PHYS 1444 – Section 003
Lecture #12

Monday, Oct. 10, 2005
Dr. Jaehoon Yu

- EMF and Terminal Voltage
- Resisters in series and parallel
- Kirchhoff's Rules
- EMFs in series and parallel
- RC Circuits

Today's homework is homework #7, due noon, next Tuesday!!

Announcements

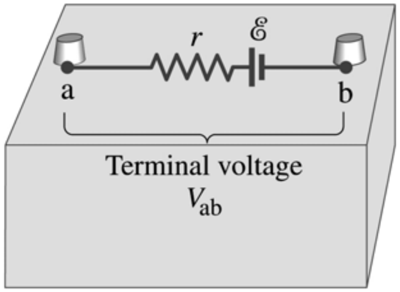
- First term exam Wednesday, Oct. 12
  - Time: 1 – 2:20 pm
  - Location: SH103
  - Coverage: CH. 21 – 25
- Reading Assignments
  - CH26 – 5
  - CH26 – 6


What did we do at
Saturday’s workshop?

EMF and Terminal Voltage

- What do we need to have current in an electric circuit?
  - A device that provides a potential difference, such as battery or generator
    - They normally convert some types of energy into electric energy
    - These devices are called source of electromotive force (emf)
      - This is does NOT refer to a real "force".
- Potential difference between terminals of emf source, when no current flows to an external circuit, is called the emf (ε) of the source.
- Battery itself has some **internal resistance** (r) due to the flow of charges in the electrolyte
  - Why does the headlight dim when you start the car?
    - The starter needs a large amount of current but the battery cannot provide charge fast enough to supply current to both the starter and the headlight

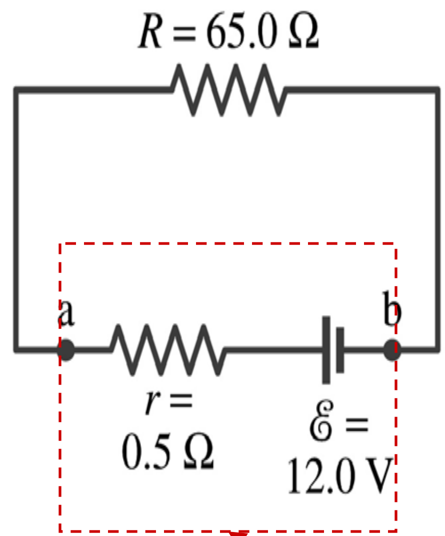
EMF and Terminal Voltage



- Since the internal resistance is inside the battery, we can never separate them out.
- So the terminal voltage difference $V_{ab}=V_a-V_b$.
- When no current is drawn from the battery, the terminal voltage equals the emf which is determined by the chemical reaction; $V_{ab}=\varepsilon$.
- However when the current I flows naturally from the battery, there is an internal drop in voltage which is equal to Ir. Thus the actual delivered terminal voltage is $\boxed{V_{ab}=\varepsilon-Ir}$

Example 26 – 1

Battery with internal resistance. A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0V and whose internal resistance is 0.5-Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistor.



What is this?

A battery or a source of emf.

(a) Since $V_{ab} = \varepsilon - Ir$ We obtain $V_{ab} = IR = \varepsilon - Ir$

Solve for I

$$I = \frac{\varepsilon}{R + r} = \frac{12.0V}{65.0\Omega + 0.5\Omega} = 0.183A$$

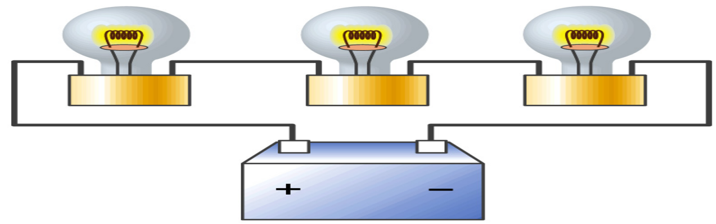
(b) The terminal voltage V_{ab} is $V_{ab} = \varepsilon - Ir = 12.0V - 0.183A \cdot 0.5\Omega = 11.9V$

