PHYS 1444 – Section 003
Lecture #4

Monday, Sept. 12, 2005

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- Quiz Problems…
- Electric Flux
- Gauss' Law
- How are Gauss' Law and Coulom's Law Related?

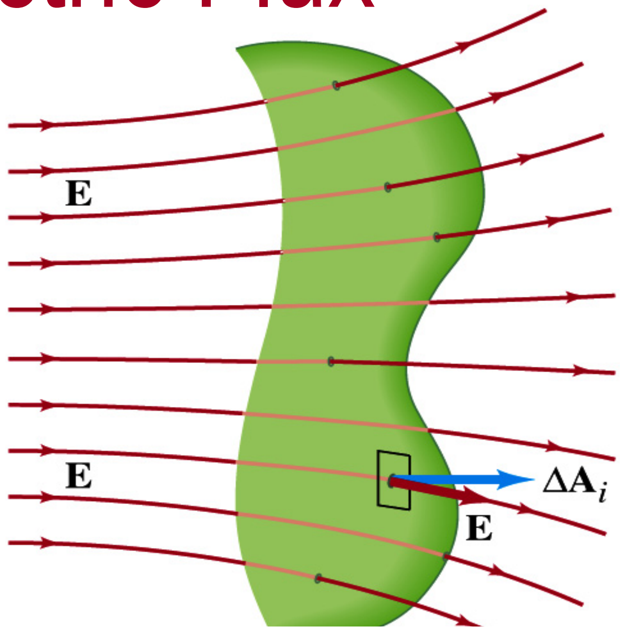
Today's homework is homework #3, due noon, next Monday!!

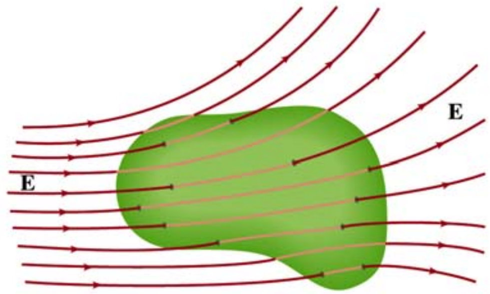
Announcements

- I sent a test message to the distribution list. I'd appreciate if you could confirm the reception of the message.
  - Ten of you have already responded.
    - Totally impressive!!!
  - Please make sure that the reply is sent only to ME not to all.
- I still have a few of you who are not on the distribution list. Please subscribe ASAP.
  - You can come and check with me on the list to make sure there is no system screw-ups….
- Quiz results
  - Do you want to know what your average is?
    - 44.2/60 → equivalent to 73.7/100
    - Not bad!!
  - Do you want to know the top score?
    - 57/100 → 95/100

Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of ΔA_i that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface can is approximately





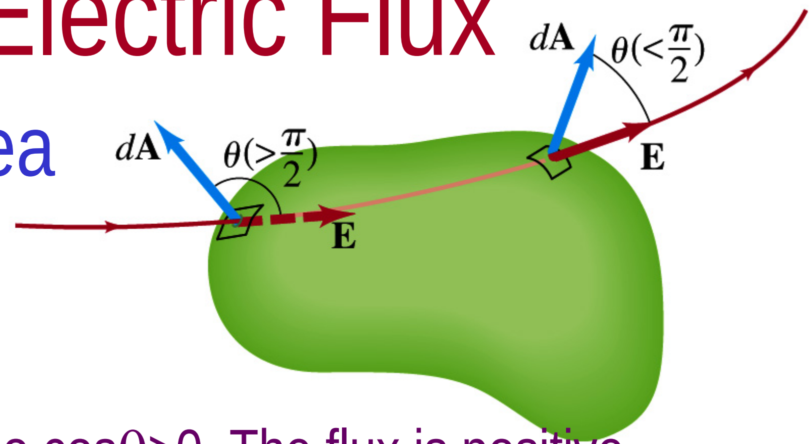
$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

- In the limit where $\Delta A_i \to 0$, the discrete summation becomes an integral.

