PHYS 1444 – Section 003
Lecture #8

Monday, Sept. 26, 2005
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- Capacitors
- Determination of Capacitance
- Capacitors in Series and Parallel
- Electric Energy Storage
- Dielectrics
- Molecular Description of Dielectrics

Today’s homework is homework #5, due noon, next Monday!!
Electrostatic Potential Energy: electron Volt

• What is the unit of electrostatic potential energy?
  – Joules

• Joules is a very large unit in dealing with electrons, atoms or molecules atomic scale problems

• For convenience a new unit, electron volt (eV), is defined
  – 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  – How many Joules is 1 eV then? $1eV = 1.6 \times 10^{-19} C \cdot 1V = 1.6 \times 10^{-19} J$

• eV however is **not a standard SI unit**. You must convert the energy to Joules for computations.

• What is the speed of an electron with kinetic energy 5000eV?
Capacitors (or Condensers)

• What is a capacitor?
  – A device that can store electric charge
  – But does not let them flow through

• What does it consist of?
  – Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  – Why can’t they touch each other?
    • The charge will neutralize…

• Can you give some examples?
  – Camera flash, UPS, Surge protectors, binary circuits, etc…

• How is a capacitor different than a battery?
  – Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.
Capacitors

• A simple capacitor consists of a pair of parallel plates of area $A$ separated by a distance $d$.
  – A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.

• How would you draw symbols for a capacitor and a battery?
  – Capacitor -||-
  – Battery (+) -|i- (-)

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Capacitors

• What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  – The capacitor gets charged quickly, one plate positive and other negative in equal amount.

• Each battery terminal, the wires and the plates are conductors. What does this mean?
  – All conductors are at the same potential. And?
  – So the full battery voltage is applied across the capacitor plates.

• So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference $V_{ba}$ between the plates. How would you write this formula?

$$Q = CV_{ba}$$

  – C is a proportionality constant, called capacitance of the device.
  – C is a property of a capacitor so does not depend on Q or V.
  – What is the unit? $C/V$ or Farad (F) Normally use $\mu$F or pF.
Determination of Capacitance

- C can be determined analytically for capacitors with simple geometry and air in between.
- Let’s consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
    - E for parallel plates is \( E = \frac{\sigma}{\varepsilon_0} \), \( \sigma \) is the surface charge density.
- E and V are related \( V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l} \)
- Since we take the integral from lower potential (a) higher potential (b) along the field line, we obtain
  \( V_{ba} = V_b - V_a = \int_a^b Edl \cos 180^\circ = +\int_a^b Edl = \int_a^b \frac{\sigma}{\varepsilon_0} dl = \int_a^b \frac{Q}{\varepsilon_0 A} dl = \frac{Q}{\varepsilon_0 A} \int_a^b dl = \frac{Q}{\varepsilon_0 A} (b - a) = \frac{Qd}{\varepsilon_0 A} \)
- So from the formula:
  - What do you notice?

\[
C = \frac{Q}{V_{ba}} = \frac{Q}{Qd / \varepsilon_0 A} = \frac{\varepsilon_0 A}{d}
\]

C only depends on the area and the distance of the plates and the permittivity of the medium between them.
Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

\[ C = \frac{\varepsilon_0 A}{d} = \]

\[ = \left( 8.85 \times 10^{-12} \, \text{C}^2 / \text{N} \cdot \text{m}^2 \right) \frac{0.2 \times 0.03 \text{m}^2}{1 \times 10^{-3} \text{m}} = 53 \times 10^{-12} \, \text{C}^2 / \text{N} \cdot \text{m} = 53 \text{pF} \]

(b) From Q=CV, the charge on each plate is

\[ Q = CV = \left( 53 \times 10^{-12} \, \text{C}^2 / \text{N} \cdot \text{m} \right) (12 \text{V}) = 6.4 \times 10^{-10} \, \text{C} = 640 \text{pC} \]
Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

\[
E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4 \times 10^{-10} \text{ C}}{6.0 \times 10^{-3} \text{ m}^2 \times 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2} = 1.2 \times 10^4 \text{ N/C} = 1.2 \times 10^4 \text{ V/m}
\]

Or, since \( V = Ed \) we can obtain \( E = \frac{V}{d} = \frac{12V}{1.0 \times 10^{-3} \text{ m}} = 1.2 \times 10^4 \text{ V/m} \)

(d) Solving the capacitance formula for \( A \), we obtain

\[
C = \frac{\varepsilon_0 A}{d}
\]

\[
A = \frac{Cd}{\varepsilon_0} = \frac{1F \cdot 1 \times 10^{-3} \text{ m}}{9 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2} \approx 10^8 \text{ m}^2 \approx 100 \text{ km}^2
\]

About 40% the area of Arlington (256km²).
Example 24 – 3

**Spherical capacitor:** A spherical capacitor consists of two thin concentric spherical conducting shells, of radius \( r_a \) and \( r_b \), as in the figure. The inner shell carries a uniformly distributed charge \( Q \) on its surface and the outer shell and equal but opposite charge \(-Q\).