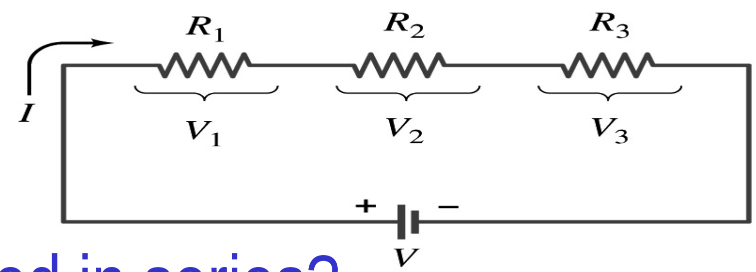
(c) The power dissipated in R and r are

$$P = I^2 R = (0.183A)^2 \cdot 65.0\Omega = 2.18W$$

$$P = I^2 r = (0.183A)^2 \cdot 0.5\Omega = 0.02W$$

Resisters in Series





- Resisters are in series when two or more resisters are connected end to end
  - These resisters represent simple resisters in circuit or electrical devices, such as light bulbs, heaters, dryers, etc
- What is common in a circuit connected in series?
  - Current is the same through all the elements in series
- Potential difference across every element in the circuit is
  - $V_1=IR_1$, $V_2=IR_2$ and $V_3=IR_3$
- Since the total potential difference is V, we obtain
  - $V=IR_{eq}=V_1+V_2+V_3=I(R_1+R_2+R_3)$
  - Thus, $R_{eq}=R_1+R_2+R_3$

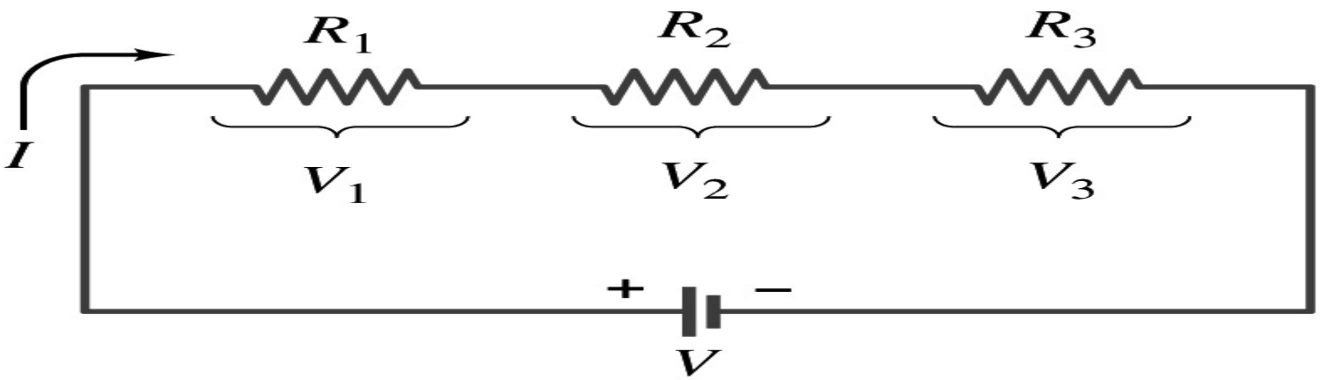
$$\boxed{R_{eq} = \sum_i R_i}$$

Resisters in series

When resisters are connected in series, the total resistance increases but the current decreases.

Energy Losses in Resisters

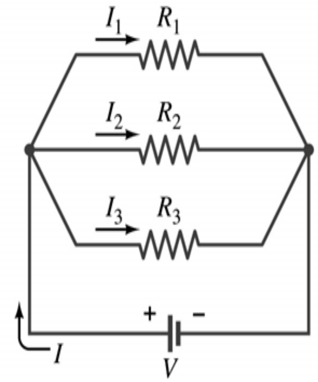
- Why is it true that $V=V_1+V_2+V_3$?

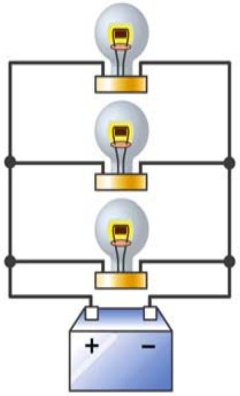


- What is the potential energy loss when charge q passes through the resister R_1, R_2 and R_3
  - $\Delta U_1=qV_1$, $\Delta U_2=qV_2$, $\Delta U_3=qV_3$
- Since the total energy loss should be the same as the energy provided to the system, we obtain
  - $\Delta U=qV=\Delta U_1+\Delta U_2+\Delta U_3=q(V_1+V_2+V_3)$
  - Thus, $V=V_1+V_2+V_3$

Resisters in Parallel

- Resisters are in parallel when two or more resisters are connected in separate branches
  - Most the house and building wirings are arranged this way.





