PHYS 1444 – Section 003
Lecture #8

Monday, Sept. 26, 2005
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- Capacitors
- Determination of Capacitance
- Capacitors in Series and Parallel
- Electric Energy Storage
- Dielectrics
- Molecular Description of Dielectrics

Today's homework is homework #5, due noon, next Monday!!

Electrostatic Potential Energy: electron Volt

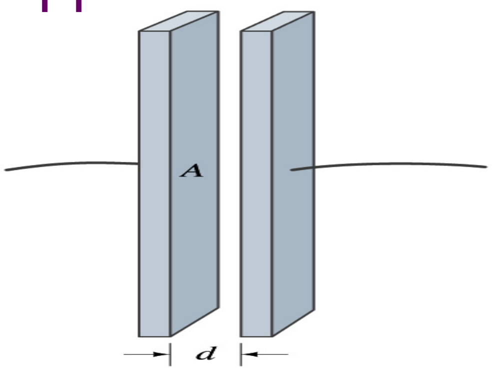
- What is the unit of electrostatic potential energy?
  - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot1V = 1.6\times10^{-19}J$
- eV however is <u>not a standard SI unit</u>. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

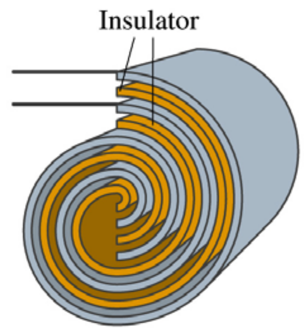
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, etc…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

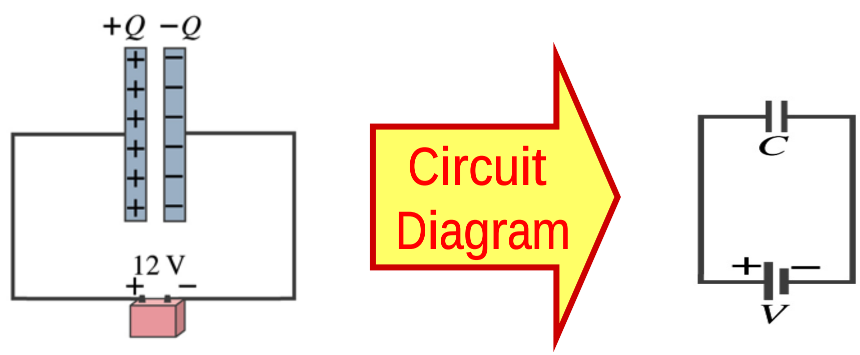
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



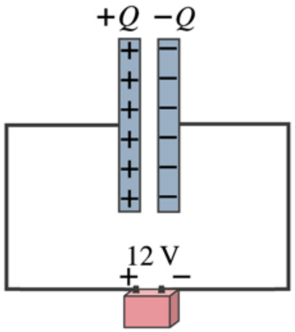


- How would you draw symbols for a capacitor and a battery?
  - Capacitor -||-
  - Battery (+) -|i- (-)



Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and other negative in equal amount.



- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

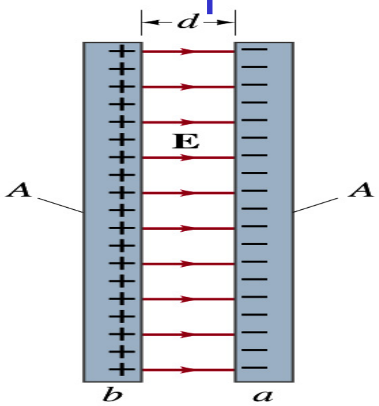
$$Q = CV_{ba}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use μF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.



- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, σ is the surface charge density.
- E and V are related $V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l}$
- Since we take the integral from lower potential (a) higher potential (b) along the field line, we obtain
- $V_{ba} = V_b - V_a = -\int_a^b Edl\cos 180^\circ = +\int_a^b Edl = \int_a^b \frac{\sigma}{\varepsilon_0} dl = \int_a^b \frac{Q}{\varepsilon_0 A} dl = \frac{Q}{\varepsilon_0 A}\int_a^b dl = \frac{Q}{\varepsilon_0 A}(b-a) = \frac{Qd}{\varepsilon_0 A}$
- So from the formula:
  - What do you notice?