Determine the capacitance of the two shells.

Using Gauss’ law, the electric field outside a uniformly charged conducting sphere is

\[
E = \frac{Q}{4\pi\varepsilon_0 r^2}
\]

So the potential difference between \( a \) and \( b \) is

\[
V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b E \cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} \, dr = -\frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0} \left( \frac{1}{r} \right)_a^b = \frac{Q}{4\pi\varepsilon_0} \left( \frac{1}{r_a} - \frac{1}{r_b} \right) = \frac{Q}{4\pi\varepsilon_0} \left( \frac{r_a - r_b}{r_b r_a} \right)
\]

Thus capacitance is

\[
C = \frac{Q}{V} = \frac{Q}{4\pi\varepsilon_0} \left( \frac{r_a - r_b}{r_b r_a} \right) = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}
\]
Capacitor Cont’d

• A single isolated conductor can be said to have a capacitance, $C$.

• $C$ can still be defined as the ratio of the charge to absolute potential $V$ on the conductor.
  – So $Q = CV$.

• The potential of a single conducting sphere of radius $r_b$ can be obtained as
  
  $$ V = \frac{Q}{4\pi \varepsilon_0} \left( \frac{1}{r_b} - \frac{1}{r_a} \right) = \frac{Q}{4\pi \varepsilon_0 r_b} \quad \text{where} \quad r_a \to \infty $$

• So its capacitance is
  
  $$ C = \frac{Q}{V} = 4\pi \varepsilon_0 r_b $$
Capacitors in Series or Parallel

- Capacitors are used in many electric circuits.
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel and in Series or in combination

\[
V = V_{ab}
\]
Capacitors in Parallel

- Parallel arrangement provides the **same voltage** across all the capacitors.
  - Left hand plates are at $V_a$ and right hand plates are at $V_b$
  - So each capacitor plate acquires charges given by the formula
    - $Q_1 = C_1V$, $Q_2 = C_2V$, and $Q_3 = C_3V$

- The total charge $Q$ that must leave battery is then
  - $Q = Q_1 + Q_2 + Q_3 = V(C_1 + C_2 + C_3)$

- Consider that the three capacitors behave like an equivalent one
  - $Q = C_{eq}V = V(C_1 + C_2 + C_3)$

- Thus the equivalent capacitance in parallel is $C_{eq} = C_1 + C_2 + C_3$

What is the net effect? The capacitance increases!!!
Capacitors in Series

- **Series arrangement is more interesting**
  - When battery is connected, \( +Q \) flows to the left plate of \( C_1 \) and \( -Q \) flows to the right plate of \( C_3 \).
  - Since the in between were originally neutral, charges get induced to neutralize the ones in the middle.
  - So the charge on each capacitor is the same value, \( Q \). *(Same charge)*

- Consider that the three capacitors behave like an equivalent one
  - \( Q = C_{eq}V \)

- The total voltage \( V \) across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - \( V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \)

- Putting all these together, we obtain:
  - \( V = \frac{Q}{C_{eq}} = Q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \)

- Thus the equivalent capacitance is

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}
\]

What is the net effect?  The capacitance smaller than the smallest \( C !!! \)
**Example 24 – 4**

**Equivalent Capacitor:** Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1 = C_2 = C_3 = C$.

We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with $C_1$.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for $C_{eq}$

$$C_{eq} = \frac{2C}{3}$$
Electric Energy Storage

• A charged capacitor stores energy.
  – The stored energy is the work done to charge it.

• The net effect of charging a capacitor is removing one type of charge from a plate and put them on to the other.
  – Battery does this when it is connected to a capacitor.

• Capacitors does not charge immediately.
  – Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    • Since there is no charge, there is no field that the external work needs to overcome.
  – When some charge is on each plate, it requires work to add more charge due to electric repulsion.
Electric Energy Storage

• The work needed to add a small amount of charge, dq, when a potential difference across the plate is V: \( dW = Vdq \).

• Since \( V = \frac{q}{C} \), the work needed to store total charge \( Q \) is

\[
W = \int_{0}^{Q} Vdq = \frac{1}{C} \int_{0}^{Q} qdq = \frac{Q^2}{2C}
\]

• Thus, the energy stored in a capacitor when the capacitor carries charges +Q and –Q is

\[
U = \frac{Q^2}{2C}
\]

• Since \( Q = CV \), we can rewrite

\[
U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV
\]
Example 24 – 7

Energy store in a capacitor: A camera flash unit stores energy in a 150µF capacitor at 200V. How much electric energy can be stored?

Using the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: 

\[ U = \frac{1}{2} CV^2 \]

\[ U = \frac{1}{2} \times 150 \times 10^{-6} \times (200)^2 = 3.0 \, J \]

How do we get J from FV^2? 

\[ FV^2 = \left( \frac{C}{V} \right)V^2 = CV = C \left( \frac{J}{C} \right) = J \]