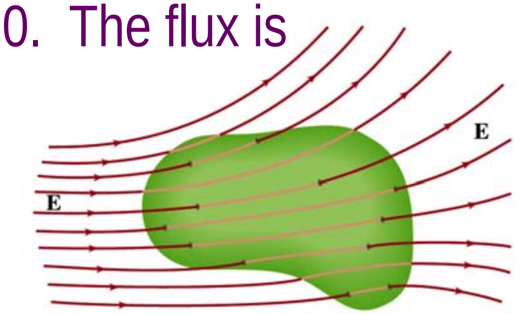
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

Generalization of the Electric Flux



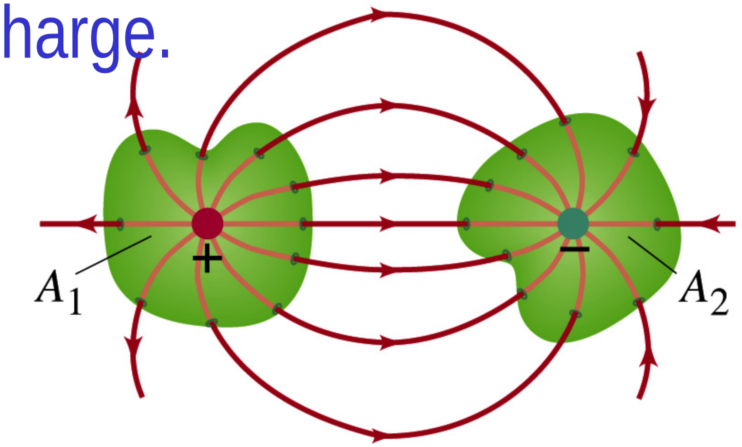
- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $\theta<\pi/2$, so $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $\theta>\pi/2$, so $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is a net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.

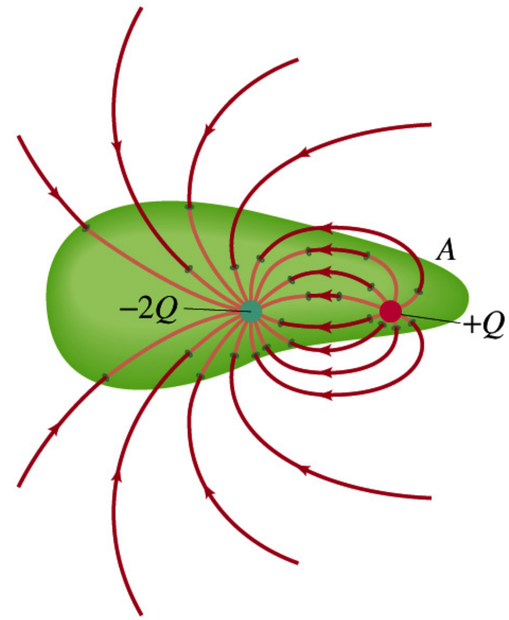


- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E}\cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

Generalization of the Electric Flux

- The field line starts or ends only on a charge.
- What is the net flux on the surface A_1?
  - The net outward flux (positive flux)
- How about A_2?
  - Net inward flux (negative flux)
- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.





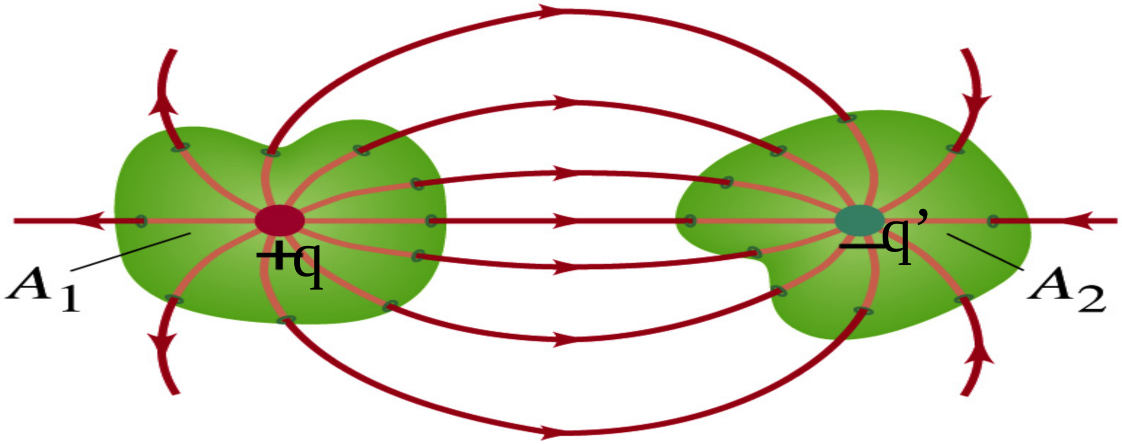
Gauss' Law

- The precise relation between flux and the enclosed charge is given by Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

