$N/C. (a) What is the magnitude of the maximum torque that the field can exert on the molecule? (b) What is the potential energy when the torque is at its maximum? (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when $\theta$=90 degrees. Thus the magnitude of the maximum torque is

$$\tau = pE\sin\theta = pE =$$

$$= \left(6.1\times10^{-30}\,C\cdot m\right)\left(2.5\times10^{5}\,N/C\right) = 1.2\times10^{-24}\,N\cdot m$$

# Example 21 – 16

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

And $\tau$ is at its maximum at $\theta$=90 degrees, the potential energy, U, is

$$U = -pE\cos\theta = -pE\cos\left(90^\circ\right) = 0$$

Is the potential energy at its minimum at $\theta$=90 degrees? No

Why not? Because U will become negative as $\theta$ increases.

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when cos$\theta$= -1, $\theta$=180 degrees.

Why is this different than the position where the torque is maximized?

The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field. to reach the equilibrium position at q=0.

Torque is maximized when the field is perpendicular to the dipole, $\theta$=90.

# Similarity Between Linear and Rotational Motions

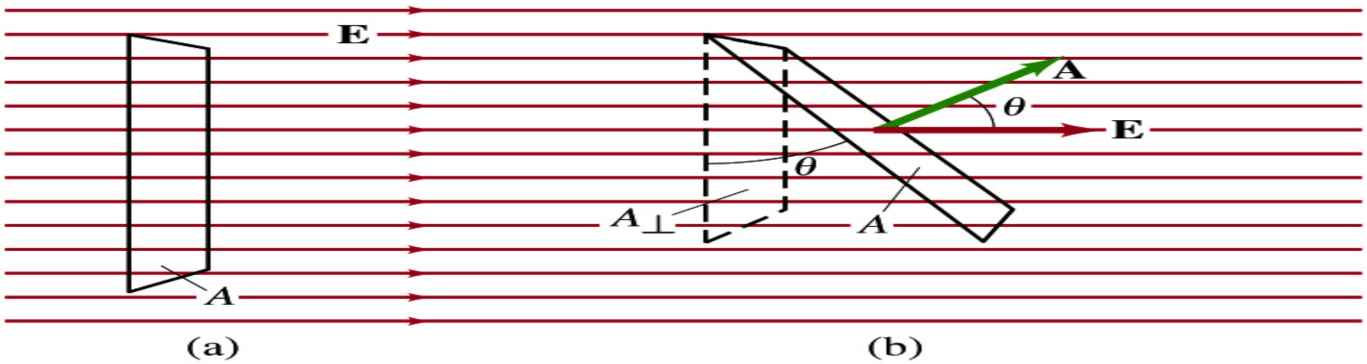
All physical quantities in linear and rotational motions show striking similarity.

| Quantities | Linear | Rotational |
|---|---|---|
| Mass | Mass $M$ | Moment of Inertia $I = mr^2$ |
| Length of motion | Distance $L$ | Angle $\theta$ (Radian) |
| Speed | $v = \frac{\Delta r}{\Delta t}$ | $\omega = \frac{\Delta \theta}{\Delta t}$ |
| Acceleration | $a = \frac{\Delta v}{\Delta t}$ | $\alpha = \frac{\Delta \omega}{\Delta t}$ |
| Force | Force $F = ma$ | Torque $\tau = I\alpha$ |
| Work | Work $W = Fd\cos\theta$ | Work $W = \tau\theta$ |
| Power | $P = \vec{F} \cdot \vec{v}$ | $P = \tau\omega$ |
| Momentum | $\vec{p} = m\vec{v}$ | $\vec{L} = I\vec{\omega}$ |
| Kinetic Energy | Kinetic $K = \frac{1}{2}mv^2$ | Rotational $K_R = \frac{1}{2}I\omega^2$ |

# Gauss' Law

- Gauss' law states the relationship between electric charge and electric field.
  - More general and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between charge and field

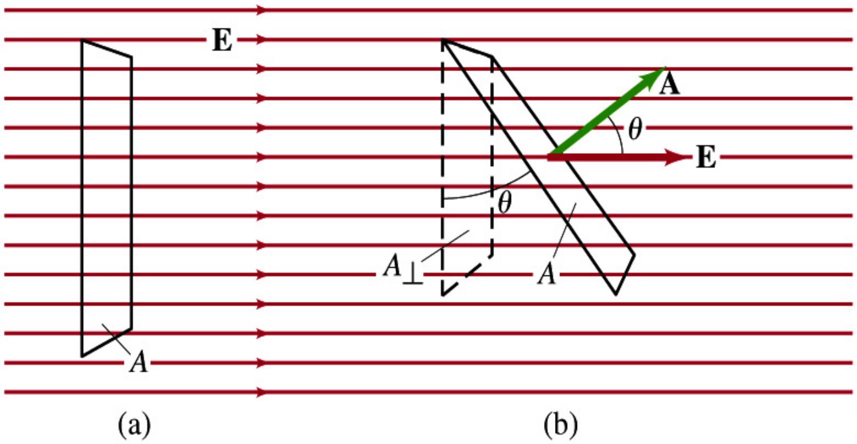
# Electric Flux




- Let's imagine a surface of area A through which a uniform electric field E passes.
- The electric flux is defined as
  - $\Phi_E$=EA, if the field is perpendicular to the surface
  - $\Phi_E$=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - Total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

# Example 22 – 1

- **Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform at 200N/C. (b) What is the flux in figure (b) if the angle is 30 degrees?




The electric flux is

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = \left(200N/C\right)\cdot\left(0.1\times 0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (a) θ=30 degrees, we obtain

$$\Phi_E = EA\cos 30^\circ = \left(200N/C\right)\cdot\left(0.1\times 0.2m^2\right)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$