

PHYS 1444 – Section 003

Lecture #9

Wednesday, Sept. 28, 2005

Dr. Jaehoon Yu

- Quiz Results and Solution
- Electric Energy Density
- Dielectrics
- Molecular Description of Dielectrics
- The Electric Battery
- Electric Current



Announcements

- Reading Assignment
 - CH24 – 6
 - Early part of 25 – 1
- Quiz results
 - Do you want to know what your average is?
 - $37.5/70 \rightarrow$ equivalent to $54/100$
 - Do you want to know what it was last time?
 - $42.8/60 \rightarrow$ equivalent to $71/100$
 - Hmm.... What do you think???
 - Do you want to know the top score?
 - $68/70 \rightarrow 97/100$



Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, $V=Ed$ and $C=\epsilon_0 A/d$
- Thus the stored energy is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 \textcircled{Ad}$$

- Since Ad is the gap volume V , we can obtain the energy density, stored energy per unit volume, as

$$u = \frac{1}{2} \epsilon_0 E^2$$

**Valid for any space
that is vacuum**

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.

Dielectrics

- Capacitors have an insulating sheet of material, called dielectric, between the plates to
 - Increase breakdown voltage than that in the air
 - Higher voltage can be applied without the charge passing across the gap
 - Allow the plates get closer together without touching
 - Increases capacitance (recall $C = \epsilon_0 A/d$)
 - Also increases the capacitance by the dielectric constant
$$C = KC_0$$
 - Where C_0 is the intrinsic capacitance when the gap is vacuum



Dielectrics

- The value of dielectric constant varies depending on material (Table 24 – 1)
 - K for vacuum is 1.0000
 - K for air is 1.0006 (this is why permittivity of air and vacuum are used interchangeably.)
- Maximum electric field before breakdown occurs is the dielectric strength. What is its unit?
 - V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$C = KC_0 = K\epsilon_0 \frac{A}{d}$$



Dielectrics

- A new quantity of the permittivity of dielectric is defined as $\epsilon = K\epsilon_0$
- The capacitance of a parallel plate with a dielectric medium filling the gap is

$$C = \epsilon \frac{A}{d}$$

- The energy density stored in an electric field E in a dielectric is

$$u = \frac{1}{2} K \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

Effect of a Dielectric Material

- Let's consider the two cases below:

Case #1 :
constant V

$$\begin{array}{ccc}
 V_0 \text{ --- } \boxed{\text{no dielectric}} \text{ --- } & +Q_0 & -Q_0 \\
 & C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & V_0 \text{ --- } \boxed{\text{with dielectric}} \text{ --- } & +Q_0 = +KQ_0 \\
 & & -Q_0 = -KQ_0 \\
 & & C = \frac{Q}{V_0} = KC_0
 \end{array}$$

(a) Voltage constant

Case #2 :
constant Q

$$\begin{array}{ccc}
 V_0 \text{ --- } \boxed{\text{no dielectric}} \text{ --- } & +Q_0 & -Q_0 \\
 & C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & \begin{array}{c} +Q_0 \\ \text{---} \\ -Q_0 \end{array} V_0, C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & \text{battery disconnected} & \longrightarrow \\
 & \begin{array}{c} +Q_0 \\ \text{---} \\ -Q_0 \end{array} V = \frac{V_0}{K} & C = KC_0 \\
 & \text{dielectric inserted} &
 \end{array}$$

(b) Charge constant

- Constant voltage: Experimentally observed that the total charge on the each plates of the capacitor increases by K as the dielectric material is inserted between the gap $\rightarrow Q=KQ_0$
 - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K $\rightarrow V=V_0/K$
 - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Effect of a Dielectric Material on Field

- What happens to the electric field within a dielectric?
- Without a dielectric, the field is
 - What are V_0 and d ?
 - V_0 : Potential difference between the two plates
 - d : separation between the two plates
- For the constant voltage, the electric field remains the same
- For the constant charge: the voltage drops to $V=V_0/K$, thus the field in the dielectric is
 - The field in the dielectric is reduced.