- What is common in a circuit connected in parallel?
  - The voltage is the same across all the resisters.
  - The total current that leaves the battery, is however, split.
- The current that passes through every element is
  - $I_1=V/R_1$, $I_2=V/R_2$, $I_3=V/R_3$
- Since the total current is I, we obtain
  - $I=V/R_{eq}=I_1+I_2+I_3=V(1/R_1+1/R_2+1/R_3)$
  - Thus, $1/R_{eq}=1/R_1+1/R_2+1/R_3$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Resisters in parallel

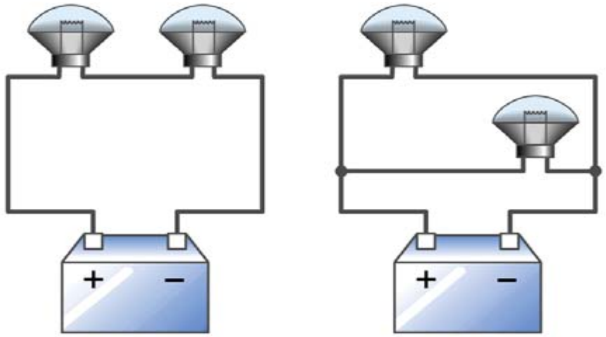
When resisters are connected in parallel, the total resistance decreases but the current increases.

Resister and Capacitor Arrangements

- Parallel Capacitor arrangements $$C_{eq} = \sum_i C_i$$
- Parallel Resister arrangements $$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$
- Series Capacitor arrangements $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$
- Series Resister arrangements $$R_{eq} = \sum_i R_i$$

Example 26 – 2

Series or parallel? (a) The light bulbs in the figure are identical and have identical resistance R. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired?



(1) Series (2) Parallel

(a) What are the equivalent resistances for the two cases?

Series ⇒ $R_{eq} = 2R$ Parallel ⇒ $\frac{1}{R_{eq}} = \frac{2}{R}$ So ⇒ $R_{eq} = \frac{R}{2}$

The bulbs get brighter when the total power transformed is larger.

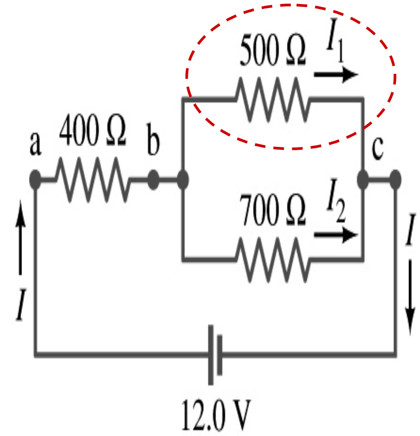
series $P_S = IV = \frac{V^2}{R_{eq}} = \frac{V^2}{2R}$ parallel $P_P = IV = \frac{V^2}{R_{eq}} = \frac{2V^2}{R} = 4P_S$

So parallel circuit provides brighter lighting.

(b) Car's headlights are in parallel to provide brighter lighting and also to prevent both lights going out at the same time when one burns out.

So what is bad about parallel circuits? Uses more energy in a given time.

Example 26 – 5



Current in one branch. What is the current flowing through the 500-Ω resister in the figure?

What do we need to find first? We need to find the total current.

To do that we need to compute the equivalent resistance.