$$C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C=\frac{\varepsilon_0 A}{d}=$$

$$=\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}m}=53\times10^{-12}\,C^2/N\cdot m=53pF$$

(b) From Q=CV, the charge on each plate is

$$Q=\ CV=\left(53\times10^{-12}\,C^2/N\cdot m\right)\left(12V\right)=6.4\times10^{-10}C=640pC$$

Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}\,C}{6.0\times10^{-3}\,m^2 \times 8.85\times10^{-12}\,C^2/N\cdot m^2} = 1.2\times10^{4}\,N/C = 1.2\times10^{4}\,V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}\,m} = 1.2\times10^{4}\,V/m$

(d) Solving the capacitance formula for A, we obtain

$$C = \frac{\varepsilon_0 A}{d}$$

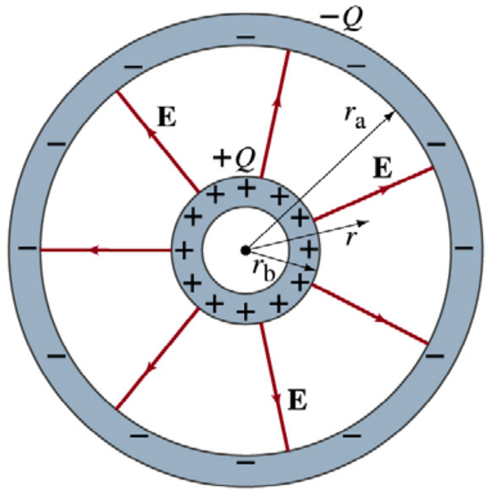
Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot 1\times10^{-3}\,m}{\left(9\times10^{-12}\,C^2/N\cdot m^2\right)} \approx 10^{8}\,m^2 \approx 100km^2$$

About 40% the area of Arlington ($256km^2$).

Example 24 – 3

Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell and equal but opposite charge –Q. Determine the capacitance of the two shells.



Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l} =$$

$$= -\int_a^b E \cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr = -\frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{r} \right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right) = \frac{Q}{4\pi\varepsilon_0} \left(\frac{r_a - r_b}{r_b r_a} \right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0} \left(\frac{r_a - r_b}{r_b r_a} \right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}$$

Capacitor Cont'd

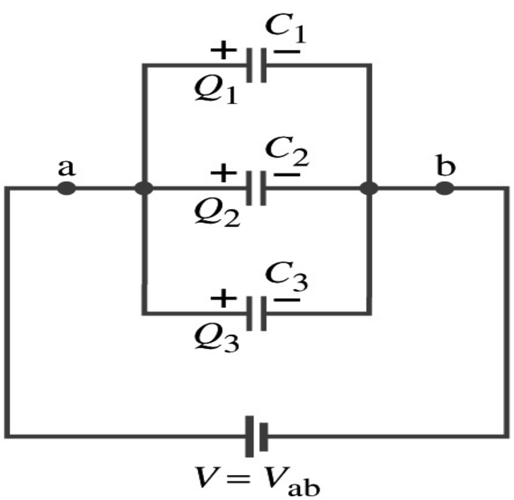
- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

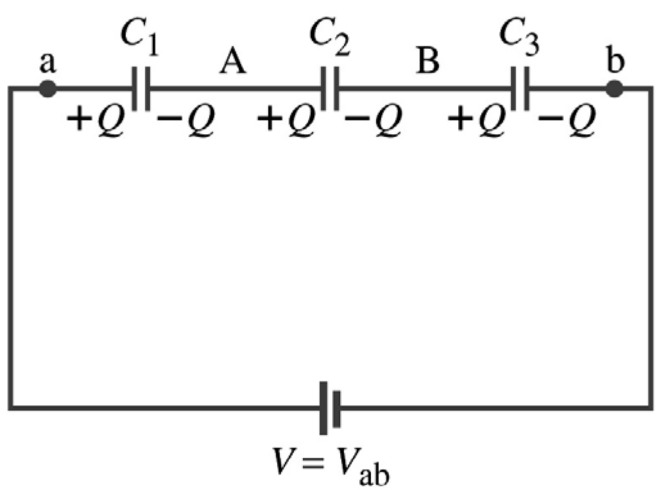
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \qquad \text{where} \quad r_a \to \infty$$

- So its capacitance is $C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b$