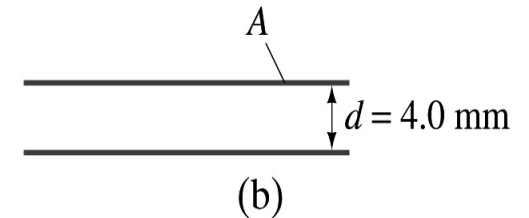
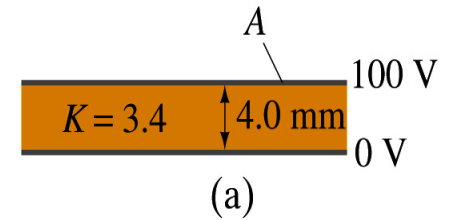
$$E_0 = \frac{V_0}{d}$$

$$E = E_D = \frac{V}{d} = \frac{V_0}{dK} = \frac{E_0}{K}$$

$$E_D = \frac{E_0}{K}$$

Example 24 – 8

Dielectric Removal: A parallel-plate capacitor, filled with a dielectric with $K=3.4$, is connected to a **100-V** battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A=4.0\text{m}^2$, and are separated by $d=4.0\text{mm}$. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



$$(a) \quad C = \frac{\epsilon A}{d} = \frac{K \epsilon_0 A}{d} = \left(3.4 \times 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \right) \frac{4.0 \text{ m}^2}{4.0 \times 10^{-3} \text{ m}} = 3.0 \times 10^{-8} \text{ F} = 30 \text{ nF}$$

$$Q = CV = (3.0 \times 10^{-8} \text{ F}) \times 100 \text{ V} = 3.0 \times 10^{-6} \text{ C} = 3.0 \mu\text{C}$$

$$E = \frac{V}{d} = \frac{100 \text{ V}}{4.0 \times 10^{-3} \text{ m}} = 2.5 \times 10^4 \text{ V/m}$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (3.0 \times 10^{-8} \text{ F}) (100 \text{ V})^2 = 1.5 \times 10^{-4} \text{ J}$$

Example 24 – 8 cont'd

- (b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0 = \frac{C}{K} = \left(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2\right) \frac{4.0 \text{ m}^2}{4.0 \times 10^{-3} \text{ m}} = 8.8 \times 10^{-9} \text{ F} = 8.8 \text{ nF}$$

Since charge is the same ($Q_0 = Q$) before and after the removal of the dielectric, we obtain

$$V_0 = Q/C_0 = K Q/C = KV = 3.4 \times 100 \text{ V} = 340 \text{ V}$$

$$E_0 = \frac{V_0}{d} = \frac{340 \text{ V}}{4.0 \times 10^{-3} \text{ m}} = 8.5 \times 10^4 \text{ V/m} = 84 \text{ kV/m}$$

$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \frac{C}{K} (KV)^2 = \frac{1}{2} K C V^2 = KU = 3.4 \times 1.5 \times 10^{-4} \text{ J} = 5.1 \times 10^{-4} \text{ J}$$

Where did the extra energy come from?

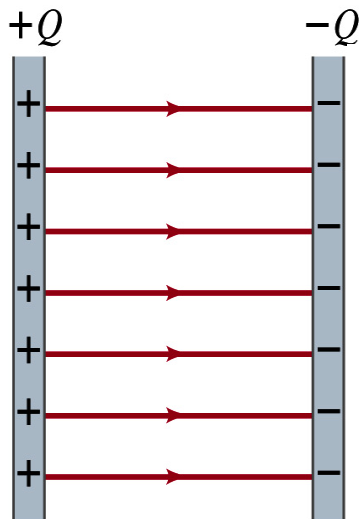
~~The energy conservation law is violated in electrostatics??~~

Wrong!
Wrong!
Wrong!

External force has done the work of $3.6 \times 10^{-4} \text{ J}$ on the system to remove dielectric!!

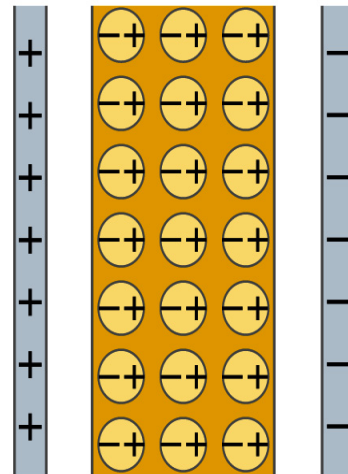
Molecular Description of Dielectric

- So what in the world makes dielectrics behave the way they do?
- We need to examine this in a microscopic scale.
- Let's consider a parallel plate capacitor that is charged up $+Q(=C_0V_0)$ and $-Q$ with air in between.
 - Assume there is no way any charge can flow in or out



(a)

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(b)