R_{eq} of the small parallel branch is: $$\frac{1}{R_P}=\frac{1}{500}+\frac{1}{700}=\frac{12}{3500} \qquad R_P=\frac{3500}{12}$$

R_{eq} of the circuit is: $$R_{eq}=400+\frac{3500}{12}=400+292=692\Omega$$

Thus the total current in the circuit is $$I=\frac{V}{R_{eq}}=\frac{12}{692}=17mA$$

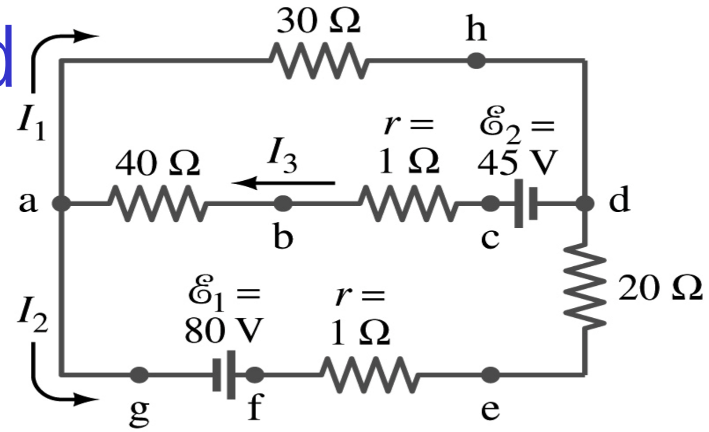
The voltage drop across the parallel branch is $$V_{bc}=IR_P=17\times10^{-3}\cdot292=4.96V$$

The current flowing across 500-Ω resister is therefore

$$V_{bc}I_{500}=\frac{V_{bc}}{R}=\frac{4.96}{500}=9.92\times10^{-3}=9.92mA$$

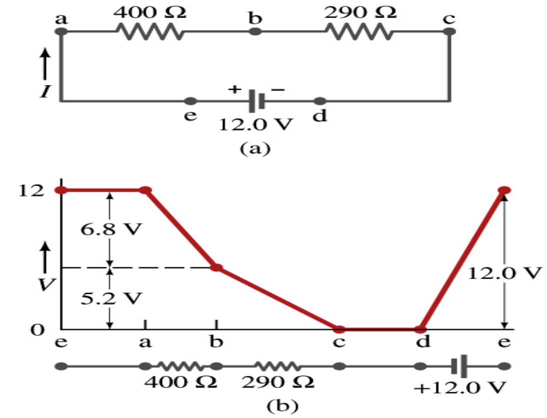
What is the current flowing 700-Ω resister? $$I_{700}=I-I_{500}=17-9.92=7.08mA$$

Kirchhoff's Rules – 1st Rule



- Some circuits are very complicated to do the analysis using the simple combinations of resisters
  - G. R. Kirchhoff devised two rules to deal with complicated circuits.
- Kirchhoff's rules are based on conservation of charge and energy
  - Kirchhoff's 1st rule: Junction rule, charge conservation.
    - At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.
    - In other words, what goes in must come out.
    - At junction *a* in the figure, I_3 comes into the junction while I_1 and I_2 leaves: $I_3 = I_1 + I_2$

Kirchhoff's Rules – 2nd Rule

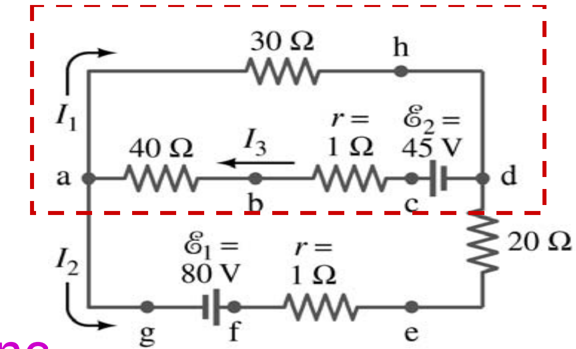


- Kirchoff's 2nd rule: Loop rule, uses conservation of energy.
  - The sum of the changes in potential around any closed path of a circuit must be zero.
- The current in the circuit in the figure is I=12/690=0.017A.
  - Point *e* is the high potential point while point d is the lowest potential.
  - When the test charge starts at *e* and returns to *e*, the total potential change is 0.
  - Between point *e* and *a*, no potential change since there is no source of potential nor any resistance.
  - Between *a* and *b*, there is a 400Ω resistance, causing IR=0.017*400 =6.8V drop.
  - Between *b* and *c*, there is a 290Ω resistance, causing IR=0.017*290 =5.2V drop.
  - Since these are voltage drops, we use negative sign for these, -6.8V and -5.2V.
  - No change between *c* and *d* while from d to e there is +12V change.
  - Thus the total change of the voltage through the loop is: -6.8V-5.2V+12V=0V.

