PHYS 1444 – Section 003
Lecture #13

Monday, Oct. 17, 2005
Dr. Jaehoon Yu

- RC Circuits
- RC time
- Exam Problem Solutions

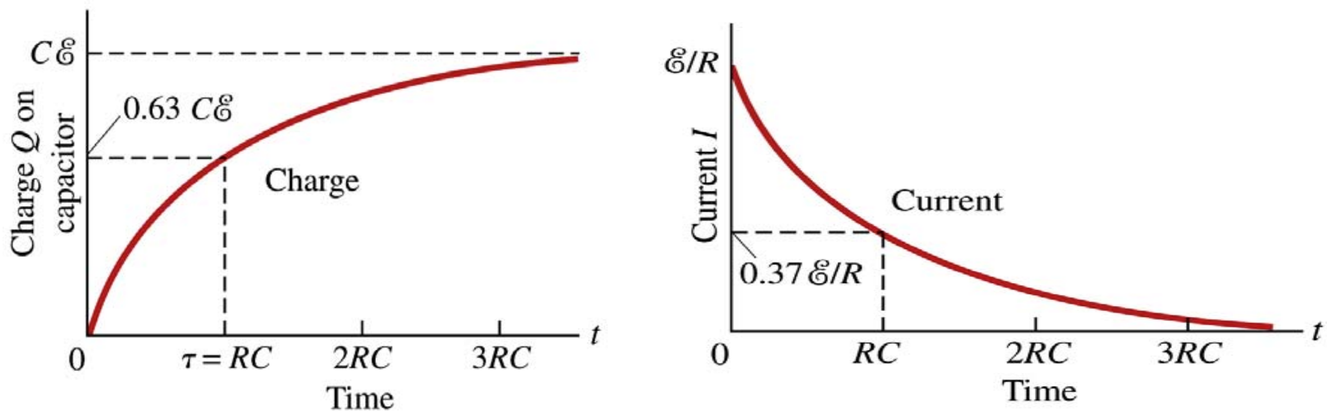
Today's homework is homework #7, due noon, next Tuesday!!

Announcements

- How was the exam?
  - Do you want to know how you did?
  - I am sure you do… Right?
  - OK enough teasing around…
  - Here goes the average
    - 52.5
  - The top score: 85
- Reading Assignments
  - CH26 – 5
  - CH26 – 6

RC Circuits

- How does all this look like in graphs?
  - Charge and the current on the capacitor as a function of time



  - From energy conservation (Kirchhoff's 2nd rule), the emf ε must be equal to the voltage drop across the capacitor and the resister
    - $\varepsilon = IR + Q/C$
    - R includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instant, and Q is the charge of the capacitor at that same instance.

Analysis of RC Circuits

- From the energy conservation, we obtain $\varepsilon = IR + Q/C$
- Which ones are constant in the above equation?
  - ε, R and C are constant
  - Q and I are functions of time
- How do we write the rate at which the charge accumulated on the capacitor?
  - We can rewrite the above equation as $\varepsilon = R\frac{dQ}{dt} + \frac{1}{C}Q$
  - This equation can be solved by rearranging the terms as $\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}$

Analysis of RC Circuits

- Now integrating from t=0 when there was no charge on the capacitor to t when the capacitor is fully charged, we obtain
- $\int_0^Q \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC}\int_0^t dt$ $\rightarrow$
- $-\ln\left(C\varepsilon - Q\right)\Big|_0^Q = -\ln\left(C\varepsilon - Q\right) - \left(-\ln C\varepsilon\right) = \frac{t}{RC}\Big|_0^t = \frac{t}{RC}$
- So, we obtain $\ln\left(1 - \frac{Q}{C\varepsilon}\right) = -\frac{t}{RC}$ $\rightarrow$ $1 - \frac{Q}{C\varepsilon} = e^{-t/RC}$
- Or $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$
- The potential difference across the capacitor is V=Q/C, so $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$

Analysis of RC Circuits

- Since $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$ and $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$
- What can we see from the above equations?
  - Q and V_C increase from 0 at t=0 to maximum value Q_{max}=C ε and V_C= ε.
- In how much time?
  - The quantity RC is called the time constant, τ, of the circuit
    - τ=RC, What is the unit? Sec.
  - What is the physical meaning?
    - The time required for the capacitor to reach $(1-e^{-1})=0.63$ or 63% of the full charge
- The current is $I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$