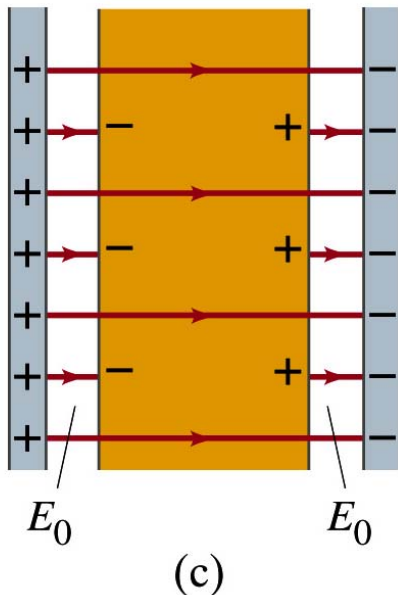


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- Now insert a dielectric
 - Dielectric can be polar → could have permanent dipole moment. What will happen?
- Due to electric field molecules may be aligned.

Molecular Description of Dielectric

- OK. Then what happens?
- Then effectively, there will be some negative charges close to the surface of the positive plate and positive charge on the negative plate
 - Some electric field do not pass through the whole dielectric but stops at the negative charge



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- So the field inside dielectric is smaller than the air
- Since electric field is smaller, the force is smaller
 - The work need to move a test charge inside the dielectric is smaller
 - Thus the potential difference across the dielectric is smaller than across the air

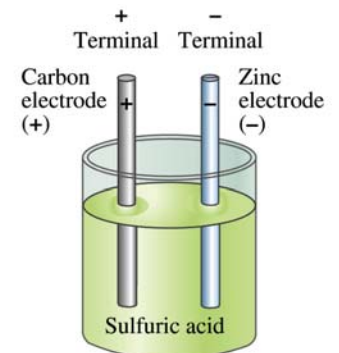
Electric Current and Resistance

- So far we have been studying static electricity
 - What the heck is the static electricity?
 - The charges so far has not been moving but staying put at the location they are placed.
- Now we will learn dynamics of electricity
- What is the electric current?
 - A flow of electric charge
 - A few examples of the things that use electric current in everyday lives?
- In an electrostatic situation, there is no electric field inside a conductor but when there is current, there is field inside a conductor
 - Electric field is needed to keep charges moving



The Electric Battery

- What is a battery?
 - A device that produces electrical energy from the stored chemical energy and produces electricity.
- Electric battery was invented by Volta in 1790s in Italy
 - It was made of disks of zinc and silver based on his research that certain combinations of materials produce a greater electromotive force (emf), or potential, than others
- Simplest batteries contain two plates made of dissimilar metals, electrodes
 - Electrodes are immersed in a solution, electrolyte
 - This unit is called a cell and many of these form a battery
- Zinc and Iron in the figure are terminals



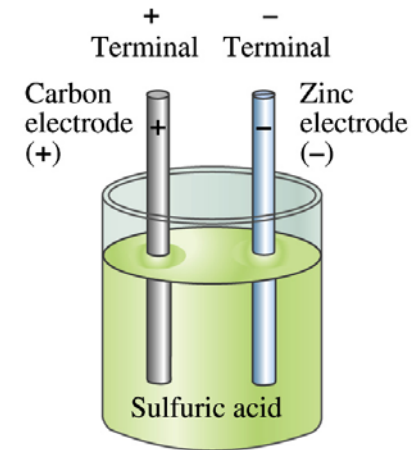
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How does a battery work?

- One of the electrodes in the figure is zinc and the other carbon
- The acid electrolyte reacts with the zinc electrode and dissolve it.
- Each zinc atom leaves two electrons in the electrode and enters into the solution as a positive ion → zinc electrode acquires negative charge and electrolyte becomes positively charged
- Thus the carbon electrode become positively charged
- Since the two terminals are oppositely charged, there is potential difference between them



How does a battery work?

- When the terminals are not connected, only small amount of zinc is dissolved into the solution.
- How is a particular potential maintained?
 - As large number of zinc ion gets produced, if the terminals are not connected
 - zinc electrode gets increasingly charged up negative
 - zinc ions get recombined with the electrons in zinc electrode
- Why does battery go dead?
 - When the terminals are connected, the negative charges will flow away from the zinc electrode
 - More zinc atoms dissolve into the electrolyte to produce more charge
 - One or more electrode get used up not producing any more charge.