Using Kirchhoff's Rules

1. Determine the flow of currents at the junctions.
   - It does not matter which direction you decide.
   - If the value of the current after completing the calculations comes are negative, you just flip the direction of the current flow.
2. Write down the current equation based on Kirchhoff's 1st rule at various junctions.
   - Be sure to see if any of them are the same.
3. Choose closed loops in the circuit
4. Write down the potential in each interval of the junctions, keeping the sign properly.
5. Write down the potential equations for each loop.
6. Solve the equations for unknowns.

Example 26 – 8



Use Kirchhoff's rules. Calculate the currents I_1, I_2 and I_3 in each of the branches of the circuit in the figure.

The directions of the current through the circuit is not known *a priori* but since the current tends to move away from the positive terminal of a battery, we arbitrary choose the direction of the currents as shown.

We have three unknowns so we need three equations.

Using Kirchhoff's junction rule at point a, we obtain $I_3 = I_1 + I_2$

This is the same for junction d as well, so no additional information.

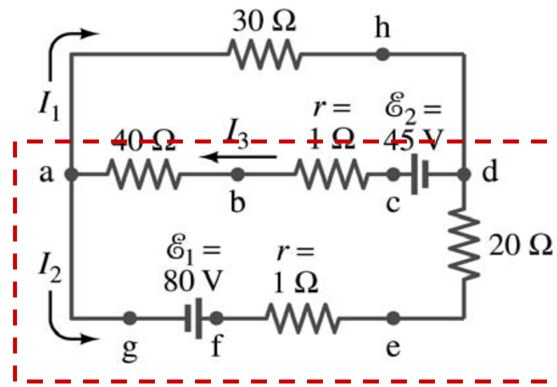
Now the second rule on the loop *ahdcba*.

$$V_{ah} = -I_1 30 \qquad V_{hd} = 0 \qquad V_{cd} = +45 \qquad V_{cb} = -I_3 \qquad V_{ba} = -40 I_3$$

The total voltage change in loop *ahdcba* is.

$$V_{ahdcba} = -30 I_1 + 45 - I_3 - 40 I_3 = 45 - 30 I_1 - 41 I_3 = 0$$

Example 26 – 8, cnt'd



Now the second rule on the other loop *agfedcba*.

$$V_{ag} = 0 \quad V_{gf} = +80 \quad V_{fe} = -I_2 \quad V_{ed} = -I_2 20$$

$$V_{cd} = +45 \quad V_{cb} = -I_3 \quad V_{ba} = -40 I_3$$

The total voltage change in loop *agfedcba* is. $V_{agfedcba} = -21I_2 + 125 - 41I_3 = 0$

So the three equations become

$$I_3 = I_1 + I_2$$

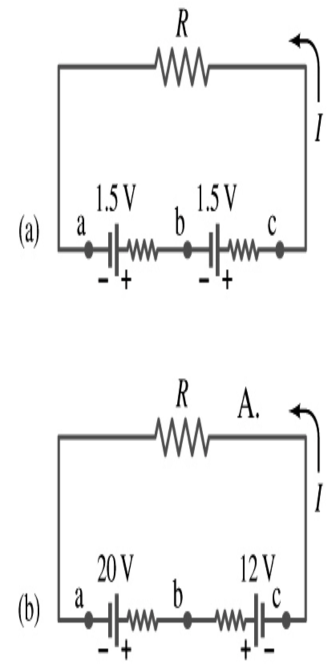
$$45 - 30I_1 - 41I_3 = 0$$

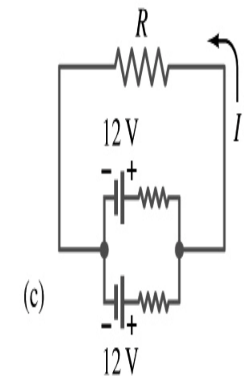
$$125 - 21I_2 - 41I_3 = 0$$

We can obtain the three current by solving these equations for I_1, I_2 and I_3.