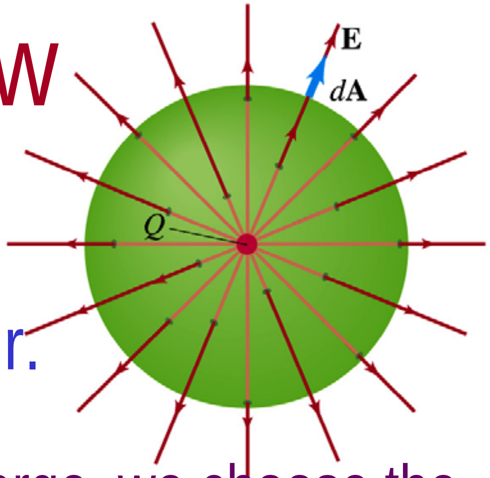
  - ε_0 is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
  - The integral is over the value of **E** on a closed surface of our choice in any given situation
  - The charge Q_{encl} is the net charge enclosed by the arbitrary close surface of our choice.
  - It does NOT matter where or how much charge is distributed inside the surface
  - The charge outside the surface does not contribute. Why?
    - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface

Gauss' Law



- Let's consider the case in the above figure.
- What are the results of the closed integral of the gaussian surfaces A_1 and A_2?
  - For A_1 $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For A_2 $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

Coulomb's Law from Gauss' Law



- Let's consider a charge Q enclosed inside our imaginary gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Must have the same magnitude at any point on the surface
    - Points radially outward / inward parallel to the surface vector dA.
- The Gaussian integral can be written as

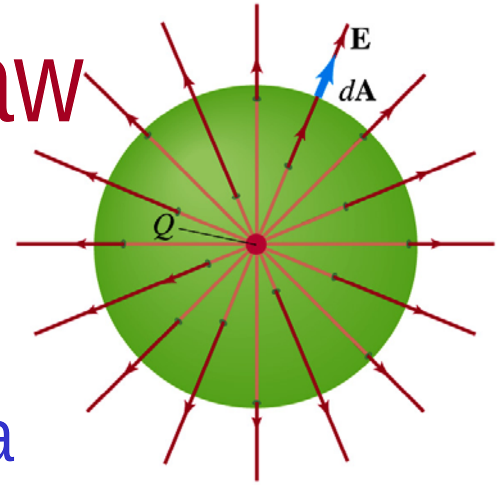
$$\oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Solve for E

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field of Coulomb's Law

Gauss' Law from Coulomb's Law



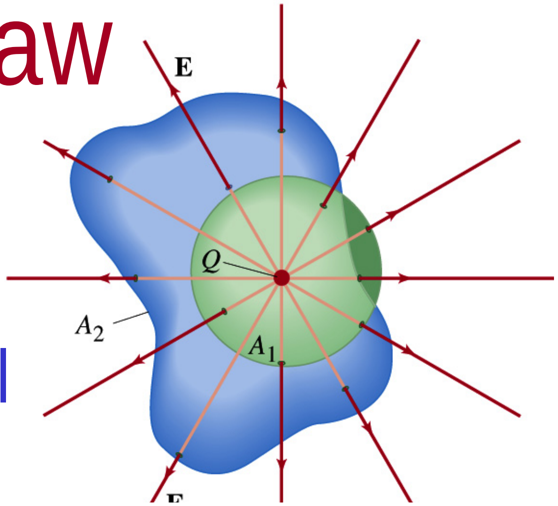
- Let's consider a single point static charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface is $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Gauss' Law

Gauss' Law from Coulomb's Law Irregular Surface



- Let's consider the same single point static charge Q surrounded by a symmetric spherical surface A_1 and a randomly shaped surface A_2.
- What is the difference in the number of field lines passing through the two surface due to the charge Q?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $$\oint_{A_1} \vec{E} \cdot d\vec{A} = \oint_{A_2} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.