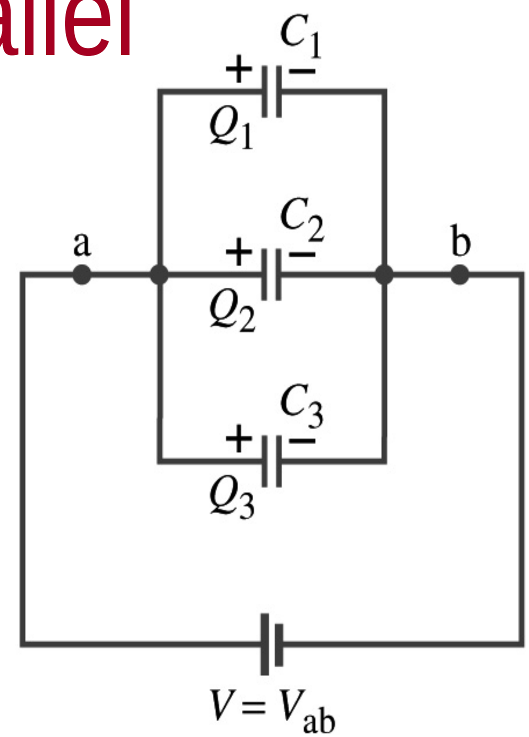
Capacitors in Series or Parallel

- Capacitors are used in may electric circuits
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel and in Series or in combination





Capacitors in Parallel

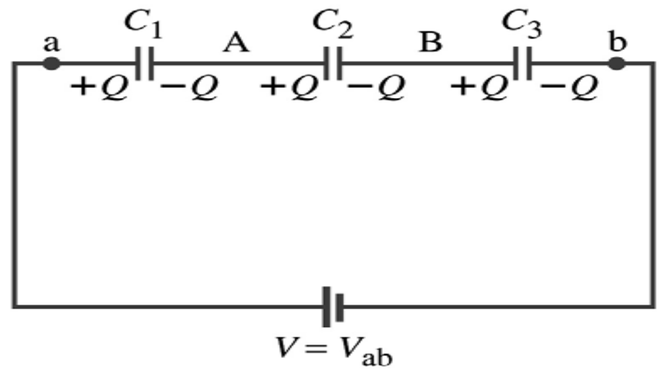


- Parallel arrangement provides the **same voltage** across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $C_{eq} = C_1 + C_2 + C_3$

What is the net effect?

The capacitance increases!!!

Capacitors in Series



- Series arrangement is more interesting
  - When battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3.
  - Since the in between were originally neutral, charges get induced to neutralize the ones in the middle.
  - So the charge on each capacitor is the same value, Q. (Same charge)
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=Q/C_1+Q/C_2+Q/C_3)$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is

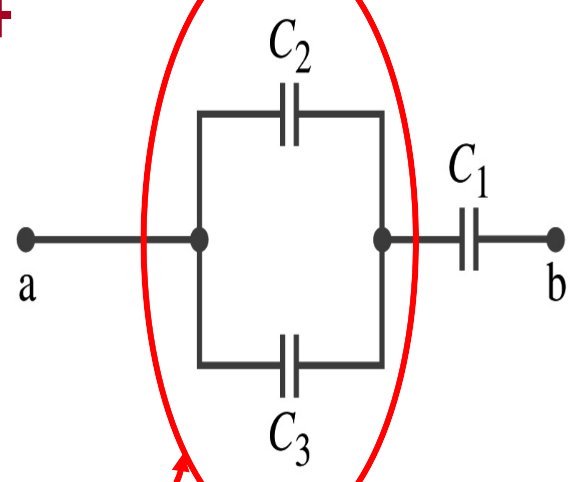
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 24 – 4

Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.



We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from a plate and put them on to the other.
  - Battery does this when it is connected to a capacitor.
- Capacitors does not charge immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external work needs to overcome.
  - When some charge is on each plate, it requires work to add more charge due to electric repulsion.

Electric Energy Storage

- The work needed to add a small amount of charge, dq, when a potential difference across the plate is V: dW=Vdq.
- Since V=q/C, the work needed to store total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries charges +Q and –Q is

$$U = \frac{Q^2}{2C}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24 – 7

Energy store in a capacitor: A camera flash unit stores energy in a 150μF capacitor at 200V. How much electric energy can be stored?

Using the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150\times10^{-6}F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$