• Gauss’ Law
• How are Gauss’ Law and Coulom’s Law Related?
• Electric Potential Energy
• Electric Potential
Announcements

- I have all but 10 of you, of which 5 on the distribution list, sent me confirmation.
  - The other five are not on the distribution list.

- Extra credit opportunities
  - Attend two Einstein lectures and get your flier signed by the lecture: 
    5 extra credit each
    - One today at noon on the 6th floor central library
    - The other at 2pm Thursday in NH 100.
  - 15 point extra credit for presenting a professionally prepared 3 page presentation on any one of the exhibits at the UC gallery (till 9/16) and the subsequent themed displays at the central library.
    - Must include what it does, how it works and where it is used. Possibly how it can be made to perform better.
    - Due: Oct. 19, 2005
Gauss’ Law from Coulomb’s Law

Irregular Surface

- Let’s consider a single point static charge $Q$ surrounded by a symmetric spherical surface $A_1$ and a randomly shaped surface $A_2$.

- What is the difference in the number of field lines passing through the two surface due to the charge $Q$?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is for the same enclosed charge.
  - So we can write: $\oint A_1 E \cdot dA = \oint A_2 E \cdot dA = \frac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the surface enclosing it is. ⇒ Gauss’ law, $\oint E \cdot dA = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge $Q$. 

\[\oint E \cdot dA = \int_A E \cdot dA = \frac{Q}{\varepsilon_0}\]
Gauss’ Law w/ more than one charge

- Let’s consider several charges inside a closed surface.
- For each charge, $Q_i$, enclosed by the chosen surface,
  \[ \oint E_i \cdot dA = \frac{Q_i}{\varepsilon_0} \]
  What is $E_i$?
  The electric field produced by $Q_i$ alone!
- Since electric fields can be added vectorially, following the superposition principle, the total field $\vec{E}$ is equal to the sum of the fields due to each separate charge $\vec{E} = \sum E_i$. So
  \[ \oint \vec{E} \cdot dA = \oint \left( \sum E_i \right) \cdot dA = \sum \frac{Q_i}{\varepsilon_0} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]
  What is $Q_{\text{encl}}$?
  The total enclosed charge!
- Gauss’ law follows from Coulomb’s law for any distribution of electric charge enclosed within a closed surface of any shape.
So what good is Gauss’ Law?

• Derivation of Gauss’ law from Coulomb’s law is only valid for static electric charge.

• Electric field can also be produced by changing magnetic fields.
  – Coulomb’s law cannot describe this field while Gauss’ law is still valid

• Gauss’ law is more general than Coulomb’s law.
  – Can be used to obtain electric field, forces or obtain charges

Gauss’ Law: Any difference between the input and output flux of the electric field over any enclosed surface is due to the charge within that surface!!!
Example 22 – 2

Flux from Gauss’ Law: Consider the two gaussian surfaces, $A_1$ and $A_2$, shown in the figure. The only charge present is the charge $Q$ at the center of surface $A_1$. What is the net flux through each surface $A_1$ and $A_2$?

• The surface $A_1$ encloses the charge $+Q$, so from Gauss’ law we obtain the total net flux:
  \[
  \oint \mathbf{E} \cdot d\mathbf{A} = \frac{+Q}{\varepsilon_0}
  \]

• The surface $A_2$ the charge, $+Q$, is outside the surface, so the total net flux is $0$.

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{0}{\varepsilon_0} = 0
\]
Example 22 – 5

Long uniform line of charge: A very long straight wire possesses a uniform positive charge per unit length, \( \lambda \). Calculate the electric field at points near but outside the wire, far from the ends.

- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of radial vector \( r \).

- Due to cylindrical symmetry, the field is the same on the gaussian surface of a cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector \( \vec{E} \) is perpendicular to the surface vector \( dA \).

- From Gauss' law
  \[
  \oint E \cdot d\vec{A} = E \oint dA = E (2\pi rl) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}
  \]

  Solving for \( E \)
  \[
  E = \frac{\lambda}{2\pi\varepsilon_0 r}
  \]
Electric Potential Energy

• Concept of energy is very useful solving mechanical problems
• Conservation of energy makes solving complex problems easier.
• When can the potential energy be defined?
  – Only for a conservative force.
  – The work done by a conservative force is independent of the path but only dependent on??
    • The difference between the initial and final positions
  – Can you give me an example of a conservative force?
    • Gravitational force
• Is the electrostatic force between two charges a conservative force?
  – Yes. Why?
  – The dependence of the force to the distance is identical to that of the gravitational force.