EMFs in Series and Parallel: Charging a Battery

- When two or more sources of emfs, such as batteries, are connected in series
  - The total voltage is the algebraic sum of their voltages, if their direction is the same
    - V_{ab}=1.5 + 1.5=3.0V in figure (a).
  - If the batteries are arranged in an opposite direction, the total voltage is the difference between them
    - V_{ac}=20 – 12=8.0V in figure (b)
    - Connecting batteries in opposite direction is wasteful.
    - This, however, is the way a battery charger works.
    - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
    - Some battery are rechargeable since their chemical reactions are reversible but most the batteries do not reverse their chemical reactions

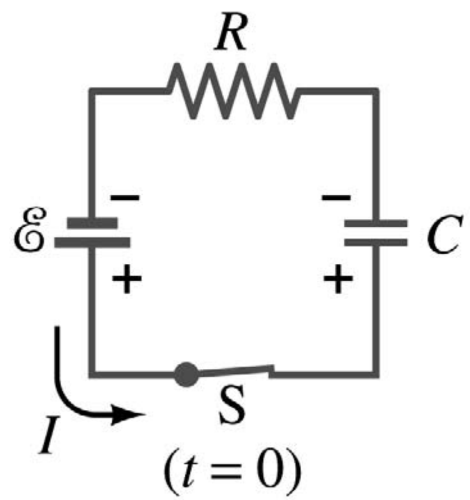




Parallel arrangements (c) are used only to increase currents.

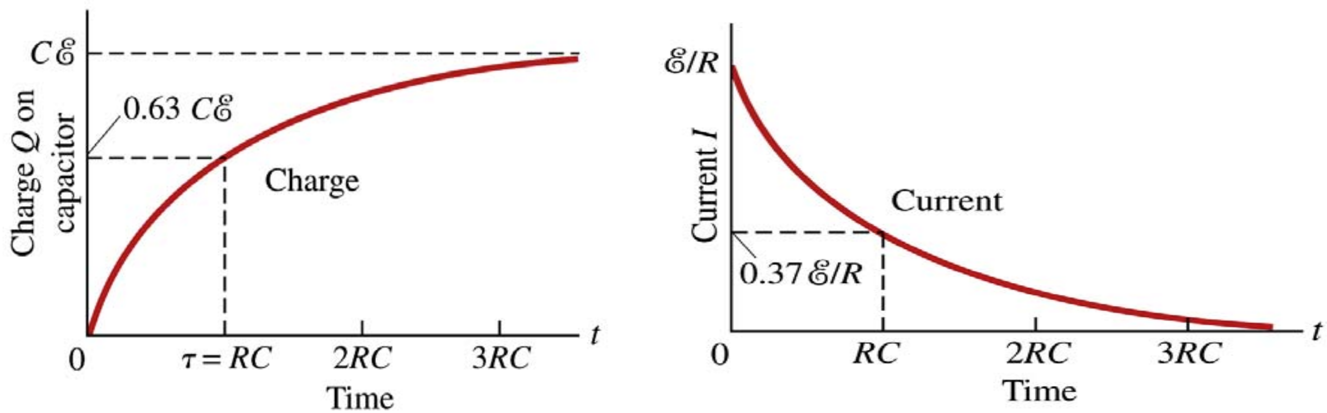
RC Circuits

- Circuits containing both resisters and capacitors
  - RC circuits are used commonly in everyday life
    - Control windshield wiper
    - Timing of traffic light from red to green
    - Camera flashes and heart pacemakers
- How does an RC circuit look?
  - There should be a source of emf, capacitors and resisters
- What happens when the switch S is closed?
  - Current immediately starts flowing through the circuit.
  - Electrons flows out of negative terminal of the emf source, through the resister R and accumulates on the upper plate of the capacitor
  - The electrons from the bottom plate of the capacitor will flow into the positive terminal of the battery, leaving only positive charge on the bottom plate
  - As the charge accumulates on the capacitor, the potential difference across it increases
  - The current reduces gradually to 0 till the voltage across the capacitor is the same as emf.
  - The charge on the capacitor increases till it reaches to its maximum $C\varepsilon$.



RC Circuits

- How does all this look like in graphs?
  - Charge and the current on the capacitor as a function of time



  - From energy conservation (Kirchhoff's 2nd rule), the emf ε must be equal to the voltage drop across the capacitor and the resister
    - $\varepsilon = IR + Q/C$
    - R includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instant, and Q is the charge of the capacitor at that same instance.